Passivity Based Control method for the diffusion process

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Outline

1. Motivation
2. Considered class of systems
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finite dimensional IPHS: Ramirez et al, Chemical Engineering Science (2013) and Ramirez et al, Automatica (2016)

$$\begin{aligned} \dot{x} &= R(x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x}) J \frac{\partial U}{\partial x} + g(x, \frac{\partial U}{\partial x}) u(t), \\ y &= g^T(x, \frac{\partial U}{\partial x}) \frac{\partial U}{\partial x}(x) \end{aligned} \tag{1}$$

$x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input, $g(x, \frac{\partial U}{\partial x}) \in \mathbb{R}^{n\times m}$, $U(x) \in \mathbb{R}$, $S(x) \in \mathbb{R}$ represent respectively the internal energy (the hamiltonian) and the entropy functions, $J \in \mathbb{R}^n \times \mathbb{R}^n$ is a structure matrix of the Poisson bracket $\{.,.\}_J$, where $\{S, U\}_J = \frac{\partial S^T}{\partial x}(x) J \frac{\partial U}{\partial x}(x)$.
$R = R(x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x}) = \gamma(x, \frac{\partial U}{\partial x})\{S, U\}_J$ with $\gamma(x, \frac{\partial U}{\partial x}) : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$, $\gamma \geq 0$, a non linear positive function.

Motivation

Ramirez et al TFMST'13 (2013) and Ramirez et al Automatica (2016)

A passivity based control (PBC) method has been presented for finite dimensional IPHS.

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A passivity based control (PBC) method has been presented for finite dimensional IPHS.

The main question now is:

How do this approach can be extended to the distributed diffusion process ?.

Considered class of systems

Ramirez and Le Gorrec(2016)

$$\begin{aligned} \frac{\partial n_i}{\partial t} &= -\frac{\partial}{\partial z}(f_{n_i}), \quad i = 1, \ldots, m \\ \frac{\partial s}{\partial t} &= -\frac{\partial}{\partial z} f_s + \sigma_s + \Sigma_{i=1}^{m} \sigma_{n_i}, \quad t > 0, z \in (c, d) \end{aligned} \tag{2}$$

where n_i is the mole number of the spicy i (and n the vector $n = (n_1, \ldots, n_m)^T$) and s represents the entropy. $f_n = -\frac{L}{T}\frac{\partial \mu}{\partial z} = [f_{n_1}, ..., f_{n_m}]^T$ is the number of mole flux and $f_s = -\frac{\lambda}{T}\frac{\partial T}{\partial z}$ is the entropy flux. $\sigma_s = -\frac{1}{T} f_s \frac{\partial T}{\partial z} = \frac{\lambda}{T^2}(\frac{\partial T}{\partial z})^2$, $\sigma_n = -\frac{1}{T} f_n \frac{\partial \mu}{\partial z} = \frac{L}{T^2}(\frac{\partial \mu}{\partial z})^2$, where λ is the heat conduction, L is the diffusion coefficient, T is the temperature and μ is the chemical potential.

Considered class of systems

Hypothesis 1

λ and L are only functions of the states (not functions of $\frac{\partial U}{\partial x}$)

IPHS representation of the diffusion process have been suggested in (Ramirez and Le Gorrec TFMST'16 (2016))

$$\frac{dx}{dt} = (R_s J + \Sigma_{i=1}^{m} R_{n_i} J_{n_i})\frac{\partial U}{\partial x} + \frac{\partial}{\partial z}(R\frac{\partial U}{\partial s}), \tag{3}$$

$$v = \begin{bmatrix} \frac{\partial u}{\partial x}(t,c) \\ \frac{\partial u}{\partial x}(t,d) \end{bmatrix}, \qquad y = \begin{bmatrix} F_n(t,c) \\ F_n(t,d) \end{bmatrix}, \tag{4}$$

where $x = [n, s]^T$ is the state vector, v and y are conjugated input output of the system evaluated at the boundaries,
$F_n = [f_{n_1}, \ldots, f_{n_m}, f_s]$,

Considered class of systems

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$$\text{and } J = \begin{bmatrix} -\frac{L_1}{\lambda} & 0 & & & 0 \\ 0 & -\frac{L_2}{\lambda} & & & \\ & & \ddots & & 0 \\ & & & -\frac{L_m}{\lambda} & 0 \\ 0 & & & 0 & 1 \end{bmatrix} \frac{\partial}{\partial z},$$

$$J_{n_1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & & 0 \\ & & \\ 1 & & 0 \end{bmatrix} \frac{\partial}{\partial z},\ J_{n_3}, \cdots, J_{n_{m-1}},\ J_{n_m} = \begin{bmatrix} 0 & & 0 \\ 0 & & 0 \\ & & 1 \\ 0 & 1 & 0 \end{bmatrix} \frac{\partial}{\partial z}$$

$R_s = \gamma_s\{S, U\}_{J_s}$, $\gamma_s = \frac{\lambda}{T^2}$, $R_{n_i} = \gamma_{n_i}\{S, U\}_{J_{n_i}}$, $R = [R_{n_1} \dots R_{n_m} R_s]$, $\gamma_{n_i} = \frac{L_i}{T^2}$.

IPHS representation of the distributed heat equation suggested in (Ramirez and Le Gorrec TFMST'16 (2016))

$$\frac{\partial s}{\partial t} = R_s J_s \frac{\partial u}{\partial s} + \frac{\partial}{\partial z}(R_s \frac{\partial u}{\partial s}) \tag{5}$$

$$v = \begin{bmatrix} (R_s \frac{\partial u}{\partial s})(t,c) \\ (R_s \frac{\partial u}{\partial s})(t,d) \end{bmatrix}, \qquad y = \begin{bmatrix} \frac{\partial u}{\partial s}(t,c) \\ \frac{\partial u}{\partial s}(t,d) \end{bmatrix}, \tag{6}$$

with $R_s = \gamma_s(s, \frac{\partial u}{\partial s}, z)\{S, U\}_{J_s}$, $\gamma_s = \frac{1}{T^2}\lambda(s, \frac{\partial u}{\partial s}, z)$, $J_s = \frac{\partial}{\partial z}$, $\{S, U\}_{J_s} = \frac{\partial T}{\partial z}$ *and* v *is the input and* y *is the output.*

These systems are constructed such that :

$$\frac{dU}{dt} = y^T v, \tag{7}$$

$$\frac{dS}{dt} = \int_c^d \sigma dz + \frac{\lambda(t,d)}{T(t,d)} \frac{\partial T}{\partial z}(t,d) - \frac{\lambda(t,c)}{T(t,c)} \frac{\partial T}{\partial z}(t,c) \tag{8}$$

Add a distributed finite rank control to system (3)

$$\frac{dx}{dt} = (R_s J + \Sigma_{i=1}^{m} R_{n_i} J_{n_i}) \frac{\partial U}{\partial x} + \\ + \frac{\partial}{\partial z}(R \frac{\partial U}{\partial s}) + G(x) v_2(t), \quad (9)$$

$$v_1(t) = \begin{bmatrix} \frac{\partial u}{\partial x}(t, c) \\ \frac{\partial u}{\partial x}(t, d) \end{bmatrix}, \qquad y_1(t) = \begin{bmatrix} F_n(t, c) \\ F_n(t, d) \end{bmatrix}, \quad (10)$$

Where v_2 is a time dependent distributed finite rank control signal, and $G(x)$ represents the spatial distribution of v_2.

Check a Lyapunov function candidate

Kondepudi, Prigogine (1998), Alonso, Ydstie Automatica (2001)

An availability energy for the distributed diffusion process (2) is given by

$$A(t) = \int_c^d a(x, x^\star)dz \tag{11}$$

where

$$a(x, x^\star) = U(x) - [U(x^\star) + \frac{\partial U^T}{\partial x}(x^\star)(x - x^\star)], \tag{12}$$

with x is the current state and $x^\star$ is the reference state.

Hypothesis 2

the availability energy A is a strictly positive function with minimum at $x^\star$.

To seek conditions under which system (9) is stable

To this end, we consider

$$\frac{dA}{dt} = \int_c^d \frac{\partial a^T(x, x^\star)}{\partial x} \frac{\partial x}{\partial t} dz, \quad (13)$$

$$= \int_c^d (M + N + L) dz, \quad (14)$$

where

$$M = R_s\{a, U\}_{J_s}. \quad (15)$$

and

$$N = R_n\{a, U\}_{J_n}. \quad (16)$$

For L we have

$$\int_c^d L dz = \frac{\partial a^T}{\partial x}(R\frac{\partial U}{\partial s})\Big|_c^d + \int_c^d \frac{\partial a^T}{\partial x} G v_2(t) dz, \quad (17)$$

$$= \tilde{y_1}^T \tilde{v_1} + \tilde{y_2}^T v_2. \quad (18)$$

where $\tilde{v_1} = \begin{bmatrix} (R\frac{\partial U}{\partial s})(t,d) \\ (R\frac{\partial U}{\partial s})(t,c) \end{bmatrix}$, $\tilde{y_1} = \begin{bmatrix} \frac{\partial a}{\partial x}(t,d) \\ -\frac{\partial a}{\partial x}(t,c) \end{bmatrix}$, and

$$\tilde{y_2} = \int_c^d G^T(x)\frac{\partial a}{\partial x}(x,x^\star)dz.$$

Finally,

$$\frac{dA}{dt} = \int_c^d [R_s\{a,U\}_{J_s} + R_n\{a,U\}_{J_n}]dz + \quad (19)$$
$$+ \tilde{y_1}^T \tilde{v_1} + \tilde{y_2}^T v_2,$$

To seek conditions under which system (9) is stable

Now if we set,

$$\int_c^d [R_s\{a,U\}_{J_s} + R_n\{a,U\}_{J_n}]dz + \tilde{y_1}^T \tilde{v_1} + \tilde{y_2}^T v_2 = -k(x,x^\star), \quad (20)$$

It may be possible to choose the additional control v_2 such that

$$\begin{aligned} k(x,x^\star) &= 0, \textit{for} \quad x = x^\star, \\ k(x,x^\star) &> 0, \textit{for} \quad x \neq x^\star, \end{aligned} \quad (21)$$

Hence A is a Lyapunov functional candidate.

Hypothesis 3

the orbits of the system are precompact.

By La Salle's Invariance principle, we have

Proposition

Consider the distributed IPHS formulation (3) *of the diffusion process* (2)*, such that :*

- $x^\star$ *be an equilibrium point.* $\tilde{v_1} = \begin{bmatrix} (R\frac{\partial u}{\partial s})(t,d) \\ (R\frac{\partial u}{\partial s})(t,c) \end{bmatrix}$ *is the input, and* $\tilde{y_1} = \begin{bmatrix} \frac{\partial a}{\partial x}(t,d) \\ -\frac{\partial a}{\partial x}(t,c) \end{bmatrix}$ *is the output, and* $\tilde{y_2} = \int_c^d G^T(x)\frac{\partial a}{\partial x}(x,x^\star)dz$ *is the output associated to the finite rank distributed control* v_2 *satisfying*

$$\int_c^d [R_s\{a,U\}_{J_s} + R_n\{a,U\}_{J_n}]dz + \tilde{y_1}^T\tilde{v_1} + \tilde{y_2}^T v_2 = -k(x,x^\star), \quad (22)$$

where $k(x,x^\star) = 0$*, for* $x = x^\star$ *and* $k(x,x^\star) > 0$ *for* $x \neq x^\star$*. Under hypothesis* 1, 2, 3 *the system is globally asymptotically stable.*

PBC method for the heat equation

$$\frac{\partial s}{\partial t} = R_s J_s \frac{\partial u}{\partial s} + \frac{\partial}{\partial z}(R_s \frac{\partial u}{\partial s}) + g(s)\tilde{v}_2(t) \tag{23}$$

$$v_1(t) = \begin{bmatrix} (R_s \frac{\partial u}{\partial s})(t,c) \\ (R_s \frac{\partial u}{\partial s})(t,d) \end{bmatrix}, \qquad y_1(t) = \begin{bmatrix} \frac{\partial u}{\partial s}(t,c) \\ \frac{\partial u}{\partial s}(t,d) \end{bmatrix}, \tag{24}$$

Where $\tilde{v}_2$ is a time dependant distributed finite rank control, and $g(s)$ represents the spatial distribution of $\tilde{v}_2$.

Proposition

Let $s^\star$ be the desired equilibrium to be stabilized. Consider the distributed IPHS formulation (5) of the heat equation, with

- $v_1 = \begin{bmatrix} R_s \frac{\partial u}{\partial s}(t,d) \\ R_s \frac{\partial u}{\partial s}(t,c) \end{bmatrix}$ *is the input, and* $\tilde{y}_1 = \begin{bmatrix} \frac{\partial a}{\partial s}(t,d) \\ -\frac{\partial a}{\partial s}(t,c) \end{bmatrix}$ *is the output.*
- $\tilde{y}_2 = \int_c^d g^T(s) \frac{\partial a}{\partial s}(s,s^\star) dz$ *is the output associated to the finite rank distributed control* $\tilde{v}_2$ *satisfying*

$$\int_c^d R_s\{a,u\}_{J_s} dz + \tilde{y}_1^T v_1(t) + \tilde{y}_2^T \tilde{v}_2(t) = -k(s,s^\star) \quad (25)$$

where $k(s,s^\star) = 0$, *for* $s = s^\star$ *and* $k(s,s^\star) > 0$ *for* $s \neq s^\star$. *Then under hypothesis 1, 2, and 3 the system is globally asymptotically stable.*

Conclusions and outlook

- We presented a PBC method for the distributed diffusion process.
- this approach is a generalization of the PBC method for finite dimensional IPHS.
- The method garantees under realistic assumptions the global asymptotic stability of the system.
- develop other approaches achieving stability of the diffusion process and compare it with this method.

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- We presented a PBC method for the distributed diffusion process.
- this approach is a generalization of the PBC method for finite dimensional IPHS.
- The method garantees under realistic assumptions the global asymptotic stability of the system.
- develop other approaches achieving stability of the diffusion process and compare it with this method.
- How do this approach generalize for other irreversible systems ?.

Thank you for your attention.