Port Hamiltonian representation of heat exchanger

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Outline

Heat EXchanger and energy-efficient systems

Energy saving from heat recovery systems: transfer of heat from a set of hot streams to a set of cold streams.

Heat networks are commonly referred to as

- District Heating Systems (DHSs) when found in urban areas
- Heat Exchanger Networks (HENs) in industrial environments.

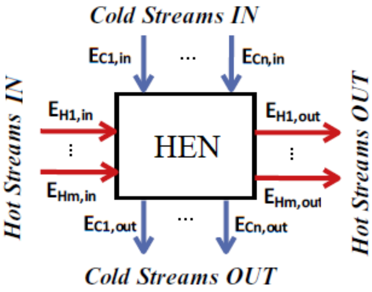

Figure 1: HEN

HEN, their design and their properties

- The design is performed for steady state conditions based for instance on the method Pinch (Linhoff,1983)

What about the adaptability and the robustness of HEN?

- a HEN should remain operable for different operating conditions
- Practically, a HEN should remain operable under variations in operating conditions without loosing stream temperature targets and, at the same time, maintain economically optimal energy integration

Some solutions to be more flexible and robust $\longrightarrow$ Control problem

- by-pass (flow rate fraction), stream split, storage tank

B. Linhoff, The pinch method for networks, 1983, Chem. Eng. sc.

Heat EXchanger HEX, HEN and their representation

Single-phase representation

- Steady state representation (Chandrashekar et Wong, 1982)
- Dynamic representation of HEX (cell representation) (Varga et al., 1995)

Two-phase representation

- Dynamic representation of HEX (Wu at al., 2015)
- PDE representation (Eldredge et al., 2008) - Moving Boundary

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Chandrashekar et Wong, Thermodynamic systems analysis -I. A Graph-Theoretic Approach, 1982, Energy
Varga et al.,Controlability and observability of heat exchanger networks in the time varying parmeter case, Control Eng. Practice, 1995
Wu et al.,A hybrid transient model for simulation of air-cooled refrigeration systems: Description and experimental validation, Int J. of refrigeration, 2015
Eldredge et al., Moving-Boundary Heat Exchanger Models With Variable Outlet Phase, 2008, Journal of Dynamic Systems, Measurement, and Control

Compartment representation of the HEX

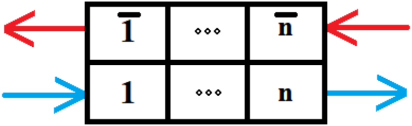

Figure 2: Compartment representation of counter current HE

The bar variables $(\bar{.})$ are associated to the hot fluid. Assumptions:

- Each compartment is homogeneous.
- The hot and cold fluids remain in a single ideal fluid phase.
- Their density ρ and heat capacity c_p $\bar{c}_p$ are constant
- The volume V, $\bar{V}$ of the matter in all the compartments is constant.
- The pressure P is assumed to be constant and uniform.
- No heat exchange with the environment through the wall.
- No energy accumulation in the wall between the two fluids
- The heat transfer coefficient λ $(J/K/s)$ is constant.

The energy balance of a HEX

The energy balance:

$$\left\{\begin{array}{l}
\dot{H}_1 = \frac{\lambda}{\rho V c_p}(\bar{H}_1 - H_1) + Q h_{in} - \frac{Q}{\rho V} H_1 \\
\dot{H}_2 = \frac{\lambda}{\rho V c_p}(\bar{H}_2 - H_2) + \frac{Q}{\rho V} H_1 - \frac{Q}{\rho V} H_2 \\
\dot{H}_3 = \frac{\lambda}{\rho V c_p}(\bar{H}_3 - H_3) + \frac{Q}{\rho V} H_2 - \frac{Q}{\rho V} H_3 \\
\dot{\bar{H}}_1 = \frac{-\lambda}{\bar{\rho} \bar{V} \bar{c}_p}(\bar{H}_1 - H_1) + \frac{\bar{Q}}{\bar{\rho} \bar{V}} \bar{H}_2 - \frac{\bar{Q}}{\bar{\rho} \bar{V}} \bar{H}_1 \\
\dot{\bar{H}}_2 = \frac{-\lambda}{\bar{\rho} \bar{V} \bar{c}_p}(\bar{H}_2 - H_2) + \frac{\bar{Q}}{\bar{\rho} \bar{V}} \bar{H}_3 - \frac{\bar{Q}}{\bar{\rho} \bar{V}} \bar{H}_2 \\
\dot{\bar{H}}_3 = \frac{-\lambda}{\bar{\rho} \bar{V} \bar{c}_p}(\bar{H}_3 - H_3) + \bar{Q} \bar{h}_{in} - \frac{\bar{Q}}{\bar{\rho} \bar{V}} \bar{H}_3
\end{array}\right. \tag{1}$$

The enthalpy vector **H**

Q mass flow rate

λ heat exchange coefficient

V the volume of compartiment

ρ the volume density

H_i enthalpy of comp. i :

$H_i = \rho V(c_p(T_i - T_{ref}) - h_{i,ref})$

T_i temperature of comp. i

T_{ref}, $h_{i,ref}$ temperature and enthalpy reference.

Quasi Port Hamiltonian representation of one HEX

For simplicity let us consider the enthalpy vector $\mathbf{H} = \begin{pmatrix} \dots & H_i & \bar{H}_i & .. \end{pmatrix}$
Let the Hamiltonian be the opposite of the total entropy: $\mathcal{H} = -\mathbb{1}^T \mathbf{S}$
with $\mathbb{1}^T = \begin{pmatrix} 1 & \dots & 1 \end{pmatrix}$ and $\mathbf{S^t} = \begin{pmatrix} \dots & S_i & \bar{S}_i & .. \end{pmatrix}$.
Consider the co vector $\nabla \mathcal{H}^T = -\begin{pmatrix} .. & \frac{1}{T_i} & \frac{1}{\bar{T}_i} & .. \end{pmatrix}$ with $\nabla . = \frac{\partial .}{\partial \mathbf{H}}$.

Entropic representation of HE

$$\frac{d\mathbf{H}}{dt} = \overbrace{\left(\begin{array}{c|c|c} A_1 & 0 & 0 \\ \hline 0 & A_2 & 0 \\ \hline 0 & 0 & A_3 \end{array}\right)}^{Heat\ \ transfer} \nabla \mathcal{H} + \overbrace{G(\mathbf{H})Q}^{Convection} + \overbrace{\bar{G}(\mathbf{H})\bar{Q}}^{Convection} \tag{2}$$

with $A_i = \lambda T_i \bar{T}_i \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$

IPHS definition

IPHS are an extension of port Hamiltonian systems (Ramirez,2013).

The dynamics of IPHS (with $x \in \mathbb{R}^n$ the thermodynamic state vector)

$$\frac{dx}{dt} = R(x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x}) J \frac{\partial U}{\partial x}(x) + W\left(x, \frac{\partial U}{\partial x}\right) + g(x, \frac{\partial U}{\partial x}) u \qquad (3)$$

The Hamiltonian function U (the energy) and the entropy function S
$\frac{\partial U}{\partial x}$ is the co-state vector

$$R(x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x}) = \gamma(x, \frac{\partial U}{\partial x}) \{S, U\}_J$$

The Poisson bracket of S and U: $\{S, U\}_J = \frac{\partial S}{\partial x}^T J \frac{\partial U}{\partial x}$
W and g are $\mathcal{C}^\infty(\mathbb{R}^n)$. They are associated with the ports of the systems.
u is the input vector of the system.

H. Ramirez et al. (2013), Irreversible port-hamiltonian systems: A general formulation of irreversible processes with application to the CSTR, 2013, Chemical Engineering Science

IPHS representation of one HE

Let us consider the enthalpy vector $x^T = \begin{pmatrix} H_1 & H_2 & H_3 & \bar{H}_1 & \bar{H}_2 & \bar{H}_3 \end{pmatrix}$ and the Hamiltonian $\mathbb{H} = \sum_i x_i$, the total entropy $S = \sum_{i=1}^{3} S_i + \bar{S}_i$ and their conjugated variables $\frac{\partial \mathbb{H}}{\partial x}^T = \mathbb{1}^T$, $\frac{\partial S}{\partial x}^T = [\frac{1}{T_1}\ \frac{1}{T_2}\ \frac{1}{T_3}\ \frac{1}{\bar{T}_1}\ \frac{1}{\bar{T}_2}\ \frac{1}{\bar{T}_3}]$.

proposition

The heat exchanger dynamics defined by (1) admits a IPHS representation:

$$\begin{aligned} \frac{dx}{dt} &= \sum_i^3 R_i(x, \frac{\partial \mathbb{H}}{\partial x}, \frac{\partial S}{\partial x}) J_i \frac{\partial \mathbb{H}}{\partial x}(x) \\ &\quad + (B_1 x_{in} + B_2 x) u + (\bar{B}_1 \bar{x}_{in} + \bar{B}_2 x) \bar{u} \\ \begin{pmatrix} y \\ \bar{y} \end{pmatrix} &= \begin{pmatrix} x_{in} B_1^T + x^T B_2^T \\ \bar{x}_{in} \bar{B}_1^T + x^T \bar{B}_2^T \end{pmatrix} \frac{\partial \mathbb{H}}{\partial x} = \begin{pmatrix} -x_3 + x_{in} \\ -\bar{x}_1 + \bar{x}_{in} \end{pmatrix} \end{aligned} \tag{4}$$

IPHS representation of one HE

$$\frac{dx}{dt} = \sum_{i}^{3} R_i(x, \frac{\partial \mathbb{H}}{\partial x}, \frac{\partial S}{\partial x}) J_i \frac{\partial \mathbb{H}}{\partial x}(x) + (B_1 x_{in} + B_2 x)u + (\bar{B}_1 \bar{x}_{in} + \bar{B}_2 x)\bar{u}$$

with $J_i = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & \delta_{i1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_{i2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_{i3} \\ \hline -\delta_{i1} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\delta_{i2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\delta_{i3} & 0 & 0 & 0 \end{array}\right]$ The Kronecker index $\delta_{ij} = 1$

if $i = j$, 0 in the other cases. $u = \frac{Q}{\rho V}$ and $\bar{u} = \frac{\bar{Q}}{\bar{\rho}\bar{V}}$

$R_i(x, \frac{\partial \mathbb{H}}{\partial x}, \frac{\partial S}{\partial x}) = \gamma_i(x, \frac{\partial \mathbb{H}}{\partial x})\{S, \mathbb{H}\}_{J_i}$ with $\gamma_i(x, \frac{\partial \mathbb{H}}{\partial x}) = \lambda T_i \bar{T}_i$.

$B_2 = \left[\begin{array}{ccc|ccc} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$ $B_1^T = \left(\begin{array}{cccc} 1 & 0 & \dots & 0 \end{array} \right)$

IPHS representation of one HE

Passivity analysis

$$\begin{aligned}
\frac{d\mathbb{H}}{dt} &= \frac{\partial \mathbb{H}}{\partial x}^T \frac{dx}{dt} \\
&= \sum_i^3 \mathbb{1}^T k(T_i - \bar{T}_i) J_i \frac{\partial \mathbb{H}}{\partial x} + \mathbb{1}^T (B_1 x_{in} + B_2 x) u \\
&\qquad + \mathbb{1}^T (\bar{B}_1 \bar{x}_{in} + \bar{B}_2 x) \bar{u} \\
&= \sum_i^3 k(T_i - \bar{T}_i) \overbrace{\mathbb{1}^T J_i \mathbb{1}}^{=0} + \overbrace{\mathbb{1}^T (B_1 x_{in} + B_2 x)}^{y^T} u \\
&\qquad + \overbrace{\mathbb{1}^T (\bar{B}_1 \bar{x}_{in} + \bar{B}_2 x)}^{\bar{y}^T} \bar{u} \\
&= y^T u + \bar{y}^T \bar{u}.
\end{aligned} \tag{5}$$

with $\mathbb{1}$ the vector of ones $\in \mathbb{R}^6$.

IPHS representation of HEN

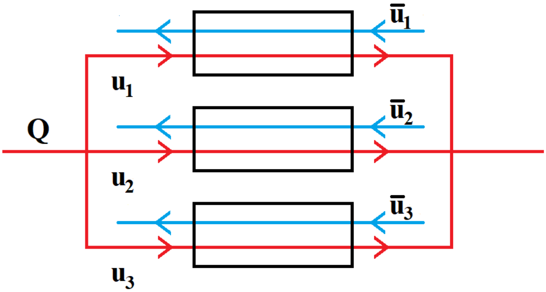

Figure 3: network of three parallel HEXs

Assumptions:

- Controlled input : volumetric flow rate per compartiment volume with $u_1 + u_2 + u_3 = \frac{Q}{\rho V}$
- The total mass flow rate Q is constant

IPHS representation of HEN

IPHS representation of the HEN ($x_e^T = [x^1, x^2, x^3]$ extended state vector)

$$\begin{array}{rcl} \dot{x}_e & = & \sum_k^3 \sum_i^3 R_i^k (\Gamma_k \otimes J_i) \nabla \mathbb{H}_e(x_e) + g(x_e)u + \bar{g}(x_e)\bar{u} \\ y & = & g(x_e)^T \nabla_{x_e} \mathbb{H}_e(x_e) \\ \bar{y} & = & \bar{g}(x_e)^T \nabla_{x_e} \mathbb{H}_e(x_e) \\ & & \mathbb{1}^T u = \frac{Q}{\rho V} \end{array}$$

with $u^T = [u_1 \;\; u_2 \;\; u_3]$, $\bar{u}^T = [\bar{u}_1 \;\; \bar{u}_2 \;\; \bar{u}_3]$. $\mathbb{H}_e$, S_e the total enthalpy and total entropy of the network. $R_i^k(x, \frac{\partial \mathbb{H}_e}{\partial xe}, \frac{\partial S_e}{\partial xe}) = \gamma_i^k(x, \frac{\partial \mathbb{H}_e}{\partial x_e})\{S_e, \mathbb{H}_e\}_{J_{ik}}$

$$\begin{array}{ccc} \mathrm{J}_{ik} = \Gamma_k \otimes J_i = -J_{ik}^T & & \\ \Gamma_k = \begin{bmatrix} \delta_{k1} & 0 & 0 \\ 0 & \delta_{k2} & 0 \\ 0 & 0 & \delta_{k3} \end{bmatrix} & & \gamma_i^k = \lambda T_i^k \bar{T}_i^k \\ g(x_e) = (I_3 \otimes B_1)x_{in,e} + (I_3 \otimes B_2)x_e & , & \bar{g}(x_e) = (I_3 \otimes \bar{B}_1)\bar{x}_{in,e} + (I_3 \otimes \bar{B}_2)x_e \\ x_{in,e}^T = [x_{in}^1, x_{in}^2, x_{in}^3] & , & \bar{x}_{in,e}^T = [\bar{x}_{in}^1, \bar{x}_{in}^2, \bar{x}_{in}^3] \end{array} \quad (6)$$

Conclusion and Perspectives

- IPHS Representation with the Hamiltonian: the total entropy
- Passivity based Control of HE
- Power Consensus Control HEN