Modelling of Tokamak plasmas as open GENERIC systems

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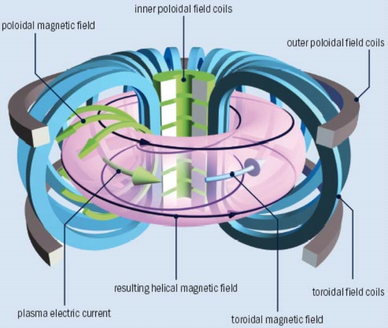
Magnetic confinement : Tokamak reactors



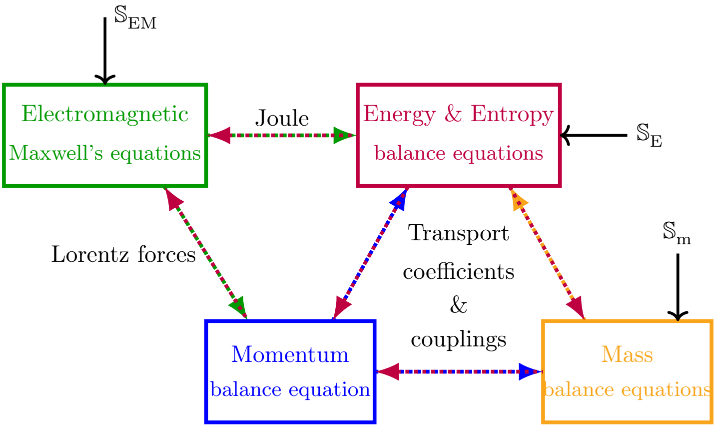
Tokamak concept (1950s)

- Lavrentiev, Tamn, & Sakharov
- Torus shape magnetic chamber
- 3 coils generate helicoidal field lines
- Auxiliary heating

Plasma : fourth state of matter

- Distributed parameter system
- Multi–physics
- Nonlinear
- Couplings (transport)
- Multi–scale
- Distributed and boundary controls

Tokamak plasmas are multi–physics systems



A structured model–based approach is considered

Early development: port-Hamiltonian plasma formulations

- Thermo-magneto **port–Hamiltonian plasma model**: 3–D, 1–D, and lumped (Vu et al., 2016)
- **IDA–PBC internal profile control** using: application to TCV (Vu et al., 2017)
- **Burning plasmas** models : 3–D, 1–D, and lumped (Vincent et al., 2018)

Geometric representations of plasma control models

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Thermodynamic formulation of systems

Considered approach : two–potentials formulation

- **Physics–based :** Coupled Hamiltonian and a dissipative metric (Morrison 1984)
- **Thermodynamic :** Total energy and entropy (Grmela et al. 1997, Ottinger et al. 1997,Ottinger 2006)

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- **Presents the open GENERIC framework**
- **Simplified 3–D thermo-magnetic plasma model**

Differential forms

- Domain $\Omega \subset \mathbb{R}^3$
- Boundary $\partial\Omega \subset \mathbb{R}^2$
- Quantities in Ω are **differential forms**: α is a k-form, then we have:

$$\alpha \in \Lambda^k(\Omega), \qquad \alpha = \alpha(z) dz_1 \wedge dz_2 \wedge \ldots \wedge dz_k$$

Differential forms and notations

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Operators on Euclidean metric

- **Exterior product** :

$$\wedge : \quad \Lambda^k(\Omega) \times \Lambda^l(\Omega) \to \Lambda^{k+l}(\Omega)$$

- **Exterior derivative** :

$$d : \quad \Lambda^k(\Omega) \to \Lambda^{k+1}(\Omega)$$

- **Hodge operator** :

$$\star : \quad \Lambda^k(\Omega) \to \Lambda^{n-k}(\Omega)$$

- **Natural pairing** :

$$< \alpha_k, \alpha_{n-k} >= \int_\Omega \alpha_k \wedge \alpha_{n-k} \quad (\in \mathbb{R})$$

A two–potential representations of closed dissipative systems

General Equilibrium Non–Equilibrium Reversible Irreversible Couplings (Ottinger, 2006)

$$\dot{x} = L(x) \cdot \frac{\delta \mathbb{E}}{\delta x} + M(x) \cdot \frac{\delta \mathbb{S}}{\delta x} + g(x)u(t)$$

- **State variables :** $x \in \mathcal{X} = C^{\infty}(\Omega, \mathbb{R}^n)$
- **Functional derivative :** $\delta/\delta x$
- **Total energy :** $\mathbb{E} : \mathcal{X} \to \mathbb{R} : x \mapsto \mathbb{E} = \int_{\Omega} e(x)$
- **Total entropy :** $\mathbb{S} : \mathcal{X} \to \mathbb{R} : x \mapsto \mathbb{S} = \int_{\Omega} s(x)$
- **Poisson operator :** L is required to be skew–symmetric
- **Dissipative operator :** M is symmetric and positive semi–definite
- **Degeneracy conditions**

$$L(x) \cdot \frac{\delta \mathbb{S}}{\delta x} = 0, \qquad \text{and} \qquad M(x) \cdot \frac{\delta \mathbb{E}}{\delta x} = 0$$

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First and second laws of thermodynamics are satisfied

- Energy conservation

$$\frac{d\mathbb{E}}{dt} = 0$$

- Positive definite irreversible entropy production

$$\frac{d\mathbb{S}}{dt} = \frac{\delta \mathbb{S}}{\delta x} \cdot M(x) \cdot \frac{\delta \mathbb{S}}{\delta x} \geq 0$$

Conservative part modelled by a Poisson bracket

The Poisson bracket represents the conservative dynamics

$$\{A, B\} = \left\langle \frac{\delta A}{\delta x}, L \cdot \frac{\delta B}{\delta x} \right\rangle = \int_\Omega \frac{\delta A}{\delta x} \wedge \left(L \cdot \frac{\delta B}{\delta x} \right)$$

Properties

- **Skew–symmetry :** $\{A, B\} = -\{B, A\}$
- **Leibniz identity :** $\{AB, C\} = A\{B, C\} + B\{A, C\}$
- **Jacobi identity :** $\{A, \{B, C\}\} + \{B, \{C, A\}\} + \{C, \{A, B\}\} = 0$

Poisson bracket is extended for open systems

Extended brackets to include boundary conditions (Ottinger 2005)

The Poisson bracket is divided into a bulk and a boundary bracket

- **Full Poisson bracket**

$$\{A,B\} = \{A,B\}_{\Omega} + \{A,B\}_{\partial\Omega}$$

- **Bulk Poisson bracket is identical as for the closed system**

$$\{A,B\}_{\Omega} = \left\langle \frac{\delta A}{\delta x}, L \cdot \frac{\delta B}{\delta x} \right\rangle$$

- **Boundary Poisson bracket is assumed under the following form**

$$\{A,B\}_{\partial\Omega} = \int_{\partial\Omega} \frac{\delta A}{\delta x} \wedge \left(L^{\partial} \cdot \frac{\delta B}{\delta x} \right) = \int_{\partial\Omega} \frac{\delta B}{\delta x} \wedge \left(L^{\partial *} \cdot \frac{\delta A}{\delta x} \right)$$

- **Bulk Poisson bracket is subject to the degeneracy condition**

$$L \cdot \frac{\delta \mathbb{S}}{\delta x} = 0$$

- **The full Poisson bracket inherits the skew–symmetry property**

$$\{A,B\}_{\Omega} + \{B,A\}_{\Omega} = -\int_{\partial\Omega} \frac{\delta B}{\delta x} \wedge \left(L^{\partial *} + L^{\partial} \right) \cdot \frac{\delta A}{\delta x}$$

Resistive part modelled by a dissipative bracket

The dissipative bracket encode the resistive dynamics

$$[A, B] = \left\langle \frac{\delta A}{\delta x}, M \cdot \frac{\delta B}{\delta x} \right\rangle = \int_\Omega \frac{\delta A}{\delta x} \wedge \left(M \cdot \frac{\delta B}{\delta x} \right)$$

Properties

- **Symmetry :** $[A, B] = [B, A]$
- **Leibniz identity :** $[AB, C] = A[B, C] + B[A, C]$
- **Positivity :** $[A, A] \geq 0$

The entropy variation is positive semi-definite

Extended brackets to include boundary conditions (Ottinger 2005)

The dissipative bracket is divided into a bulk and a boundary bracket

- **Full dissipative bracket**

$$[A,B] = [A,B]_\Omega + [A,B]_{\partial\Omega}$$

- **The bulk dissipative bracket is identical as for the closed system**

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- **Bulk dissipative bracket is subject to the degeneracy condition**

$$M \cdot \frac{\delta \mathbb{E}}{\delta x} = 0$$

- **The full dissipative bracket inherits the symmetry property**

$$[A,B]_\Omega - [B,A]_\Omega = \int_{\partial\Omega} \frac{\delta B}{\delta x} \wedge \left(M^{\partial} - M^{\partial *} \right) \cdot \frac{\delta A}{\delta x}$$

Balance equations for open GENERIC systems

Total energy balance equation

$$\frac{d\mathbb{E}}{dt} = \underbrace{\int_{\partial\Omega} \left(-\frac{\delta\mathbb{E}}{\delta x} \cdot L^{\partial} + \frac{\delta\mathbb{S}}{\delta x} \wedge \left(M^{\partial} - M^{\partial *} \right) \right) \cdot \frac{\delta\mathbb{E}}{\delta x}}_{\text{Boundary source of energy}} + \underbrace{\int_{\Omega} \frac{\delta\mathbb{E}}{\delta x} \wedge g(x)u(t)}_{\text{distributed source of energy}}$$

- Conservation of energy for the closed system
- Skew–symmetric contributions of dissipative boundary inputs produce energy

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$$\frac{d\mathbb{S}}{dt} = [\mathbb{S}, \mathbb{S}] - \underbrace{\int_{\partial\Omega} \left(\frac{\delta\mathbb{E}}{\delta x} \wedge \left(L^{\partial} + L^{\partial *} \right) + \frac{\delta\mathbb{S}}{\delta x} \wedge M^{\partial *} \right) \cdot \frac{\delta\mathbb{S}}{\delta x}}_{\text{Boundary source of entropy}} + \underbrace{\int_{\Omega} \frac{\delta\mathbb{S}}{\delta x} \wedge g(x)u(t)}_{\text{Distributed source of entropy}}$$

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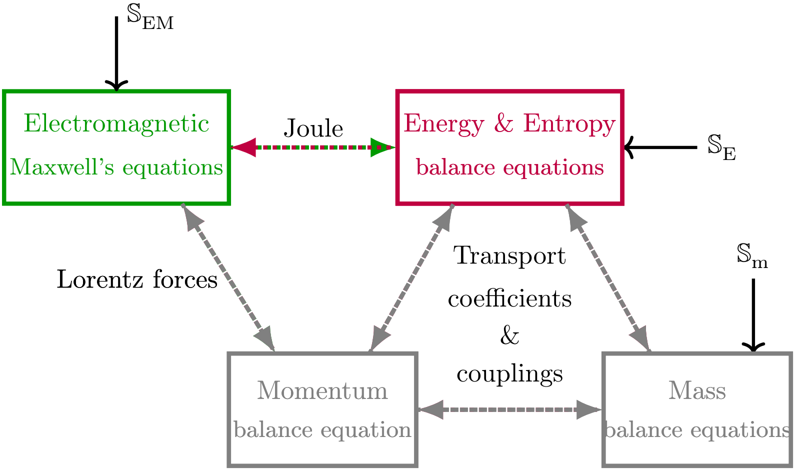
Application to Tokamak plasmas


$\mathbb{S}_{\mathrm{EM}}$
Electromagnetic
Maxwell's equations
Joule
Energy & Entropy
balance equations
$\mathbb{S}_{\mathrm{E}}$
Lorentz forces
Transport
coefficients
&
couplings
$\mathbb{S}_{\mathrm{m}}$
Momentum
balance equation
Mass
balance equations

Maxwell's equations (on $\Omega \subset \mathbb{R}^3$ with $\partial\Omega \subset \mathbb{R}^2$)

- **Faraday and Ampere's laws :**

$$\frac{\partial B}{\partial t} = -dE, \qquad \frac{\partial D}{\partial t} = dH - j, \qquad dB = 0, \qquad \text{and} \qquad dD = 0$$

with B and $D \in \Lambda^2(\Omega)$ *magnetic* and *electric* fields, E and $H \in \Lambda^1(\Omega)$ *electric* and *magnetic* fluxes, and $j \in \Lambda^2(\Omega)$ the total *plasma current*

Plasma balance equations – Electromagnetic part

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- **Phenomenological relations :**

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- **Total current :** $j = j_{\text{Ohm}} + \bar{j}$, $\bar{j} \in \Lambda^2(\Omega)$ the distributed source of current.
- **Ohm's law :** $j_{\text{Ohm}} = \star_{\sigma} E$ with *conductivity* σ

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- **Boundary conditions:** $tr(E) = E_{\partial\Omega}$ and $tr(H) = H_{\partial\Omega}$
- **Electromagnetic energy :**

$$\mathcal{H}(D, B) = \frac{1}{2}(E \wedge D + H \wedge B)$$

Plasma balance equations – Internal energy

Internal energy balance equation (on $\Omega \subset \mathbb{R}^3$ with $\partial\Omega \subset \mathbb{R}^2$)

- **Balance equation :**

$$\frac{\partial u}{\partial t} = -dj_q + E \wedge j_{\text{Ohm}} + \overline{\sigma}_u$$

- **Internal energy :** $u(s) \in \Lambda^3(\Omega)$
- **Distributed source of heat :** $\overline{\sigma}_u$
- **Heat flux :** defined with Fourier's law

$$j_q = \star_{\Gamma_{qq} T^2} d \left(\frac{1}{T} \right)$$

where $\Gamma_{qq} \in \Lambda^0$ denotes the heat diffusion coefficient and $T \in \Lambda^0(\Omega)$ the temperature

- **Gibb's relation :** $\delta u = T \wedge \delta s$
- **Entropy density :** $s \in \Lambda^3(\Omega)$

The Maxwell's and the heat diffusion equations are coupled

Plasma balance equations – Couplings

Two dissipative processes

- **Joule effect** through the plasma resistivity is the main coupling
- **Heat diffusion** process through j_q

Energy functions

- **Total energy density :**

$$e(s, D, B) = u(s) + \mathcal{H}(D, B)$$

- **Total energy variation (extended Gibbs formulation) :**

$$\delta e = \delta u + E \wedge \delta D + H \wedge \delta B \qquad \Leftrightarrow \qquad \delta e = T \wedge \delta s + E \wedge \delta D + H \wedge \delta B$$

Four state equations governing: D, B, u, and s

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Four state equations governing: D, B, u, and s

At least two possible representations

- Entropy representation $x = \begin{pmatrix} s & D & B \end{pmatrix}$
- Energy representation $x = \begin{pmatrix} e & D & B \end{pmatrix}$

Entropy formulation

- **State equations**

$$\dot{x} = L \cdot \frac{\delta \mathbb{E}}{\delta x} + M \cdot \frac{\delta \mathbb{S}}{\delta x} + g(x)u(t)$$

- **State variables**

$$x = \begin{pmatrix} s & D & B \end{pmatrix}^\top$$

Entropy GENERIC representations

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- **State variables**

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- **Potentials**

$$\mathbb{E} = \int_\Omega e(u, D, B)$$

$$\mathbb{S} = \int_\Omega s$$

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- **Potentials**

$$\mathbb{E} = \int_\Omega e(u, D, B)$$

$$\mathbb{S} = \int_\Omega s$$

- **Functional derivatives**

$$\frac{\delta \mathbb{E}}{\delta x} = \begin{pmatrix} T \\ E \\ H \end{pmatrix}$$

$$\frac{\delta \mathbb{S}}{\delta x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Full brackets encode the bulk dynamics

Poisson Bracket construction

- Skew–symmetric structure of the Maxwell's equations
- Heat diffusion is not a conservative process
- We expect the first row and column of the operator L to be full of zeros
- L^∂ results from the adjoint of L

$$L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -d(\cdot) \\ 0 & d(\cdot) & 0 \end{pmatrix} \qquad L^\partial = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & tr(\cdot) \\ 0 & 0 & 0 \end{pmatrix}$$

- $tr(\cdot)$ is the trace operator on the boundary $\partial\Omega$

Dissipative Bracket construction

- Gathers all resistive processes : heat diffusion and resistivity

$$M = \begin{pmatrix} M_{11} & -\star_{1/\eta} E & 0 \\ -\star_{1/\eta} E & T\star_{1/\eta}(\cdot) & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad M^\partial = \begin{pmatrix} tr(\star_{\Gamma_{qq}T^2} d(\cdot)) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where $M_{11}^{(s)} = -\frac{1}{T} d\left(j_q\right) + E \wedge j_{\text{ohm}}$

Entropy formulation

GENERIC state space representation

$$\dot{x} = L\frac{\delta \mathbb{E}}{\delta x} + M\frac{\delta \mathbb{S}}{\delta x} + g(x)u(t)$$

Energy balance

$$\frac{d\mathbb{E}}{dt} = -\int_{\partial\Omega} tr(E) \wedge tr(H) - \int_{\partial\Omega} tr\left(J_q\right) + \int_{\Omega} \frac{\delta \mathbb{E}}{\delta x^{(s)}} B^{(s)} u_d$$

Total entropy balance equation

$$\frac{d\mathbb{S}}{dt} = \int_{\Omega} \left[\begin{pmatrix} d\left(\frac{1}{T}\right) & E \end{pmatrix} \begin{pmatrix} \star_{\Gamma_{qq}T^2} & 0 \\ 0 & \star_{1/\eta} \end{pmatrix} \begin{pmatrix} d\left(\frac{1}{T}\right) \\ E \end{pmatrix} \right] - \int_{\partial\Omega} tr\left(\frac{J_q}{T}\right) + \int_{\Omega} \frac{\delta \mathbb{S}^{(s)}}{\delta x^{(s)}} B^{(s)} u_d$$

The model is consistent since we recover the balance equations

GENERIC thermo–magnetic plasma models are proposed

Two GENERIC formulations

- Entropy representation of thermo–magnetic plasmas
- Energy representation of thermo–magnetic plasmas

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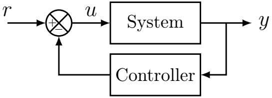
Open questions

- Passivity of open boundary GENERIC systems
- Interconnection of GENERIC systems
- Feedback control



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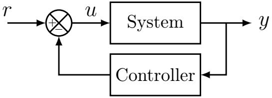

Future works

- Stability and passivity study with the GENERIC formalism
- Structure preserving geometric reduction (1–D system)
- Spatial discretization schemes (Lumped parameter system)

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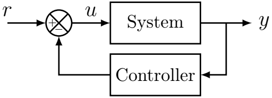

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