VISUALISATION AND ANALYSIS OF LIQUID FILM SURFACE PATTERNS FORMED ON ULTRASONIC ATOMISERS

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ABSTRACT

The ultrasonic atomisation mechanism involves the formation of regular waves on liquid surface which will break-up into droplets when resonance is reached. Spray characteristics are closely related to this wave structure developed just before atomisation. An experimental study has been undertaken in order to measure free surface wavelength. The results are compared with values predicted by linear stability analysis. The influence of several parameters (liquid properties, resonance frequency) on the wavelength is analysed. Finally the relationship between free surface wavelength and spray mean droplet diameter is investigated.

NOTATION

\[ A \] Liquid film wave amplitude
\[ a_0 \] Atomiser tip vibration amplitude
\[ D_{10} \] Number mean diameter
\[ f \] Ultrasonic resonance frequency
\[ g \] Gravity acceleration
\[ h \] Liquid film height
\[ m \] Liquid film mass
\[ N \] Number of droplets
\[ S \] Mean droplet surface
\[ \nu_p \] Wave pattern velocity
\[ V \] Mean droplet volume
\[ W_e \] Wave pattern Weber number
\[ \lambda \] Liquid film oscillation wavelength - measured
\[ \lambda_c \] Liquid film oscillation wavelength - calculated
\[ \rho \] Density
\[ \sigma \] Surface tension
\[ \mu \] Viscosity
\[ \omega \] Atomising surface wave angular frequency

INTRODUCTION

A thin liquid film formed on a high-frequency vibrating surface (30-70 kHz for the devices designed at UCL) will break-up in a fine uniform spray. The ultrasonic vibration induces surface waves in the liquid film. First, as frequency is tuned, very regular square cells will form on the free surface just before reaching the resonance frequency. Then, when resonance is reached, the amplitude grows till droplets are ejected through a crest break-up mechanism. The same instability mechanism is present in many types of atomization. The instability cause is nevertheless specific. Wave instability may result from internal causes (nozzle vibrations, liquid swirling, liquid pressure drop,...) or/and external causes (interactions with surrounding media). For the ultrasonic atomization the main source of wave growth is the vibration of the piezoelectric nozzle.

The very regular square cells generate uniform size droplets. Understanding the pattern formation on the liquid film surface is essential to further spray characterization. Droplets size in ultrasonic spray depends strongly on surface waves characteristics. Our previous work on droplets size prediction by maximum entropy formalism involves a calculated wavelength. As this wavelength prediction is achieved by linear stability analysis, there might be significant difference between theory and reality. In order to validate the theoretical values and to further investigate pattern formation phenomenon an experimental study must be performed.

The formation of regular patterns on a vertically vibrating plate is known as the Faraday instability phenomena. Several patterns have first been observed by Faraday [1] in 1831. More then a century later patterns (squares, hexagons, rolls or fivefold patterns) were highlighted by Christiansen & al. [2] and Edwards & Fauve [3]. Experiments conducted in the past years use low frequency (10-50 Hz) vibration to generate these patterns. For high frequencies (~50 kHz as used in ultrasonic atomization) only square patterns were observed (Sindayihebura & Bolle [4]).

Previous experiments intend to highlight the formation of regular wave patterns. The purpose of our experimental
work is to measure the wavelength and to investigate the correlation between the ultrasonic spray mean droplet diameter and the wave structure.

A theoretical solution to the non-linear instability problem is not achieved by now although there are several papers reporting that work is in progress (a synthesis of latest results may be found in Linghansen and Alstrom [5]). A linear stability analysis has been made by Sindayhebura [6]. The results are compared with the experimental measurements of the wavelength and with the simple wavelength prediction formulae proposed by Lang [7].

EXPERIMENTAL SET-UP

Surface of the liquid film formed on the ultrasonic atomiser tip is filmed using a high-speed camera (at 250 images/s). Average wavelength values are about 120µm so a 40 times magnification microscope is inserted between the camera and the liquid film surface. An optical fibre light source is needed in order to supply a spotlight on liquid film surface.

Figure 1 shows a photograph of the experimental set-up. The ultrasonic atomiser is basically made of two piezoelectric discs tight between two cylinders designed for maximum amplification of the ultrasonic wave amplitude. The piezoelectric elements are driven by an amplified ultrasonic frequency wave. An ultrasonic atomiser is designed to work at a unique resonance frequency. In order to estimate the influence of the frequency on square waves wavelengths, four different design atomisers (working at 30, 33, 50 and 58 kHz) are used.

In order to analyse the influence of the liquid properties several mixtures of water-methanol and water-glycerol have been prepared. The properties of these liquids are presented in Table 3 (mass fractions are considered). These liquids have been chosen in order to dissociate the surface tension effects from those of the viscosity.

A few liquid is dropped on the atomiser tip. Wave frequency is tuned up till the wave pattern forms. Images are then transferred to a computer where further wavelength measurements are done using image analysis.

RESULTS AND DISCUSSIONS

An example of liquid surface pattern is presented in Fig 2. The two images are taken at different resonance frequencies. A 1mm scale is also shown on the figure.

**Parameters affecting wavelength**

**Forcing frequency**

Four different atomisers are used in order to investigate the influence of resonance frequency on wavelength. The results (table 1, water) confirm that increasing the frequency leads to a decrease of the wavelength.

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>Wavelength (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>157.54</td>
</tr>
<tr>
<td>33</td>
<td>134.36</td>
</tr>
<tr>
<td>50</td>
<td>105.7</td>
</tr>
<tr>
<td>58</td>
<td>95.92</td>
</tr>
</tbody>
</table>

Table 1. Measured wavelengths for different forcing frequencies.
Liquid properties

Influence of viscosity and surface tension on the measured wavelength for a fixed resonance frequency is examined using two data sets for two different frequencies (33kHz and 50kHz). The properties of tested liquids (table 3) allow to separate the effects of viscosity from those of surface tension, as we have couples of fluids with same viscosity but different values of surface tension for example. Surface tension and viscosity acts the same way: when their values increase larger wavelength square patterns may be observed.

Correlation factors $R_{\text{corr}}$ (which measure the relationship between two data sets that are scaled to be independent of the unit of measurement) are calculated in table 2 for all the influencing parameters. We can notice that wavelength increases when surface tension is increased and decreases when frequency is increased. The correlation between viscosity and wavelength for all available data is not significant.

Comparison with theory

The measured wavelengths have then to be compared with predictions obtained from linear stability analysis. The first expression of wavelength in capillary waves was established by Kelvin:

$$\lambda_s f_s = \sqrt{\frac{\lambda_s g}{2\pi \rho \lambda_s}} \tanh \left( \frac{2\pi \rho h}{\lambda_s} \right)$$

Where $\lambda_s$, $f_s$, $g$, $\rho$, $\sigma$ are the free surface wavelength and $h$ the liquid film thickness. Lang [7] applied it to ultrasonic sprays by replacing the frequency $f_s$ of liquid film vibration with frequency of atomiser $f$. Two other hypothesis are made in Lang formulation:

- The liquid film is thin $\Rightarrow \tanh \left( \frac{2\pi h}{\lambda_s} \right) \approx 1$
- The influence of gravity forces is insignificant compared to capillary forces $\Rightarrow \frac{\lambda_s g}{2\pi \rho} \ll \frac{2\pi \sigma}{\rho \lambda_s}$

Lang expresses in a much simpler way the wavelength function of liquid properties and forcing ultrasonic frequency (square pattern formation is a subharmonic phenomena $\Rightarrow f_s = f/2$):

$$\lambda_{s,\text{Lang}} = \left( \frac{8\pi \sigma}{\rho f^2} \right)^{1/4}$$

A formula for the prediction of the most rapidly growing surface wavelength, which takes into account all the main parameters, has been established by Sindayihebura [6] in a stability analysis of the liquid free surface just before the ultrasonic atomisation process. This analysis was based on the Navier-Stokes equations and the following equation was found:

$$\frac{2}{\pi f^2 \lambda_s} \left[ \frac{4 \pi \rho^2}{\sigma} + g \right] \tanh \left( \frac{2\pi h}{\lambda_s} \right) + 0.02 \left[ \frac{\pi a_0}{\lambda_s} \tanh \left( \frac{2\pi h}{\lambda_s} \right) \right]^{1/4} - 1.04 = 0$$

Table 2. Correlation between wavelength and influencing factors.

<table>
<thead>
<tr>
<th>Influencing factors</th>
<th>$R_{\text{corr}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>-0.96</td>
</tr>
<tr>
<td>Surface tension</td>
<td>0.85</td>
</tr>
<tr>
<td>Viscosity</td>
<td>0.40</td>
</tr>
</tbody>
</table>

The analysis of experimental available data leads to the conclusion that differences between calculated and measured wavelengths vary with liquid properties. As viscosity increases the difference becomes more important (see Fig. 4). As pointed out by Barrio & al. [8] the linearized equation has decaying oscillatory solutions or unstable bounded ones (depending on frequency and amplitude of the forcing wave). There are no stable patterns if one do
not consider the mechanism that dissipates energy at the same rate as feed into the system. Dissipative viscous forces are neglected in equations (1) to (3).

<table>
<thead>
<tr>
<th>Mixture -mass fraction-</th>
<th>Viscosity ( (10^{-3} \text{ kg/ms}) )</th>
<th>Surface tension ( (10^{-3} \text{ N/m}) )</th>
<th>Density ( (\text{kg/m}^3) )</th>
<th>( \lambda_c \text{ (calculated)} ) ( (\mu\text{m}) )</th>
<th>( \lambda \text{ (measured)} ) ( (\mu\text{m}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1.00</td>
<td>72.75</td>
<td>1000</td>
<td>91.7</td>
<td>105.6</td>
</tr>
<tr>
<td>10% Methanol</td>
<td>1.33</td>
<td>59.04</td>
<td>981</td>
<td>70.1</td>
<td>106.6</td>
</tr>
<tr>
<td>16% Methanol</td>
<td>1.50</td>
<td>53.97</td>
<td>972</td>
<td>86.1</td>
<td>106.0</td>
</tr>
<tr>
<td>30% Methanol</td>
<td>1.79</td>
<td>43.02</td>
<td>951</td>
<td>71.4</td>
<td>104.4</td>
</tr>
<tr>
<td>47% Methanol</td>
<td>1.80</td>
<td>36.08</td>
<td>921</td>
<td>83.9</td>
<td>103.2</td>
</tr>
<tr>
<td>64% Methanol</td>
<td>1.50</td>
<td>31.84</td>
<td>885</td>
<td>72.8</td>
<td>98.0</td>
</tr>
<tr>
<td>72% Methanol</td>
<td>1.32</td>
<td>29.41</td>
<td>866</td>
<td>78.4</td>
<td>102.6</td>
</tr>
<tr>
<td>81% Methanol</td>
<td>1.10</td>
<td>27.04</td>
<td>843</td>
<td>74.8</td>
<td>94.0</td>
</tr>
<tr>
<td>15% Glycerol</td>
<td>1.49</td>
<td>72.67</td>
<td>1033</td>
<td>90.7</td>
<td>112.0</td>
</tr>
<tr>
<td>21% Glycerol</td>
<td>1.80</td>
<td>72.36</td>
<td>1048</td>
<td>90.2</td>
<td>114.0</td>
</tr>
</tbody>
</table>

Table 3. Predicted and measured wavelengths for different liquid properties and fixed forcing frequency \( f = 50\text{kHz} \).

A correction factor has to be applied to eqns (1)-(3) in order to predict an accurate wavelength. As the three equations give almost identical results for a wide range of forcing frequencies and liquid properties, it is obvious that one should use the simplest one.

The wavelength may be calculated using:

\[
\lambda = 1.26\lambda_{Lang} = \left( \frac{16\pi\sigma}{\rho f^2} \right)^{1/4}
\]  

(4)

**SPRAY MEAN DIAMETER**

In previous predictions of ultrasonic spray mean diameter based on square pattern wavelength [9], equation (3) was used. The corrected and simpler eqn (4) established above will further be applied to spray characteristics prediction.

Ultrasonic atomisation is a two stage process: first square waves develop on the liquid film surface then, when resonance is reached, the amplitude growth causes droplet detachment from wave crests. Factors that affect each stage are presented in Fig 5.
Figure 5. Two stage ultrasonic atomisation process and characteristic parameters.

Droplet break-up phenomenon is driven by surface tension and oscillation forces. The liquid film wave energy transforms (with losses) in droplets energy (kinetic and surface tension). Droplets velocity is low (~1 m/s) in ultrasonic sprays so most of the wave energy is transformed in surface tension energy of drops:

\[
\frac{1}{2} m \omega^2 A^2 + N \sigma S
\]

(5)

where \(m\) is the mass of the liquid film, \(A\) the wave amplitude, \(\omega = 2\pi f\) the wave angular frequency, \(N\) the number of droplets and \(S\) the mean droplets surface.

The formulation of mass conservation between liquid film and spray droplets leads to:

\[
m = NpV
\]

(6)

where \(V\) is the mean droplet volume. Equation (5) then becomes:

\[
D_{10} \propto \frac{3}{\pi^2} \cdot \frac{\sigma}{\rho f^2 A^2}
\]

(7)

Where \(D_{10}\) is the arithmetic mean droplets diameter (spray number mean diameter). If we further assume that wavelength and amplitude are proportional:

\[
D_{10} = const \cdot \frac{\sigma}{\rho f^2 \lambda^2}
\]

(8)

If we define a “pattern velocity” \(v_p = \lambda f\) eqn (8) express a constant Weber number for wave pattern:

\[
We_p = \frac{\rho v_p^2 D_{10}}{\sigma}
\]

(9)

Measured droplet mean diameter for the 50kHz atomiser and for all the tested liquids are plotted in Fig 6 (circles). The constant proportionality factor in equation (8) is \(const \cong 10\). So for the range of liquid properties tested \(We_p \cong 0.1\).

As our available diameter data did not allow to verify the influence of forcing frequency we used data reported by Lacas & al. [10]. As wavelength was not available for this data, eqn (4) established above is used. One may see in figure 4 that Lacas data (stars) fits well correlation (8).
Combining eqn (8) with wavelength prediction eqn (4) results in:

\[ D_{10} = 10 \cdot \frac{\sigma}{\rho f^2} \left( \frac{\sigma}{\rho f^2} \right)^2 = 0.73 \left( \frac{\sigma}{\rho f^2} \right)^3 \Rightarrow D_{10} = \lambda_s \]  

(10)

In previous studies ultrasonic spray mean diameter is assumed to be proportional with calculated wavelength using geometrical reasoning. The drop volume is considered to be equal to the volume of a wave crest (see ref.). We have shown before that the same proportionality is obtained only from physical phenomenon description.

CONCLUSIONS

In ultrasonic atomization, spray generation is preceded by a liquid film instability phenomenon. This two stage behaviour, wave formation followed by drops break-up from wave crests, is common to almost every types of atomisation. The peculiarity with an ultrasonic driven atomizer is that wave structure is very regular (square pattern). Progress in visualisation techniques allows us to photograph and analyse wave characteristics just before atomization. Theoretical investigations of wave instability phenomena lead to more or less complex wavelength prediction equations. The effects of forcing frequency and liquid properties (surface tension and density) are reflected in the same way by all the authors. Depending on the hypothesis, the prediction equation may involve gravity effects or ultrasonic input power effects (via the atomizer surface amplitude).

In order to verify the validity of these equations an experimental study has been achieved. Different frequencies and several liquids were used. The liquid film square pattern was filmed and wavelength was then measured. Correlations between available wavelength data and factors influencing the wave formation indicate that the effects of surface tension and frequency are well reflected in theoretical analysis.

Viscosity is neglected in all theoretical approaches so that differences between measured and calculated wavelengths increases when high viscosity liquids are used. For the considered ranges of liquid properties and forcing frequencies, three proposed equations give the same result. The simplest one is correlated to experimental data. The wavelength of the square pattern formed just before atomisation is well predicted by eqn (4).

The second stage of atomisation, i.e. the liquid film break-up, involves energy and mass conservation. A fraction of wave energy transforms in droplets surface energy. Developing further this proportionality and using mass conservation number mean droplet diameter can be predicted from eqn (8). A constant adimensional Weber number for the wave pattern is defined and evaluated from experimental data.

Finally combining mean droplet diameter equation and wavelength simplified equation, a proportionality between mean diameter and predicted wavelength is found. This equation is identical to expressions derived previously from geometrical considerations. The present study succeeds in establishing on purely physical basis the relation between mean droplet diameter and square pattern wavelength in an ultrasonic atomisation process.

REFERENCES

1. Faraday M., On the forms and states assumed by fluids in contact with vibrating elastic surfaces. Phil.Trans. R. Soc. Lond. 52, 319-340, 1831.