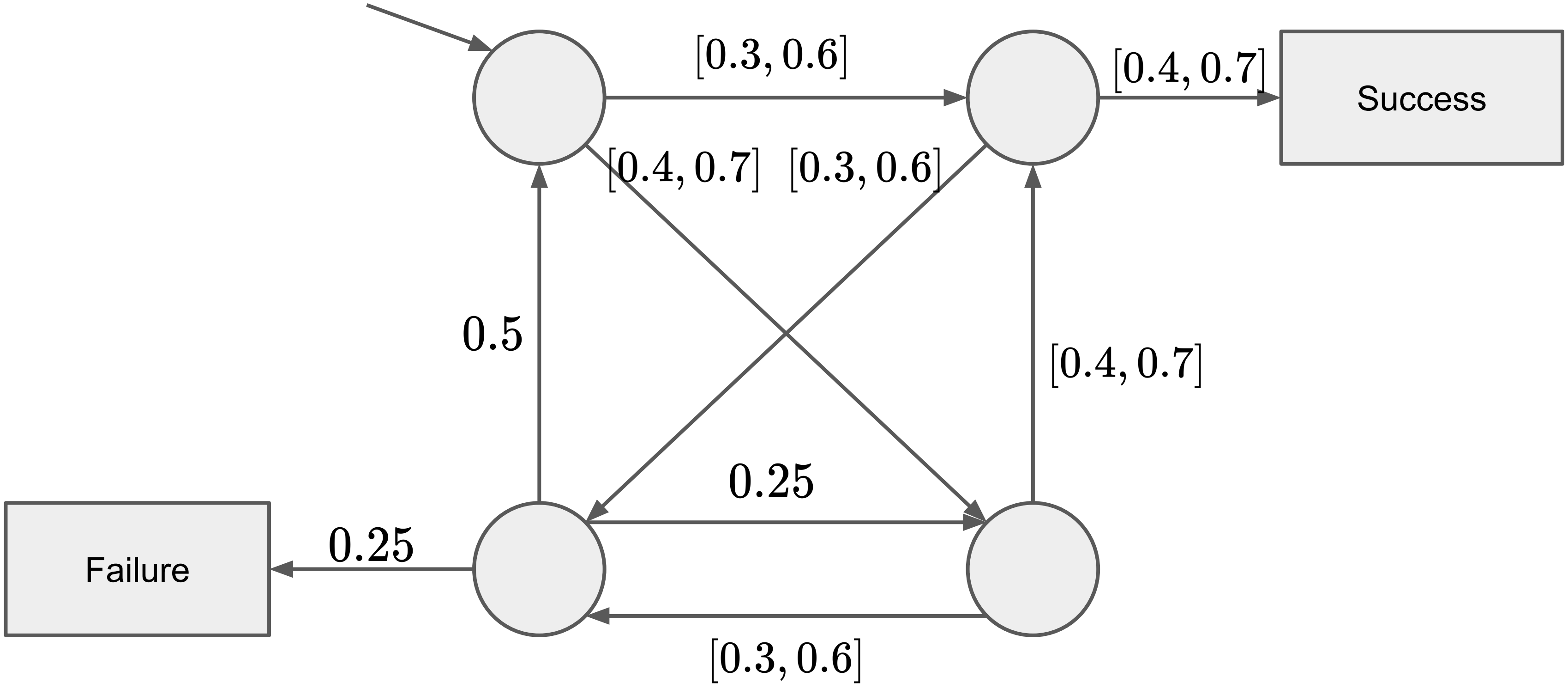
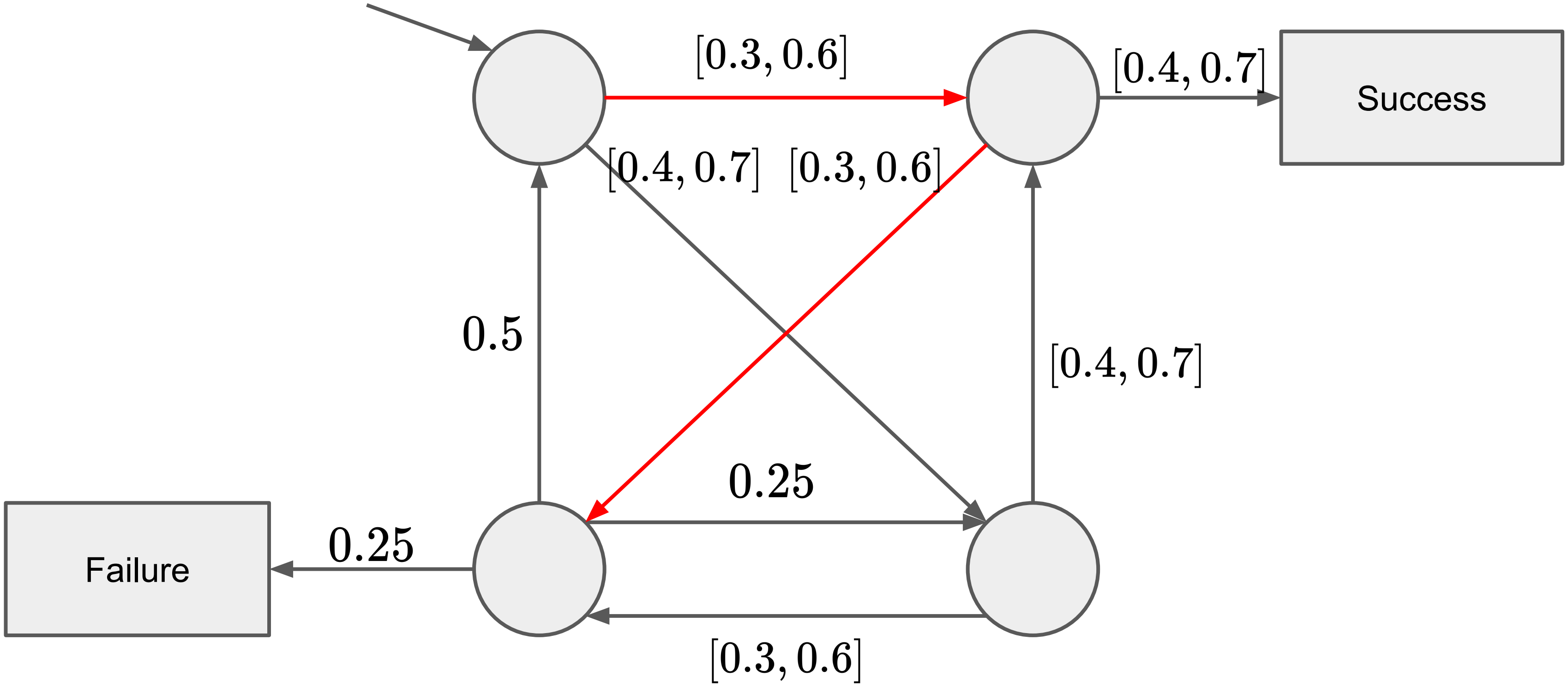
Reachability in Augmented Markov Chains

Ventsislav Chonev



[0.3, 0.6]
[0.4, 0.7]
Success
[0.4, 0.7]
[0.3, 0.6]
0.5
[0.4, 0.7]
0.25
0.25
Failure
[0.3, 0.6]



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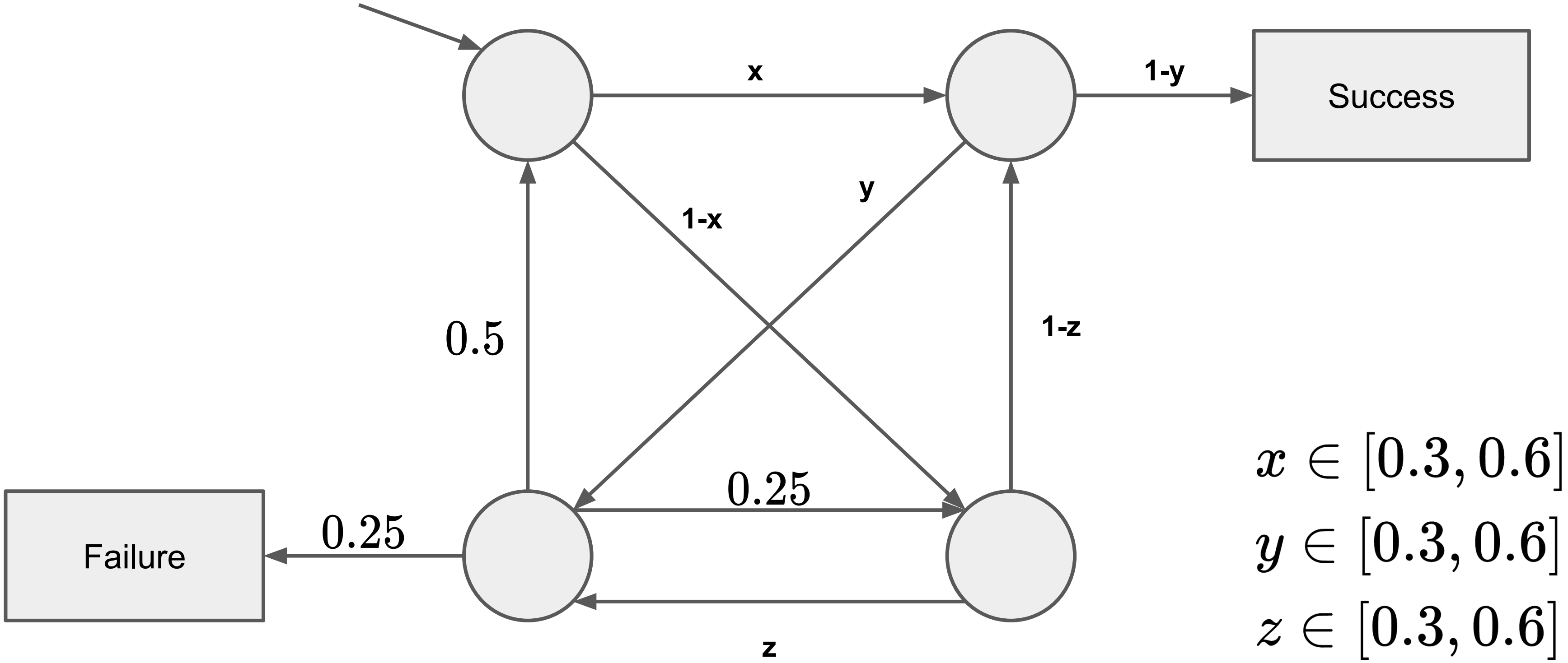
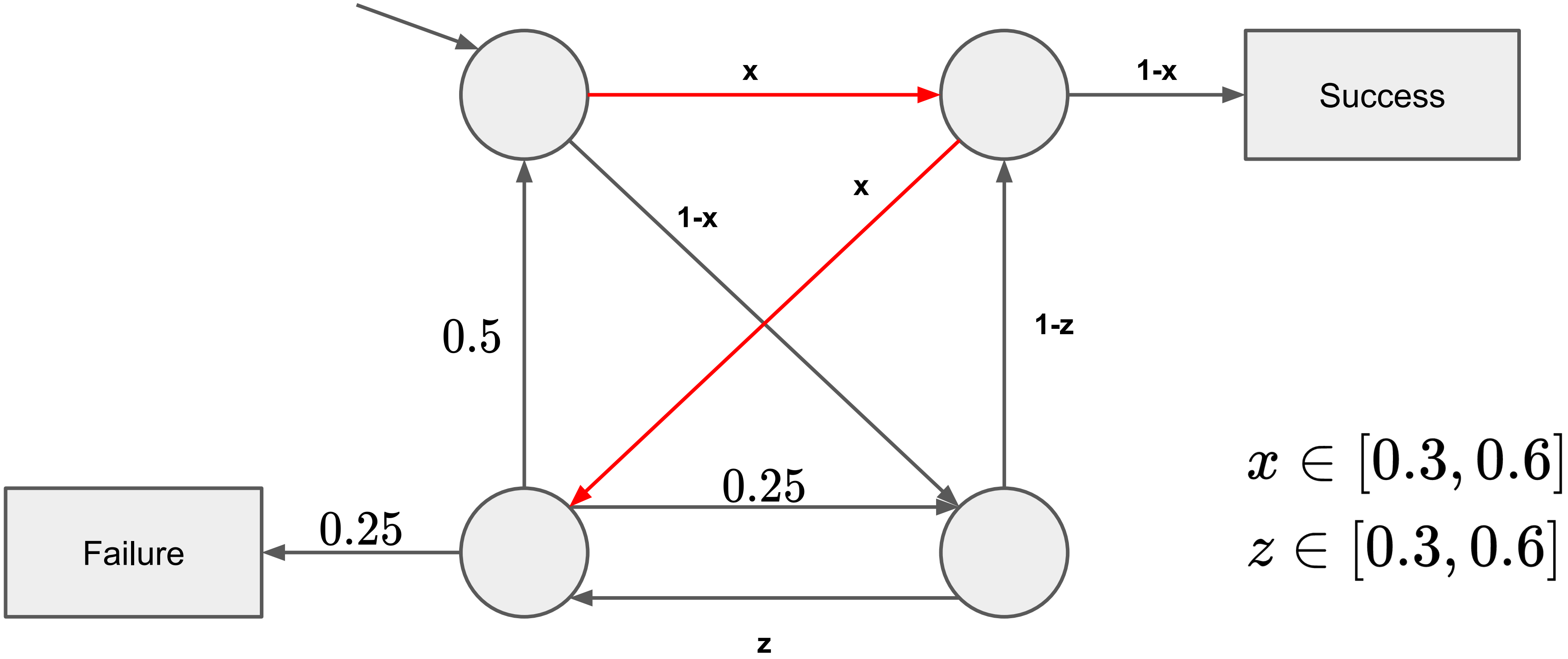

x
1-y
Success
y
1-x
0.5
1-z
0.25
0.25
Failure
z
$x \in [0.3, 0.6]$
$y \in [0.3, 0.6]$
$z \in [0.3, 0.6]$



x
1-x
Success
x
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0.5
1-z
0.25
0.25
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Augmented IMC = parametric Markov Chains = Markov chains with polynomials on edges

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Exact problem:

$$\exists \text{ refining MC} \,.\, \mathbb{P}(\text{ reach } t \text{ from } s \,) \sim \tau$$

Qualitative subproblem:

$$\text{As above, with } \tau \in \{0, 1\}.$$

Approximation problem:

$$\text{Assuming } |\mathbb{P}_{opt}(\text{ reach } t \text{ from } s \,) - \tau| \geq \epsilon,$$

$$\text{does there exist such a refining MC?}$$

all in $\exists\mathbb{R}$

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SQRT-hard

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NP-complete

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NP

with known structure

SQRT : given $a_1, \ldots, a_{n+1}$, decide $\sum_{i=1}^{n} \sqrt{a_i} \geq a_{n+1}$.

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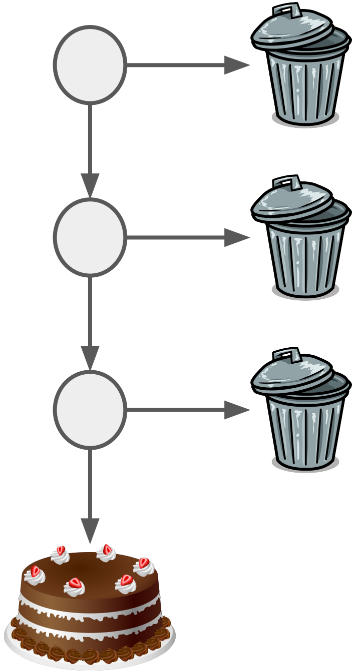

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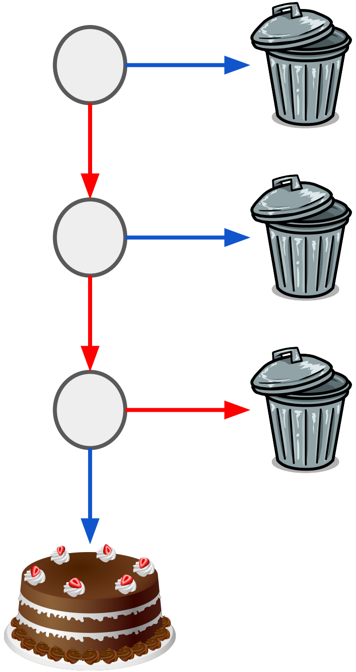

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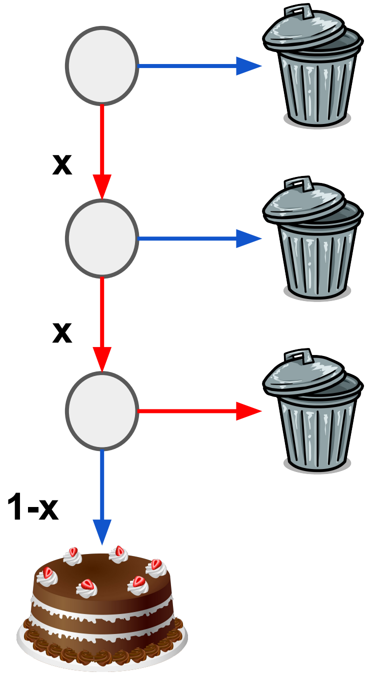
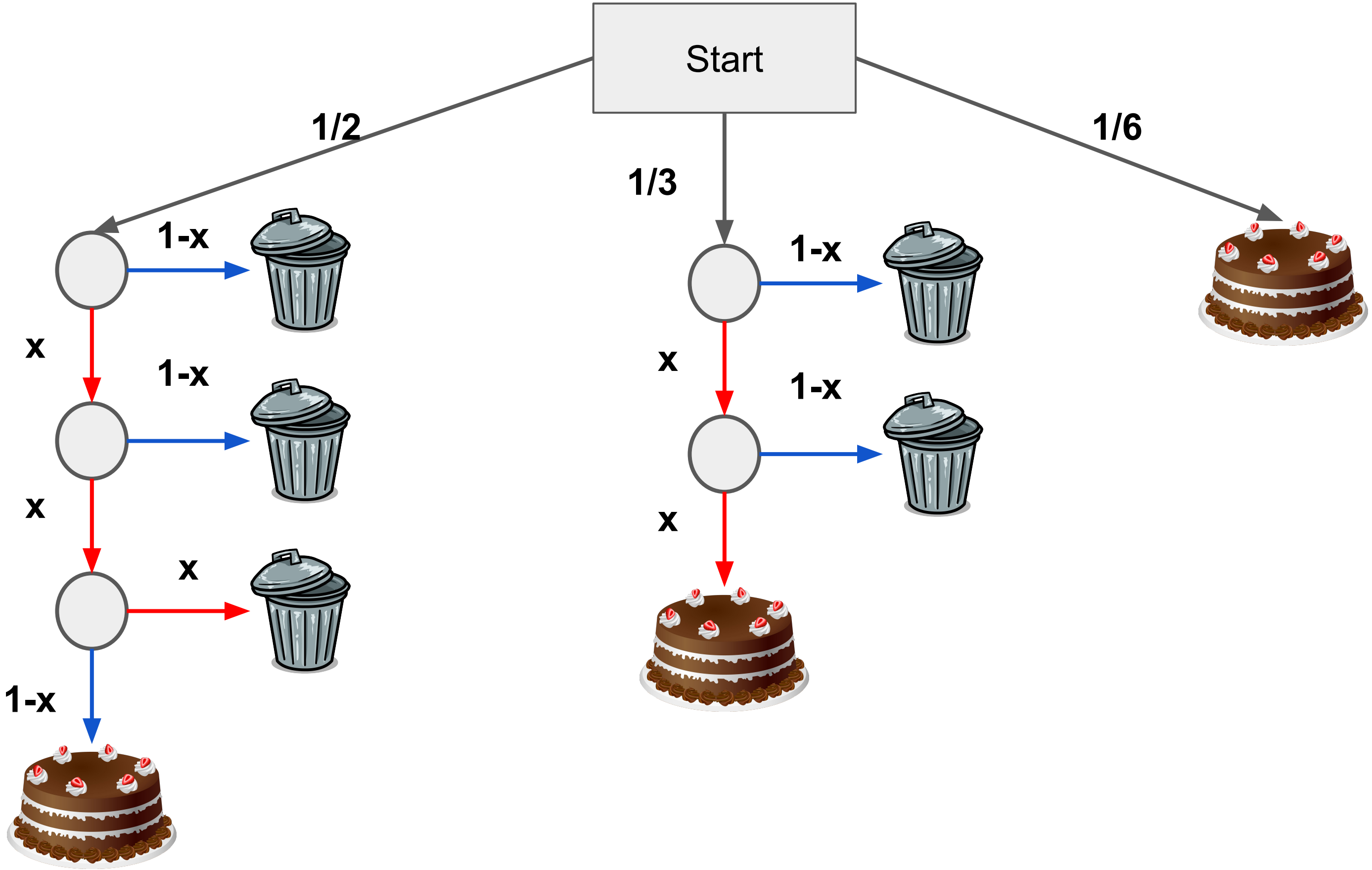


$x^2(1-x)$

$\mathbf{SQRT}$: given $a_1, \ldots, a_{n+1}$, decide $\sum_{i=1}^{n} \sqrt{a_i} \geq a_{n+1}$.



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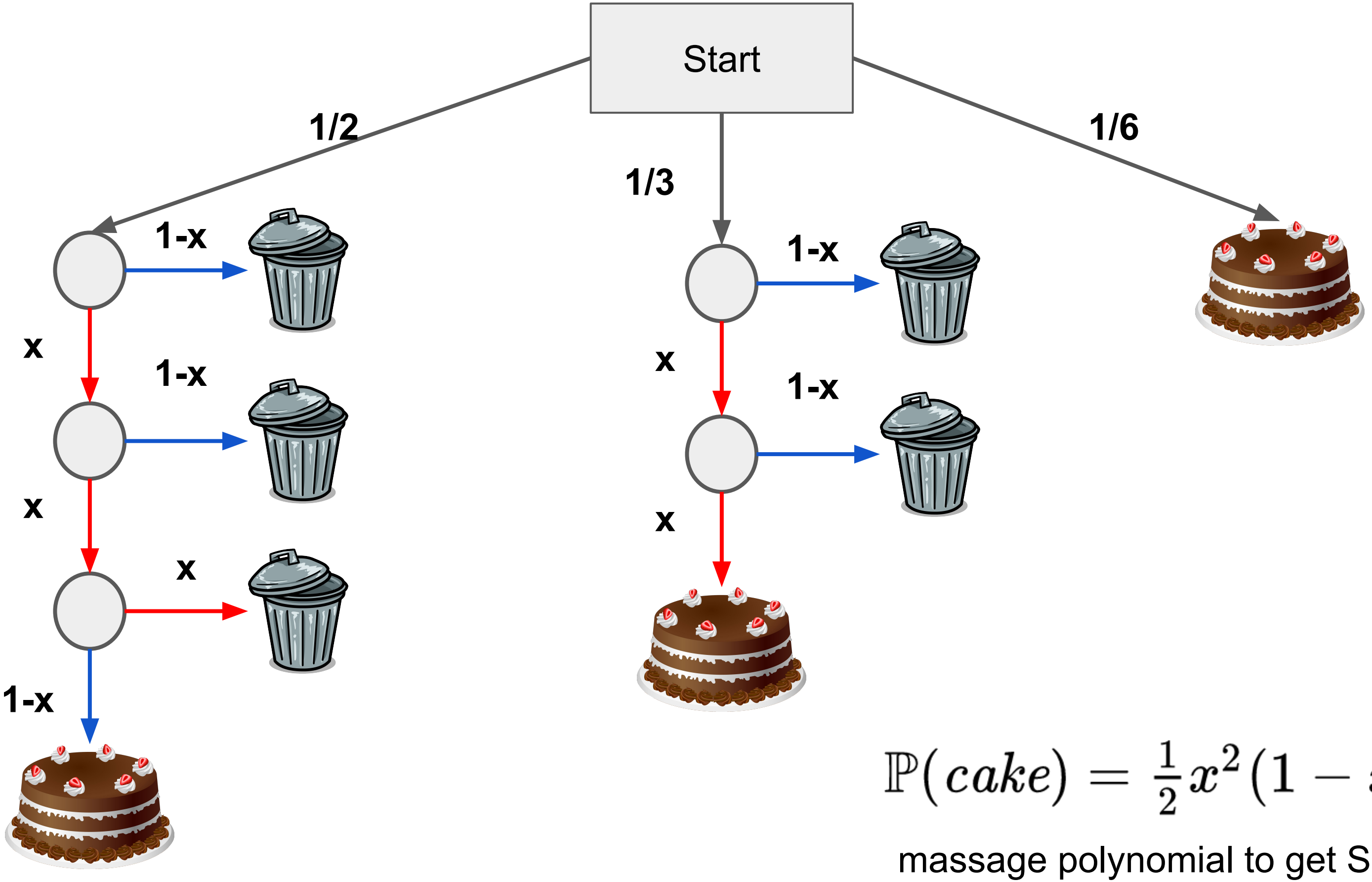

$$\mathbb{P}(cake) = \frac{1}{2}x^2(1-x) + \frac{1}{3}x^2 + \frac{1}{6}$$

massage polynomial to get SQRT-hardness

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does there exist such a refining MC? with known structure

all in $\exists\mathbb{R}$