Partial Order Reduction for Reachability Games

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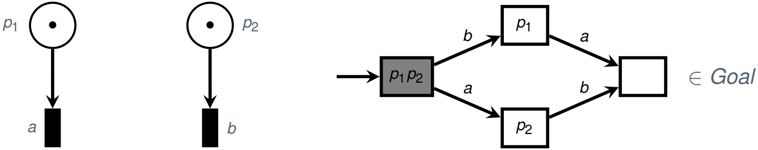
Overview

- (Generalized) Partial Order Reductions for games.
- Stable (Strategy Preserving) Reductions.
  - Stable reductions for Petri net games.
- Experiments
- Conclusion

Setting

2-player Reachability games

- Pruning of redundant action interleavings.



Partial Order Reductions

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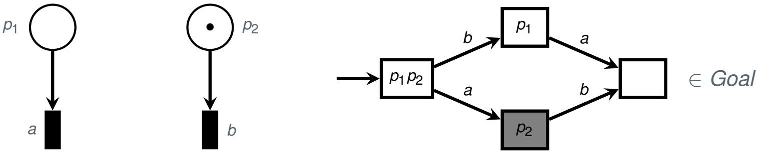

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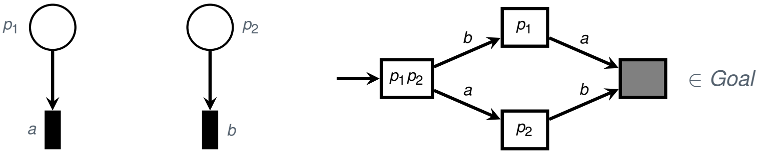

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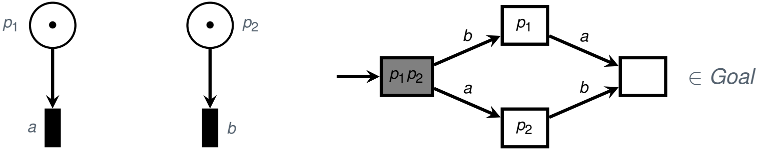

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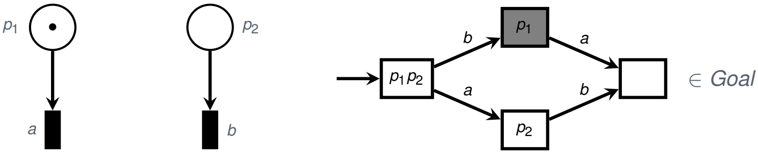

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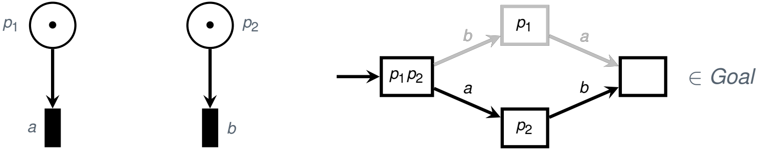

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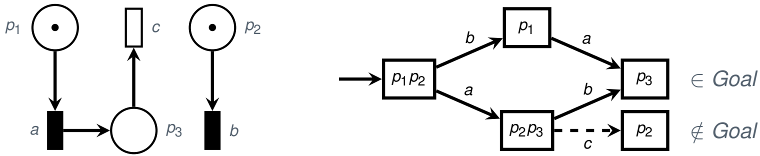

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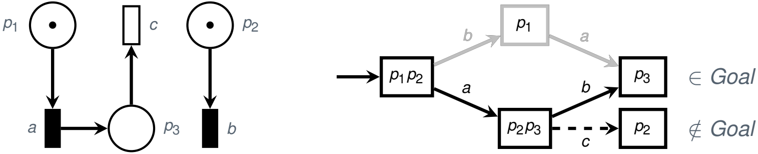
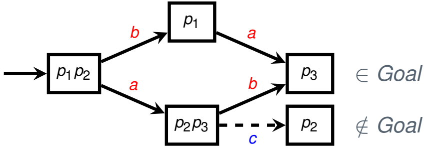


Preliminaries - Game Labelled Transition System

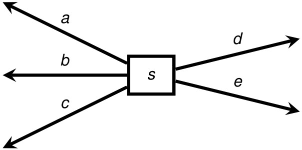
- Game Labelled Transition System (GLTS): $G = (\mathcal{S}, A_1, A_2, \rightarrow, Goal)$



- $en_1(s) = \{a \in A_1 \mid \exists s' \in \mathcal{S}.\ s \xrightarrow{a} s'\}$ for $s \in \mathcal{S}$.
- Similar definition for $en_2(s)$.
- **Example**: $en_1(p_2p_3) = \{b\}$ and $en_2(p_2p_3) = \{c\}$.
- $en(s) = en_1(s) \cup en_2(s)$.

Preliminaries - Reduction

- Game Labelled Transition System (GLTS): $G = (\mathcal{S}, A_1, A_2, \rightarrow, Goal)$
- Reduction: $St : \mathcal{S} \rightarrow 2^A$



- **Example**: $St(s) = \{c, e\}$
- Reduced GLTS: $G_{St} = (\mathcal{S}, A_1, A_2, \underset{St}{\longrightarrow}, Goal)$
  - $s \overset{a}{\underset{St}{\longrightarrow}} s'$ iff $s \overset{a}{\rightarrow} s'$ and $a \in St(s)$

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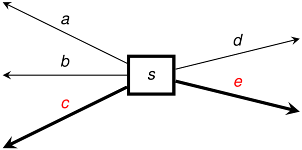

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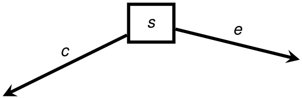

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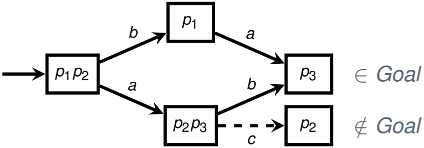
  - $s \xrightarrow[St]{a} s'$ iff $s \xrightarrow{a} s'$ and $a \in St(s)$

Strategy: $\sigma : \mathcal{S} \to A_1 \cup \{\perp\}$

- $\sigma(s) = \perp$ implies $en_1(s) = \emptyset$



Example

$\sigma(p_1 p_2) = b$, $\sigma(p_1) = a$ and $\sigma(p_3) = \perp$.

Preliminaries - Winning Strategies

- Paths: $\pi = s_0 s_1 \cdots$ and $i \in \mathbb{N}^0$
- Maximal runs from a state s subject to a strategy: $\Pi^{max}_{G,\sigma}(s)$

Winning Strategies

σ is winning from s, iff

- for all $\pi \in \Pi^{max}_{G,\sigma}(s)$ there exists a position i s.t. $\pi_i \in \mathit{Goal}$

Reachability Conditions

Purpose

Iff $s \in S$ is winning in G then s is winning in G_{st}

Conditions

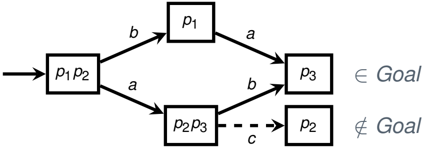
- **I** : No reduction for mixed states
- **W** : Non-stubborn transitions commute
- **R** : (Sufficient) Preservation of reachability
- **G1** and **G2** : Preservation of paths to mixed states
- **S** : Safety from mixed states
- **C** : Preservation of $P2$-cycles
- **D** : Preservation of Deadlocks

Theorem

If St *satisfies* ***I*****,** ***W*****,** ***R*****,** ***G1*****,** ***G2*****,** ***S*****,** ***C*****,** ***D*** *then for all* $s \in \mathcal{S}$ *state s is winning for the controller in G iff state s is winning for the controller in* G_{St}*.*

Reachability Conditions - Mixed States

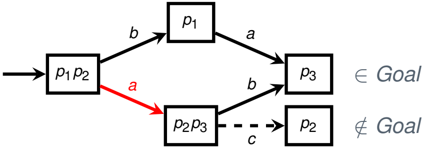
- **I**: If s is a mixed state then $en(s) \subseteq St(s)$.



- **Example**: State $p_2 p_3$ is a mixed state.

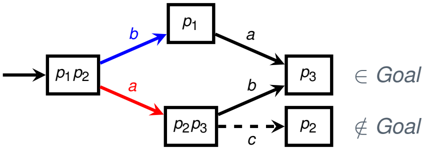
Reachability Conditions - Commutativity

- **W**: For all $w \in \overline{St(s)}^*$ and all $a \in St(s)$ if $s \xrightarrow{wa} s'$ then $s \xrightarrow{aw} s'$.



- **Example**: $St(p_1 p_2) = \{a\}$ is sufficient to satisfy **W**.

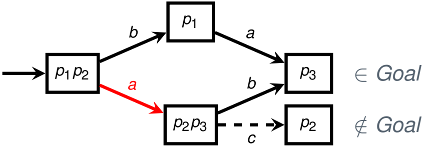
- **R**: $A_s(Goal) \subseteq St(s)$ for some interesting set of actions $A_s(Goal)$.



- If $s \notin Goal$, $w = a_1 \cdots a_n$, $s \xrightarrow{w} s'$, and $s' \in Goal$ then there exists $1 \leq i \leq n$ s.t. $a_i \in A_s(Goal)$.
- **Example**: Either $A_{p_1 p_2}(\{p_3\}) = \{a\}$ or $A_{p_1 p_2}(\{p_3\}) = \{b\}$ are both viable interesting sets.

Reachability Conditions - Game 1

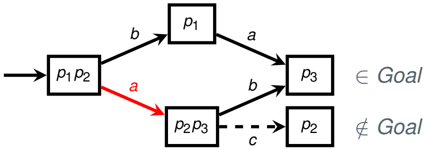
- **G1**: If $en_2(s) = \emptyset$, then for all $w \in \overline{St(s)}^*$ where $s \xrightarrow{w} s'$ then $en_2(s') = \emptyset$.



- Paths to mixed or environment states are preserved.
- **G2** is symmetric.

Reachability Conditions - Safe

- **S**: $en_1(s) \cap St(s) \subseteq safe(s)$ or $en_1(s) \subseteq St(s)$.



- $a \in A_1$ is safe in s if whenever $w \in (A_1 \setminus \{a\})^*$ and $s \xrightarrow{w} s'$ and $en_2(s') = \emptyset$ and $s \xrightarrow{aw} s''$ then $en_2(s'') = \emptyset$.
- Actions shifted to the front due to **W** may never lead to mixed or environment states.

Reachability Conditions - Deadlock & Cycle

- **C**: For all $a \in A_2$ if there exists $w \in A_2^{\omega}$ s.t. $s \xrightarrow{w}$ and a occurs infinitely often in w then $a \in St(s)$.
  - In order to preserve infinite paths of environment actions in the reduced GLTS.
- **D**: If $en_2(s) \neq \emptyset$ then there exists $a \in en_2(s) \cap St(s)$ s.t. for all $w \in \overline{St(s)}^{*}$ where $s \xrightarrow{w} s'$ we have $a \in en_2(s')$.
  - In order to preserve deadlocks in the reduced GLTS.

Instationation to Petri Net Games

Mixed states

- If $en_1(M) \neq \emptyset$ and $en_2(M) \neq \emptyset$ then $en(M) \subseteq St(M)$.

Transition Commutativity

- **W**: For all $w \in \overline{St(M)}^*$ and all $t \in St(M)$ if $M \xrightarrow{wt} M'$ then $M \xrightarrow{tw} M'$.

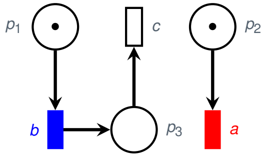
Petri Net Games - Preserving Environment Cycles

Conditions G1 and G2

If $en_2(M) = \emptyset$ then $T_2 \subseteq St(M)$.
Symmetric for Player 2.

Condition R

- If goals are all markings where there is 0 tokens in p_2 then all the transitions that decrease the number of tokens in p_2 are interesting.
- $\{a\}$ is a sufficient interesting set for this goal.



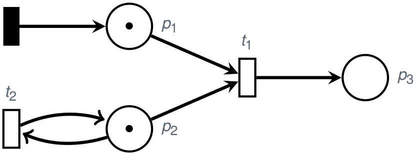
Condition S

Lemma

If $t^+ \cap {}^\bullet T_2 = \emptyset$ and ${}^-t \cap {}^\circ T_2 = \emptyset$ then t is safe in any marking.

- b is *not* safe since it contributes tokens to p_3.

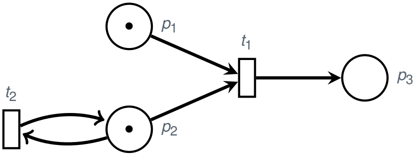
Condition C



- $\mathit{Finite} = \{\}$
- $\mathit{Marked} = \{\}$

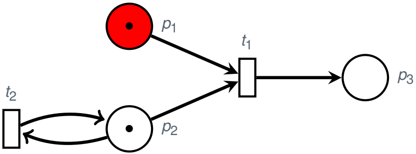
Petri Net Games - Preserving Environment Cycles

Condition C



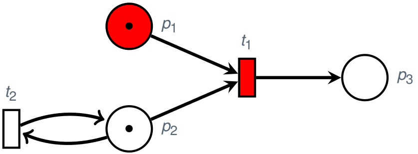
- *Finite* $= \{\}$
- *Marked* $= \{\}$

Condition C



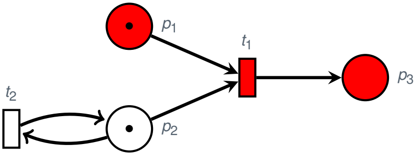
- $\mathit{Finite} = \{p_1\}$
- $\mathit{Marked} = \{\}$

Condition C



- *Finite* $= \{p_1, t_1\}$
- *Marked* $= \{\}$

Condition C



- $Finite = \{p_1, t_1, p_3\}$
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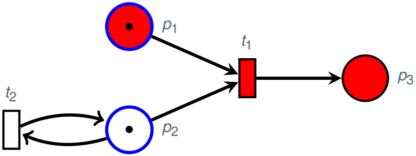
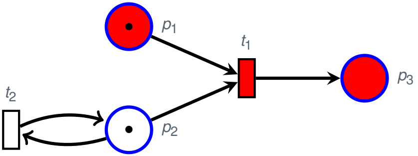
Condition C



- *Finite* $= \{p_1, t_1, p_3\}$
- *Marked* $= \{p_1, p_2\}$

Condition C



- $Finite = \{p_1, t_1, p_3\}$
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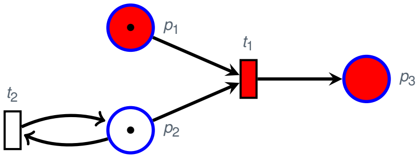
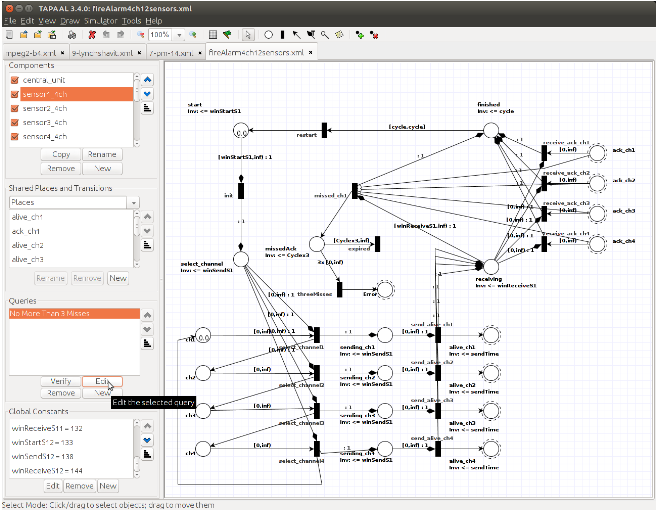


- $Finite = \{p_1, t_1, p_3\}$
- $Marked = \{p_1, p_2, p_3\}$

- A transition t may occur infinitely often if $t \notin Finite$ and all its pre-places can be marked ${}^\bullet t \subseteq Marked$.
- If t can occur infinitly often, $t \in St(s)$.

Implementation & TAPAAL

- Implemented in the TAPAAL verification tool suite.



Models

- (MW) Manufacturing Workflow.
  - Scaled on the number of requested features.
- (OW) Order Workflow.
  - Scaled on re-initialising.
- (NIM) Nim.
  - Scaled on the number of allowed pebbles and the allowable amount to add each round.
- (PCP) Producer Consumer System.
  - Scaled on the number of producers and consumers.
- (AIM) Autonomous Intersection Management.
  - Scaled on the number of cars, intersections, lane length, and speeds.
- (Lyngby) Railway Scheduling Problem.
  - Scaled on the number of moving trains.

Experiments

	Time (seconds)		Markings ×1000		Reduction	
Model	NORMAL	POR	NORMAL	POR	%T	%M
MW-40	735.2	0.2	69439	9	100	100
MW-50	1952.0	0.2	135697	11	100	100
MW-60	4417.0	0.3	234570	13	100	100
OW-10000	0.9	0.7	320	240	22	25
OW-100000	11.1	7.8	3200	2400	30	25
OW-1000000	137.7	109.8	32000	24000	20	25
NIM-5-49500	9.2	3.4	5054	892	63	82
NIM-7-49500	32.7	3.9	24039	1159	88	95
NIM-9-49500	165.1	4.7	114235	1522	97	99
NIM-11-49500	710.7	8.2	533516	1877	99	100

	Time (seconds)		Markings $\times 1000$		Reduction	
Model	NORMAL	POR	NORMAL	POR	%T	%M
PCS-2-2	24.0	19.9	9629	6554	17	32
PCS-2-3	116.1	90.9	61990	39114	22	37
PCS-2-4	399.1	283.3	240510	145109	29	40
AIM-13-100-6-11	117.9	46.6	1702	514	60	70
AIM-13-100-6-16	173.9	63.2	2464	746	64	70
AIM-13-150-9-16	337.0	219.9	3696	2454	35	34
AIM-13-150-9-21	408.0	294.1	4853	3331	28	31
AIM-14-150-9-16	449.9	278.4	4259	2865	38	33
AIM-15-150-9-16	534.5	384.9	4861	3204	28	34
LyngbySmall-2	3.4	1.0	1803	444	71	75
LyngbySmall-3	23.1	26.1	11473	10701	-13	7
LyngbySmall-4	144.3	193.3	65371	70008	-34	-7
Lyngby-2	3292.0	215.5	1511749	87214	93	94

Conclusion

- Partial Order Reduction for Games
  - Stable (Strategy Preserving)
- Reductions for both players
- Implemented in TAPAAL
- Encouraging experimental results

Future Work

- Timed Games?

Related Work

Start Pruning When Time Gets Urgent:
Partial Order Reduction for Timed Systems (CAV'18)

F. M. Bønneland, P. G. Jensen, K. G. Larsen, M. Muñiz and J. Srba