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Reachability Problems on (Partially Lossy) Queue Automata

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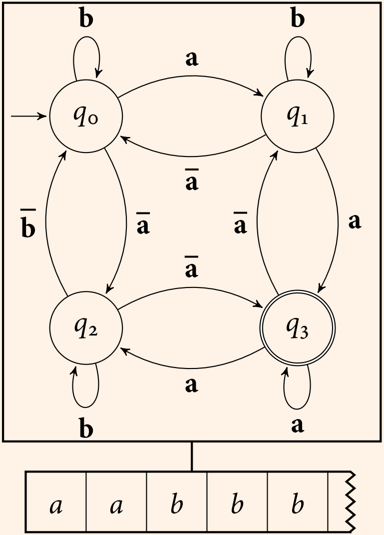
Queue Automata

- Let A be an alphabet.
- Two actions for each $a \in A$:
  - write letter $a \rightsquigarrow \mathbf{a}$
  - read letter $a \rightsquigarrow \overline{\mathbf{a}}$
- $\mathbf{A} := \{\mathbf{a} \mid a \in A\}$, $\overline{\mathbf{A}} := \{\overline{\mathbf{a}} \mid a \in A\}$
- $\Sigma := \mathbf{A} \uplus \overline{\mathbf{A}}$

Example



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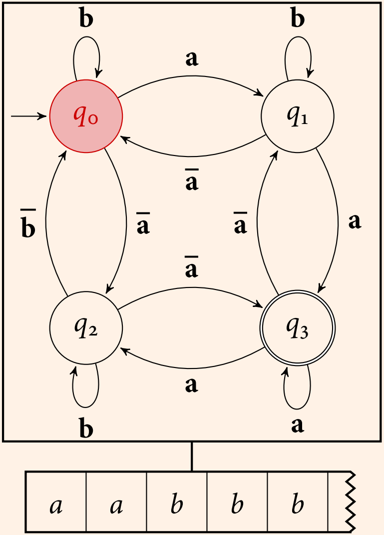

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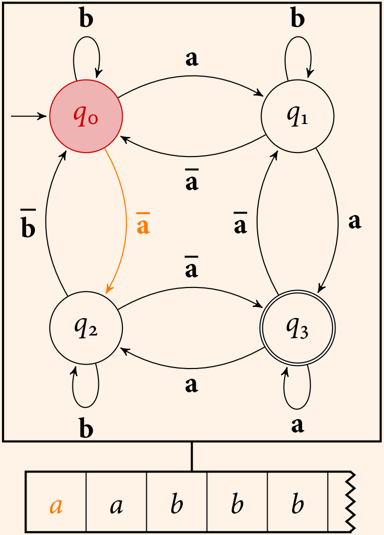

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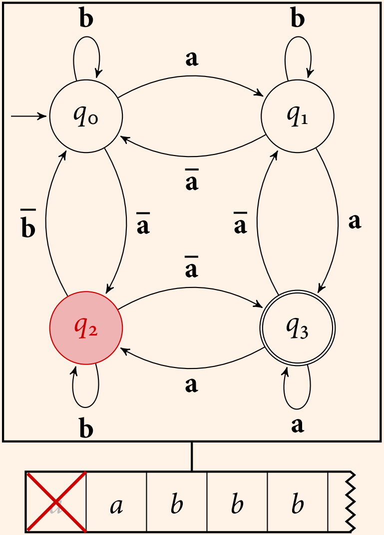

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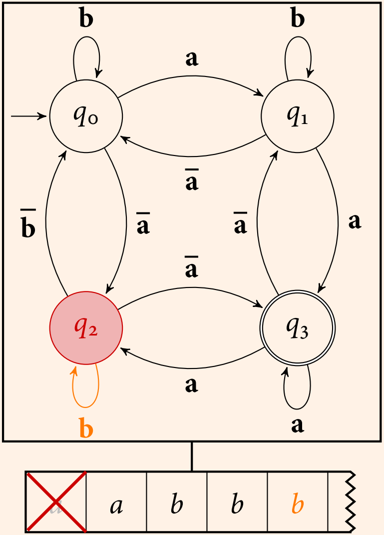

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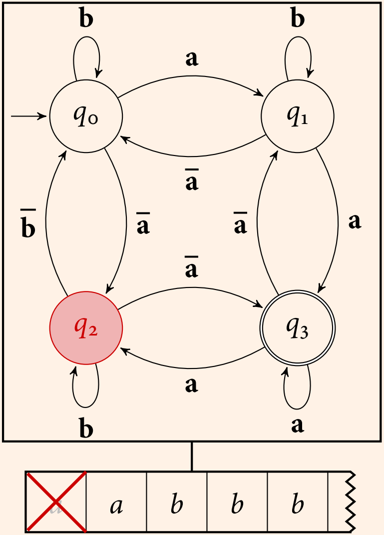

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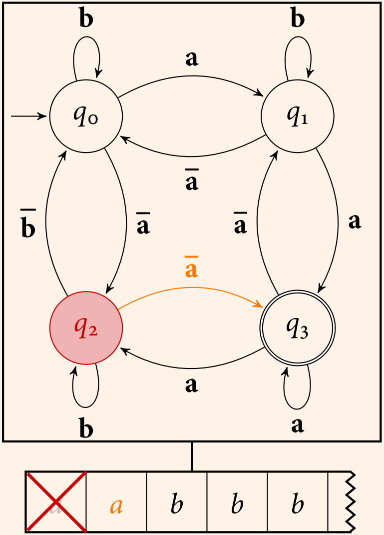

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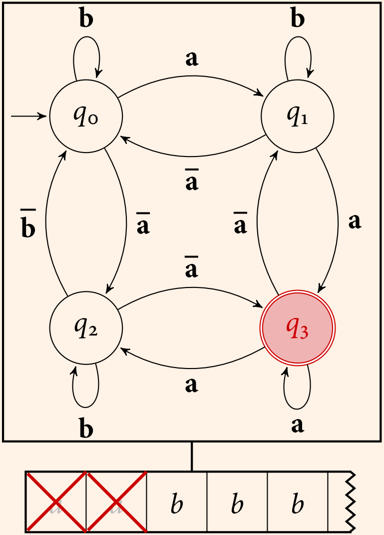

Inputs:

- $\mathbf{T} \subseteq \Sigma^*$ regular language of transformation sequences
- $L \subseteq A^*$ regular language of queue contents

Compute:

- $\textsc{Reach}(L, \mathbf{T})$:= the set of all queue contents after application of $\mathbf{T}$ on L

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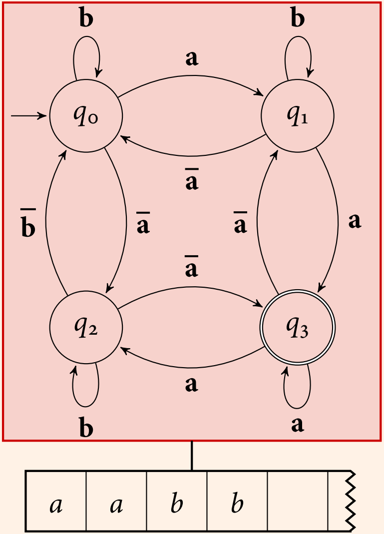

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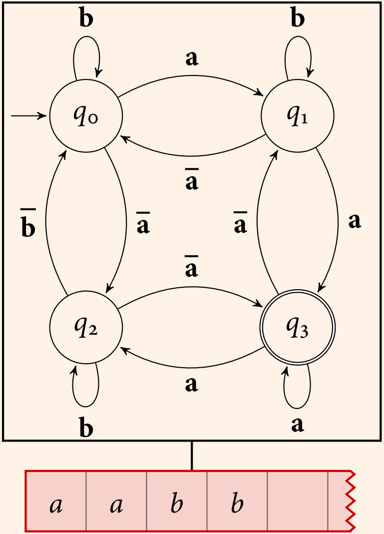

Turing-Completeness

Theorem (Brand, Zafiropulo 1983)

Queue Automata can simulate Turing-machines.

- $\textsc{Reach}(L, \mathbf{T})$ can be any recursively enumerable language
- holds already for some fixed $\mathbf{T} = \{\mathbf{t_1}, \ldots, \mathbf{t_n}\}^*$ with $\mathbf{t_1}, \ldots, \mathbf{t_n} \in \mathbf{\Sigma^*}$

Approximations of the Reachability Problem

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- Iterative approach: for $i = 0, 1, 2, \ldots$ do
  - compute the prefixes $\mathbf{T_i}$ of length i from $\mathbf{T}$
  - apply $\mathbf{T_i}$ on L
- Faster approach:

Theorem (Boigelot, Godefroid, Willems, Wolper 1997)

Let $L \subseteq A^*$ be regular and $\mathbf{t} \in \mathbf{\Sigma^*}$. Then $\textsc{Reach}(L, \mathbf{t^*})$ *is effectively regular.*

$\Rightarrow$ Combine multiple iterations of a loop to a meta-transformation

Aim

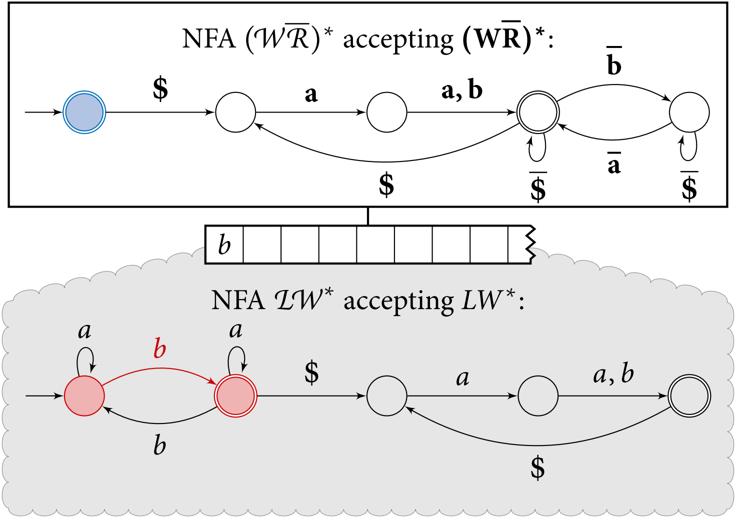
Generalize this result.

Theorem

Let $L, W, R \subseteq A^*$ be regular. Then $\textsc{Reach}(L, (\mathbf{W}\overline{\mathbf{R}})^*)$ *is effectively regular (in polynomial time).*

- We slightly modify W and R:
  - Let $\$ \notin A$ be some new letter.
  - Set $\mathbf{W'} := \mathbf{\$W}$ and $\overline{\mathbf{R'}} := \text{shuffle}(\overline{\mathbf{R}}, \overline{\mathbf{\$}}^*)$.
  - Easy: $\textsc{Reach}(L, (\mathbf{W}\overline{\mathbf{R}})^*) = \text{proj}_A(\textsc{Reach}(L, (\mathbf{W'}\overline{\mathbf{R'}})^*))$.
  - We prove that $\textsc{Reach}(L, (\mathbf{W'}\overline{\mathbf{R'}})^*)$ is regular.
- From now on, we write W and $\overline{R}$ instead of W' and $\overline{R'}$, resp.

- Consider the following example:



Proof Idea (1)

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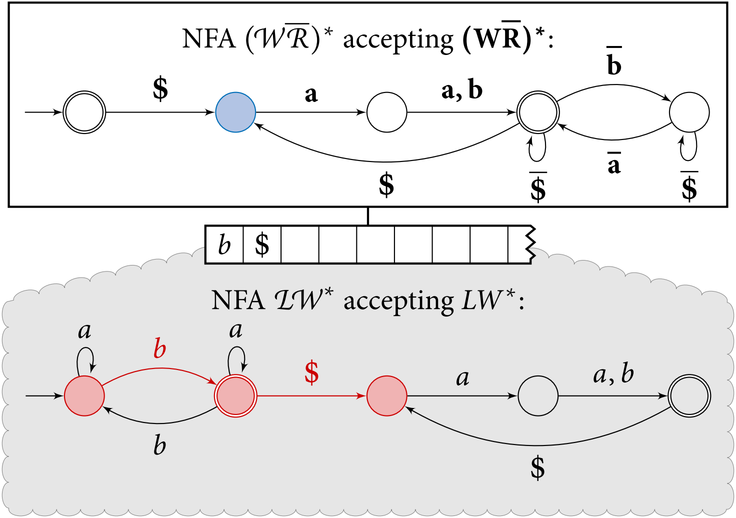

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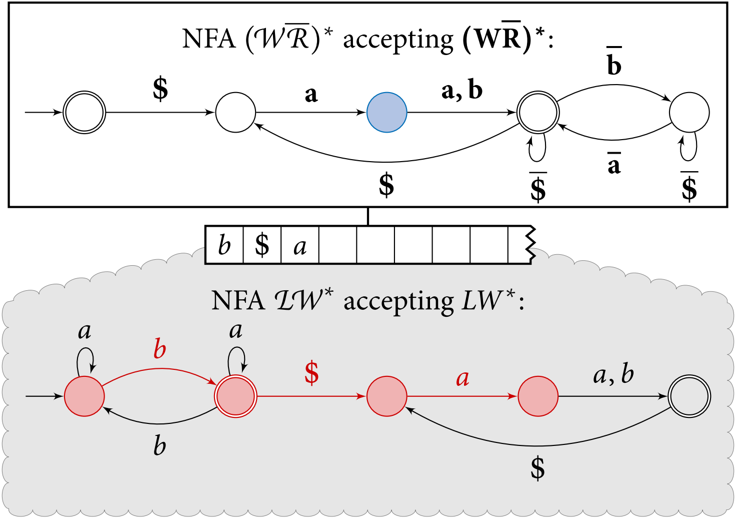

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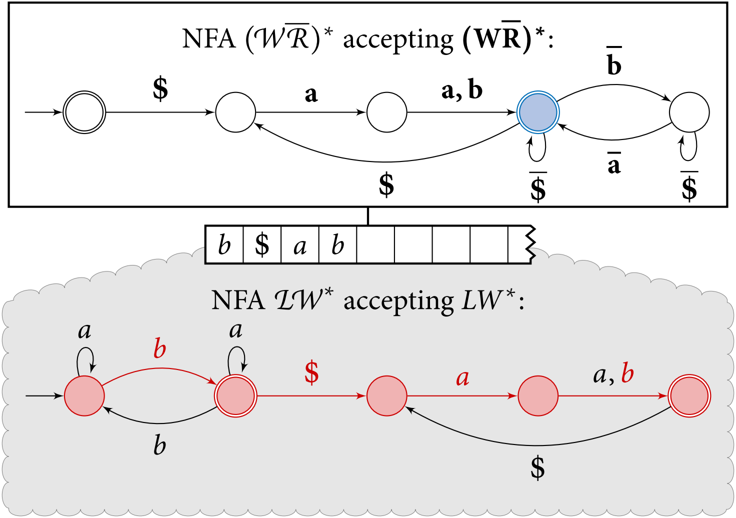

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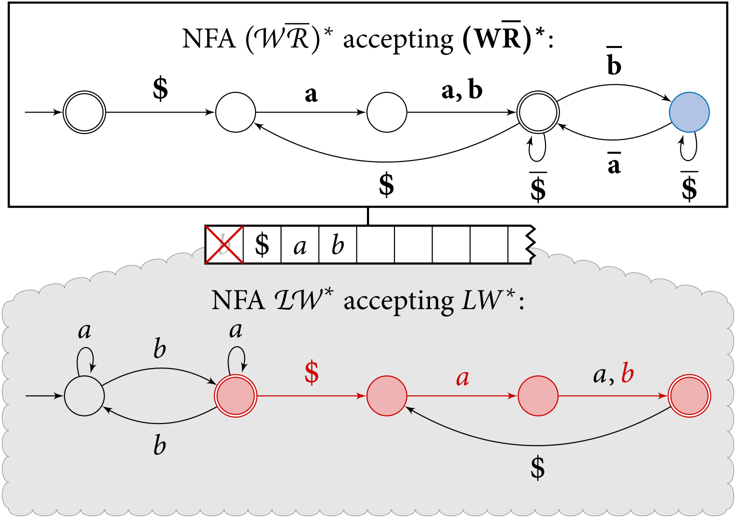

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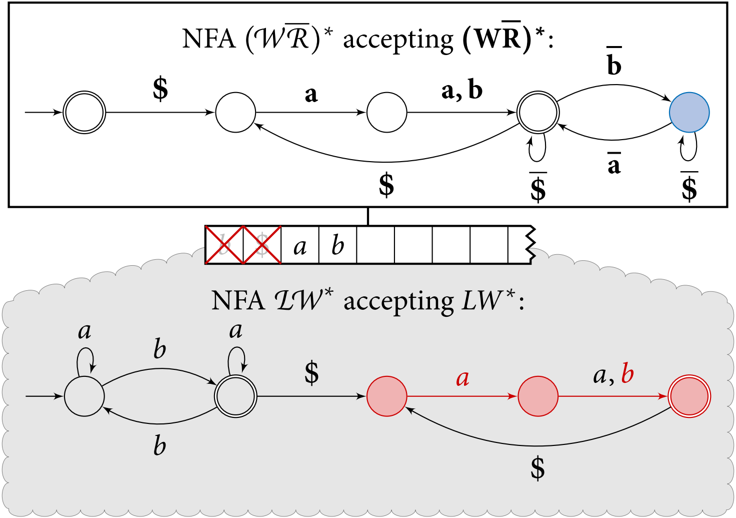

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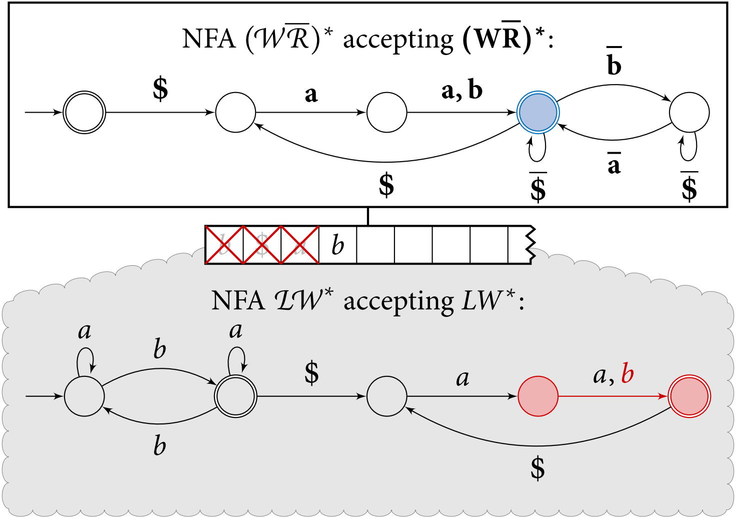

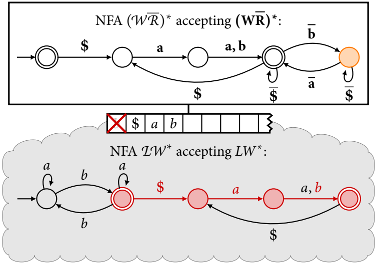

- A configuration of the queue automaton can be abstracted as follows:
  1. the current state in $(\mathcal{W}\overline{\mathcal{R}})^*$
  2. the starting state of the path in $\mathcal{LW}^*$
  3. the ending state of the path in $\mathcal{LW}^*$
  4. the number of \$s on the path

  (items 1–3: control state of $\mathcal{C}$; item 4: counter of $\mathcal{C}$)

⇒ The queue automaton can be simulated by a one-counter automaton $\mathcal{C}$

Proof Idea (2)

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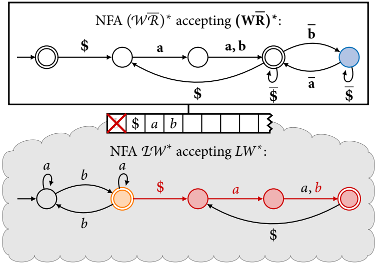

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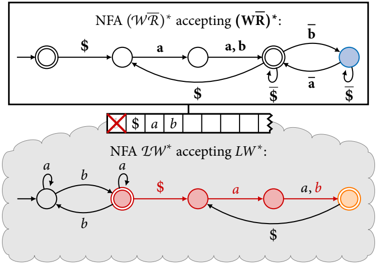

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$\Rightarrow$ The queue automaton can be simulated by a one-counter automaton C

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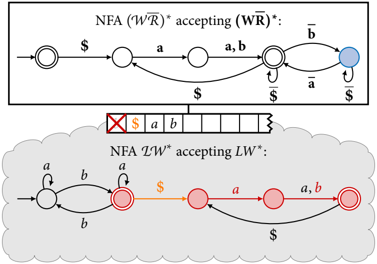

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Semantics of $\mathcal{C}$

- $\mathcal{C}$'s configurations consist of:
  1. the current state in $(\mathcal{W}\overline{\mathcal{R}})^*$
  2. the starting state of the path in $\mathcal{LW}^*$
  3. the ending state of the path in $\mathcal{LW}^*$
  4. the number of \$s on the path

  (1–3: control state of $\mathcal{C}$; 4: counter of $\mathcal{C}$)
- Let $(p, q, r, n) \in \mathrm{Conf}_{\mathcal{C}}$ be a configuration of $\mathcal{C}$.
- $[\![p, q, r, n]\!] := L(\mathcal{LW}^*_{q \to r}) \cap \mathrm{shuffle}(\$^n, A^*)$

Proposition

$$\textsc{Reach}(L, (\mathbf{W}\overline{\mathbf{R}})^*) = \bigcup_{\sigma \in \mathrm{Conf}_{\mathcal{C}},\ \textit{reach.} + \textit{acc.}} [\![\sigma]\!]\,,$$

i.e., $\textsc{Reach}(L, (\mathbf{W}\overline{\mathbf{R}})^*)$ *is a rational image of the set of reachable and accepting configurations of* $\mathcal{C}$.

Finishing the Proof

- Consider the set of reachable and accepting configurations of C.
- By [Bouajjani, Esparza, Maler 1997] this set is semilinear.
- Using a rational transduction implies effective regularity of $\textsc{Reach}(L, (\mathbf{W\overline{R}})^*)$. □

⇒ We have seen:

Theorem (Main Theorem)

Let $L, W, R \subseteq A^*$ be regular. Then $\textsc{Reach}(L, (\mathbf{W\overline{R}})^*)$ is effectively regular (in polynomial time).

Consequences

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Corollary

Let $L \subseteq A^*$ and $\mathbf{T} \subseteq \mathbf{\Sigma}^*$ be regular. Then $\textsc{Reach}(L, \mathbf{T}^*)$ is regular if

1. $\mathbf{T} = \overline{\mathbf{R_1}}\mathbf{W}\overline{\mathbf{R_2}}$ *for regular* $W, R_1, R_2 \subseteq A^*$,
2. $\mathbf{T} = \mathbf{W} \cup \overline{\mathbf{R}}$ *for regular* $W, R \subseteq A^*$,
3. $\mathbf{T} = \{\mathbf{t}\}$ *for* $\mathbf{t} \in \mathbf{\Sigma}^*$ *(cf. [Boigelot et al. 1997]), or*
4. $\mathbf{T} = \text{shuffle}(\mathbf{W}, \overline{\mathbf{R}})$ *for regular* $W, R \subseteq A^*$.

- Remark: Proofs of 3 and 4 use some result from [K. 2018, cf. STACS'18]

Thank you!