On the Computation of the Minimal Coverability Set of Petri Nets

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Minimal Coverability Set

$\mathrm{MCS}(\mathcal{N})$ is an accurate over-approximation of the reachability set of $\mathcal{N}$.

It allows to decide coverability, boundedness, place-boundedness, regularity...

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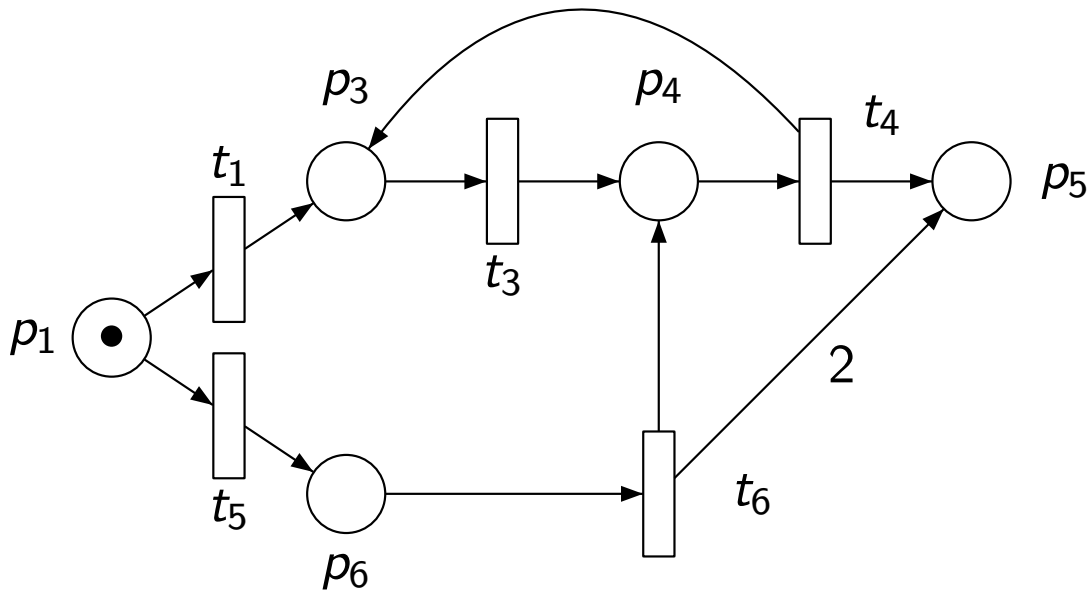

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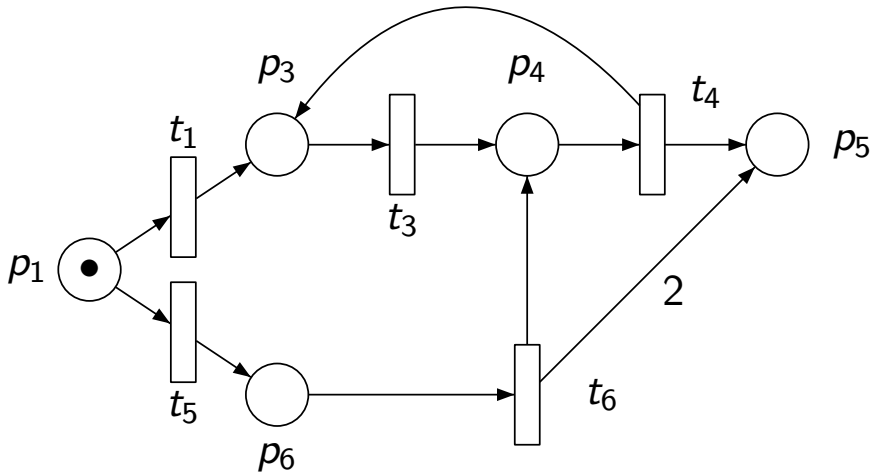
Karp & Miller Tree by example



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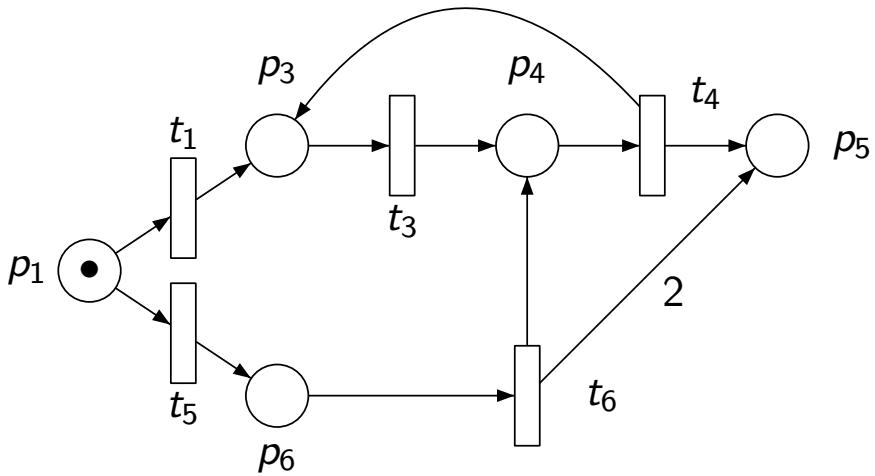

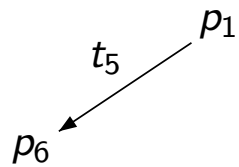

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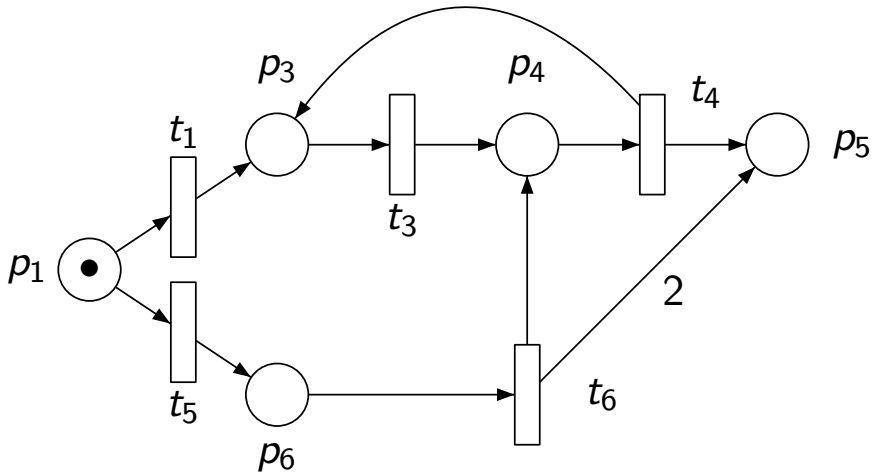

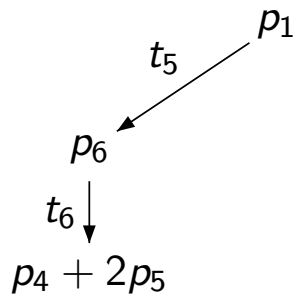

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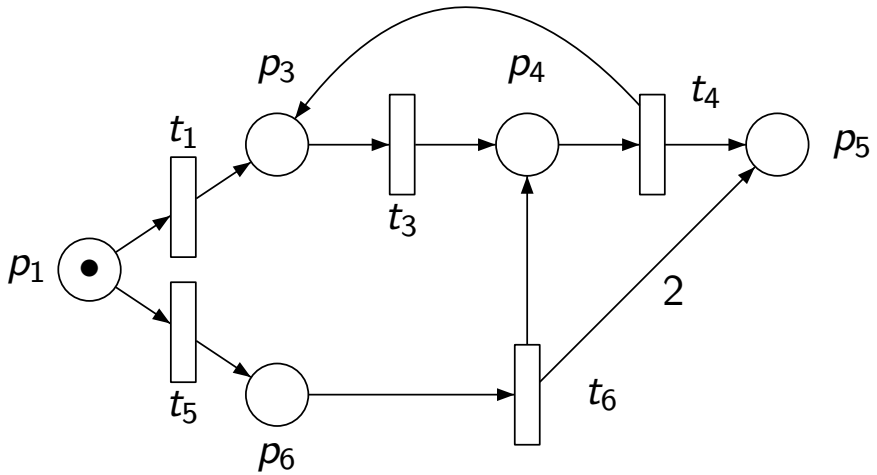

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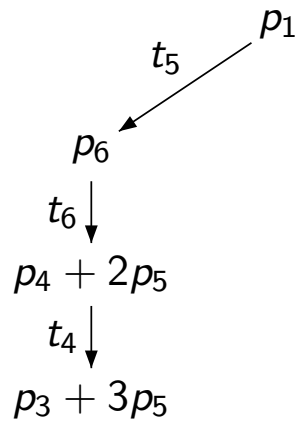

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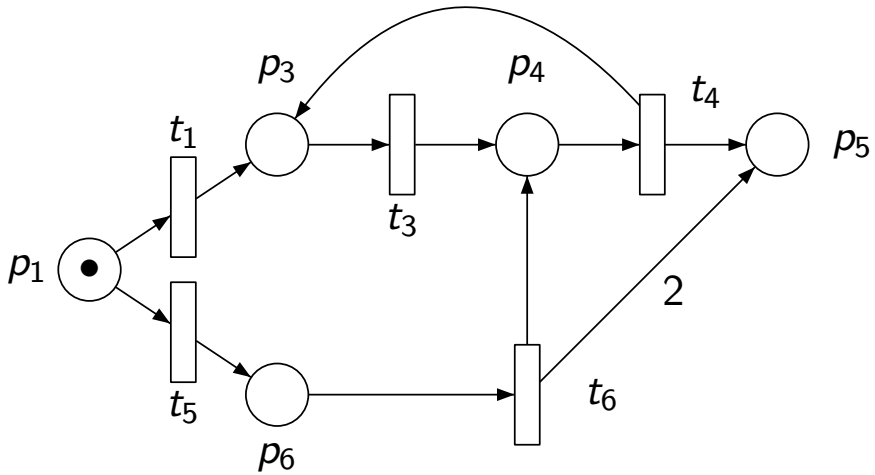

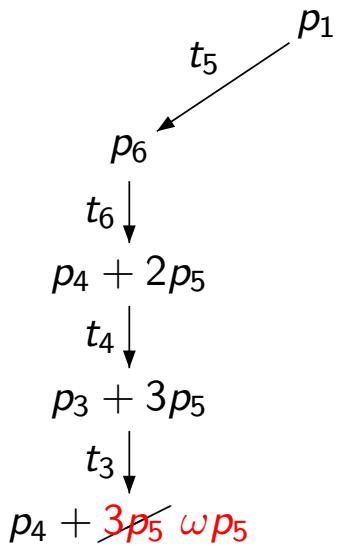

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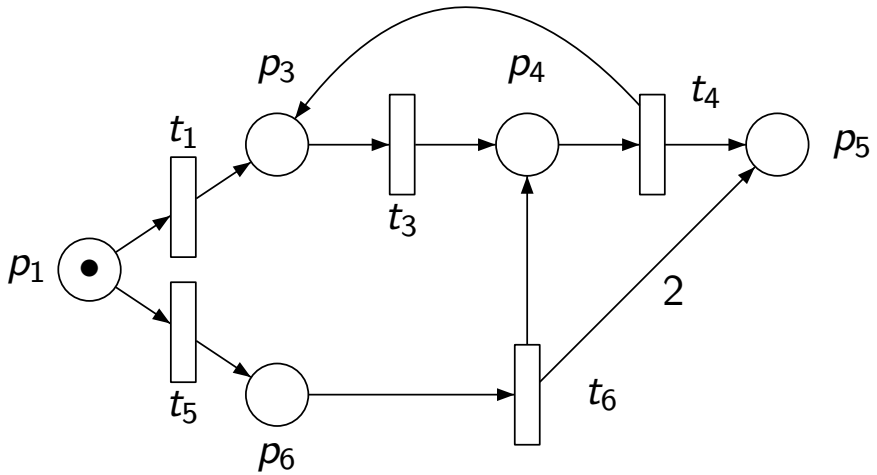

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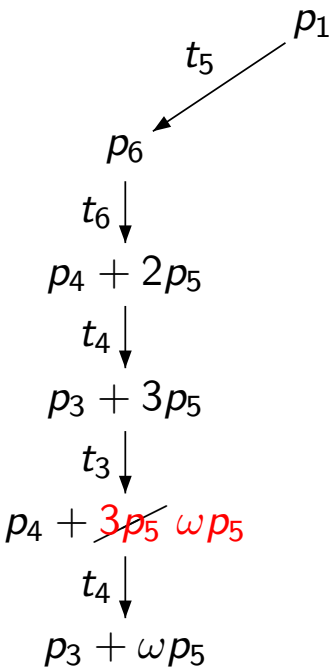

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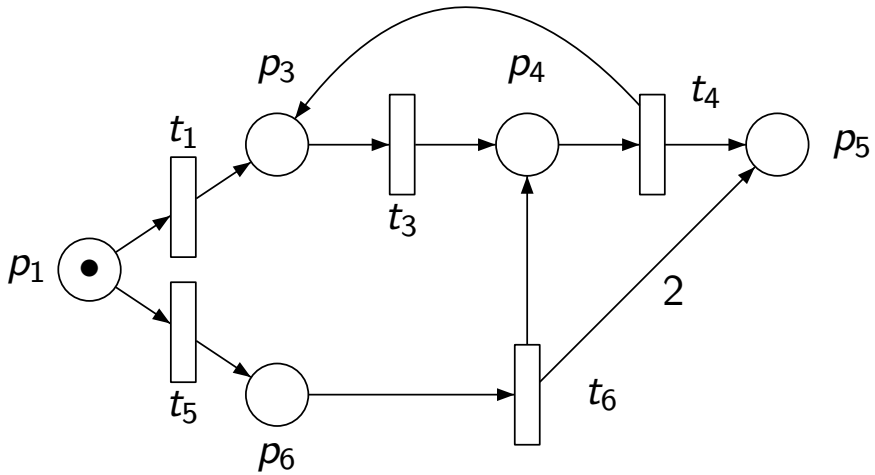

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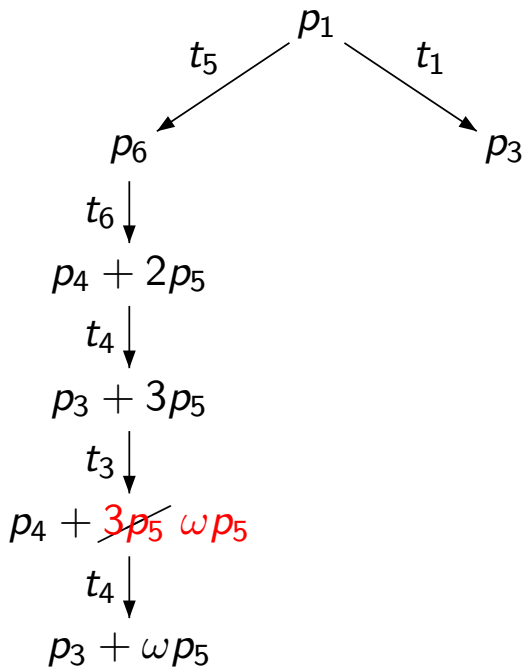

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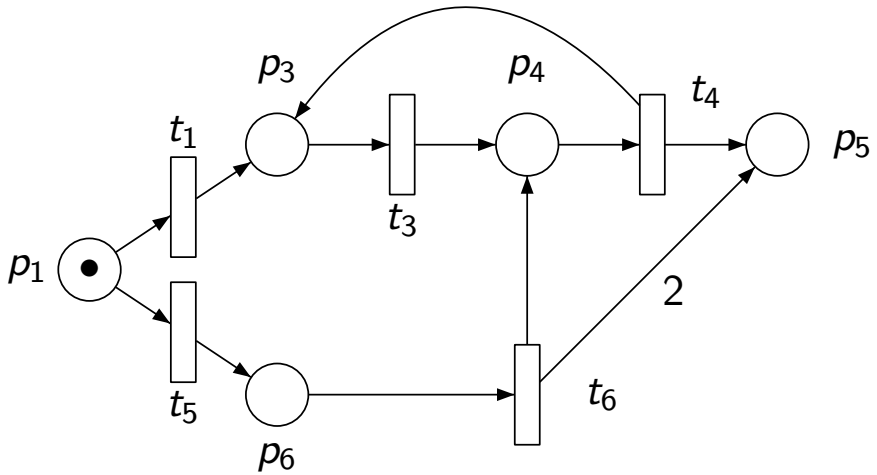

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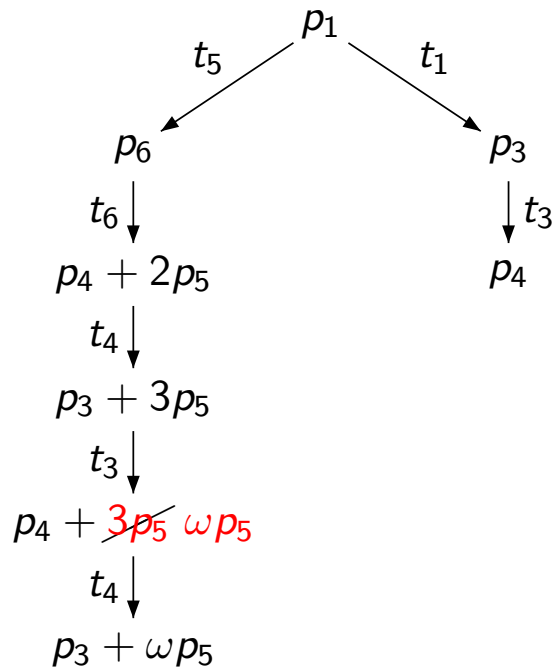

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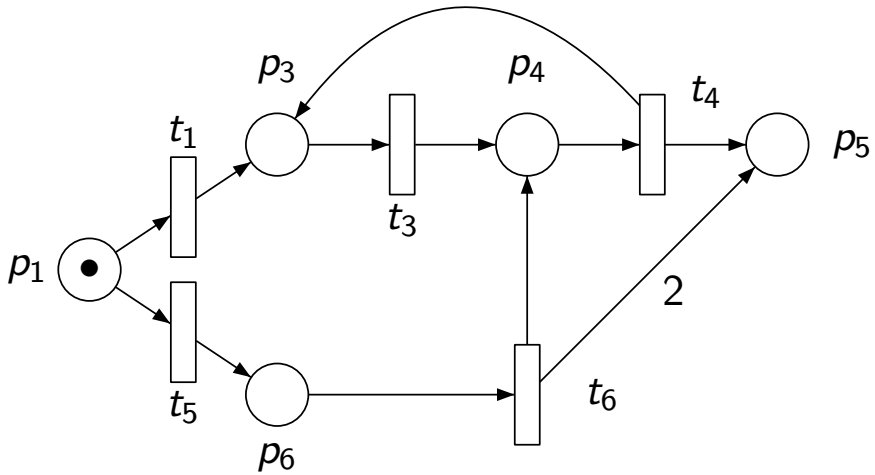

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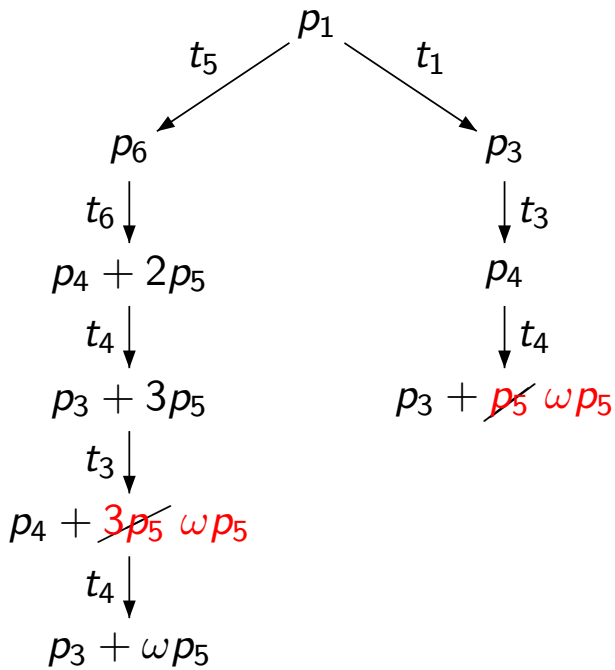

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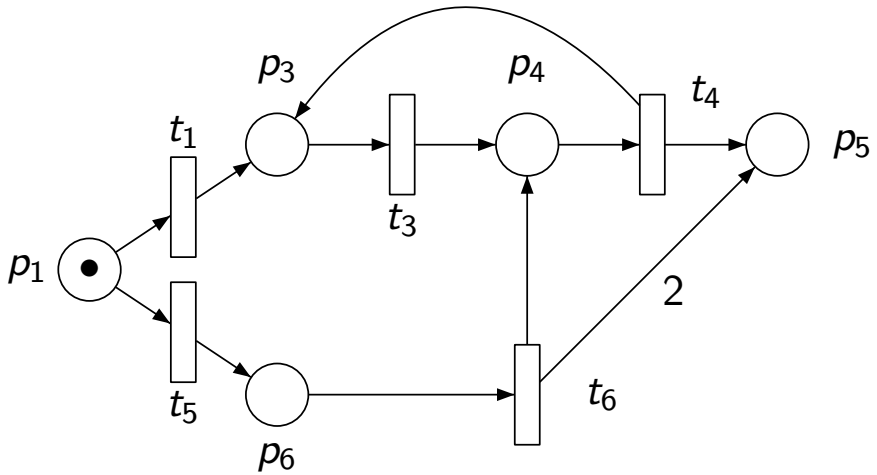

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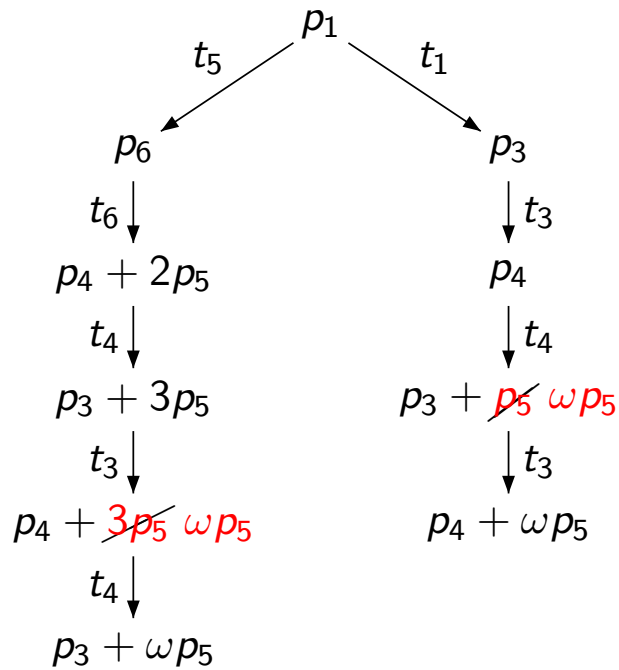

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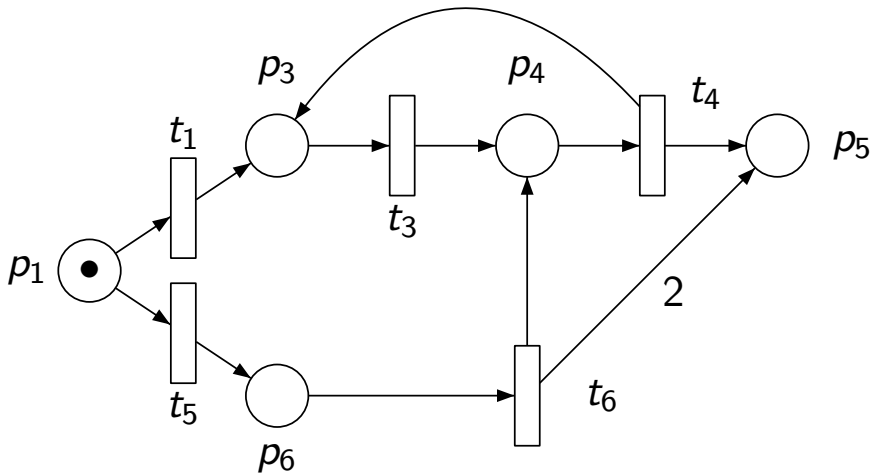

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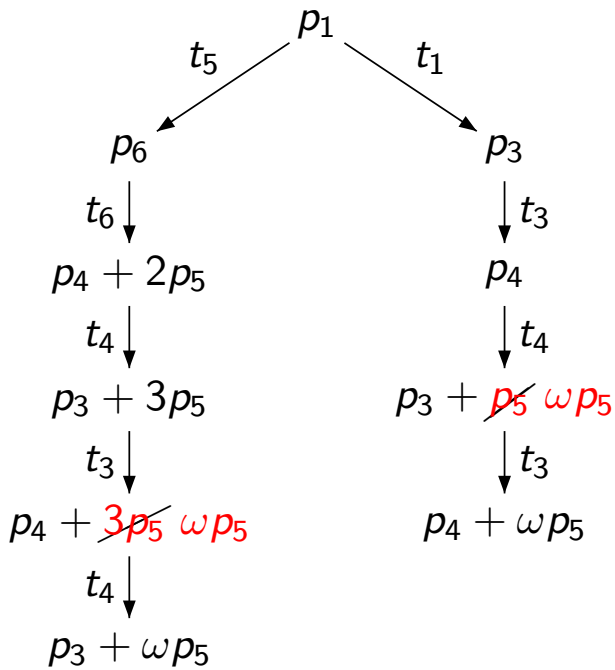

K&M Algorithm performs redundant computations: all branches are developped independently.

Computation of the MCS

- Karp & Miller Tree (1969) : impractical
- Minimal Coverability Tree (MCT) [Finkel, 1991] : incomplete
  improved $\mathrm{K\&M}$ Algorithm with pruning
- MCT Patch [Luttge, 1995]: does not terminate
- CoverProc procedure [Geeraerts et al, 2005] : not $\mathrm{K\&M}$-based
- Our algorithm: $\mathrm{K\&M}$ Algorithm with pruning
- Piipponen and Valmari, 2016: new algorithm, very efficient implementation
- Others...

K&M Algorithm

Waiting List composed of pairs (n', t).

1. Pop an element (n', t) of the Waiting List.
2. Build a new node n such that $n' \xrightarrow{t} n$
3. If n is already covered go to 1.
4. Perform acceleration if possible.
5. Add n to the tree and (n, t) to Waiting List for all possible t.

Our Algorithm

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2. Build a new node n such that $n' \xrightarrow{t} n$
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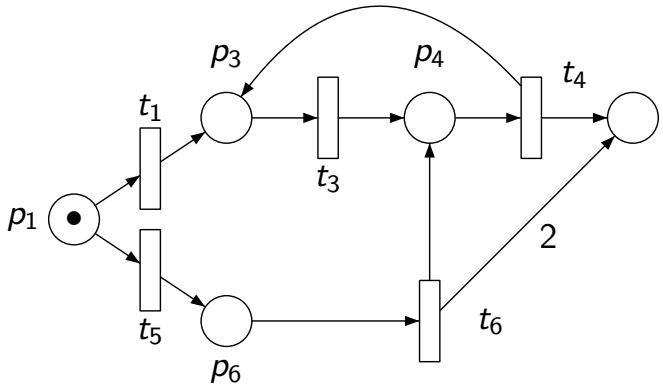
4. Perform acceleration if possible.
5. Pruning:

   for each node x covered by n such that $x \in \text{Act} \vee x \notin \text{Ancestor}(n)$, deactivate the subtree rooted in x
6. Add n to the tree and (n, t) to Waiting List for all possible t.

Monotone Pruning Algorithm by example



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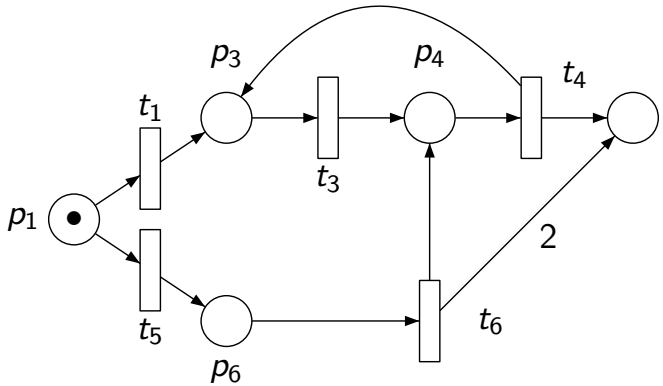

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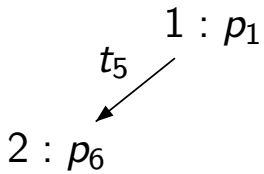

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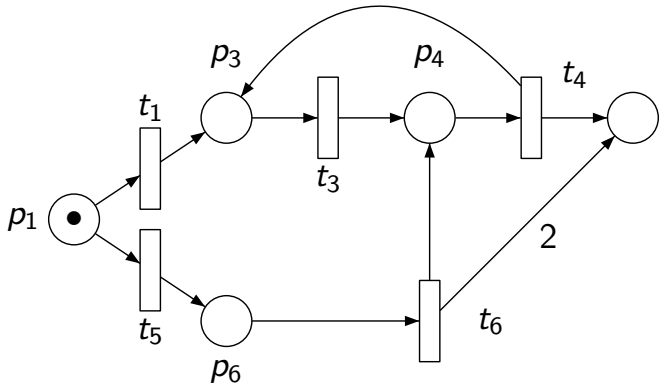

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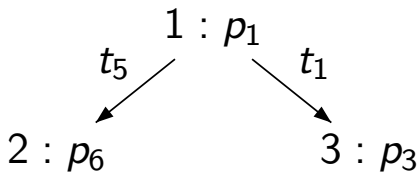

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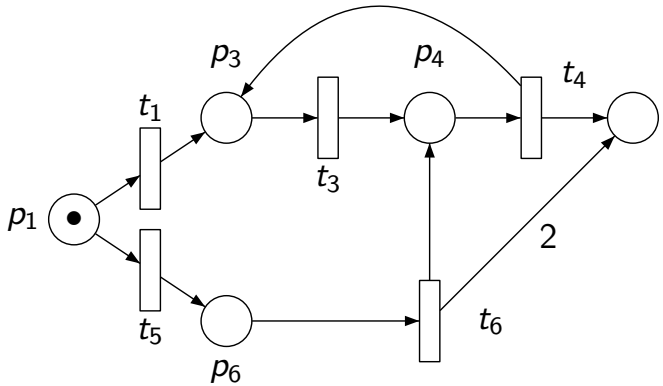

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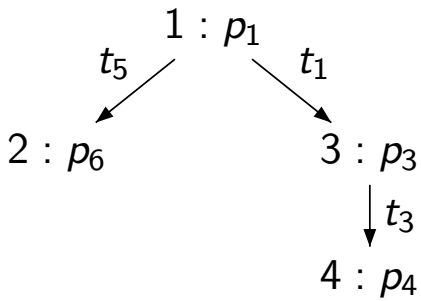

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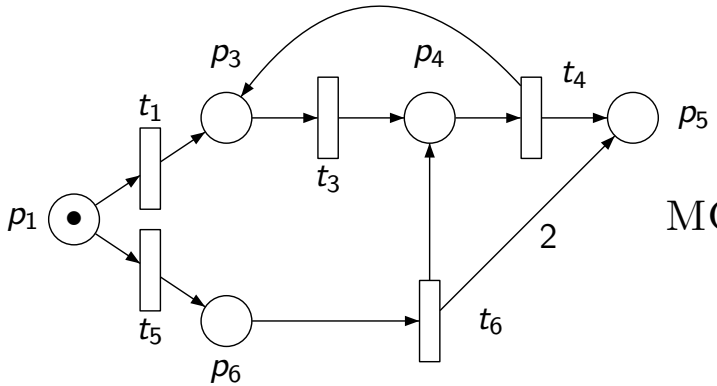

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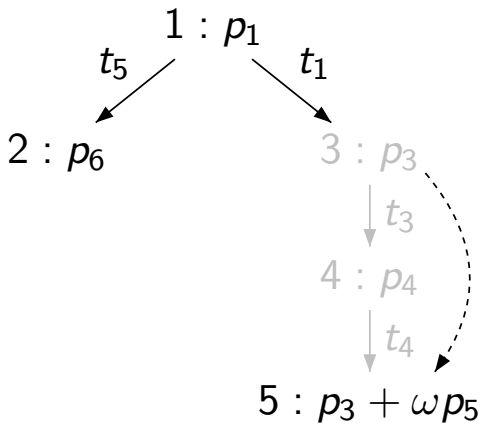

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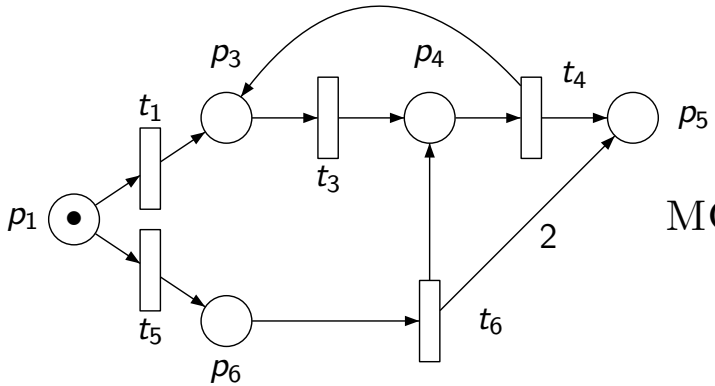

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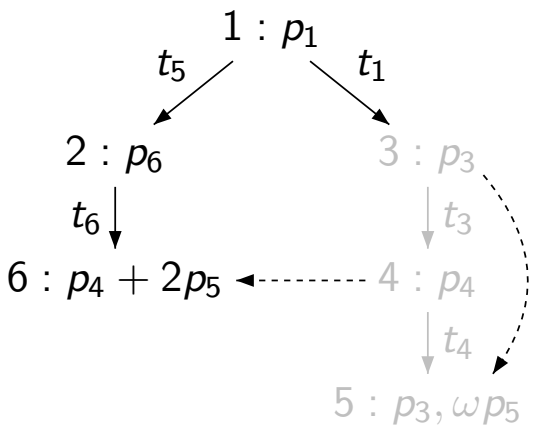

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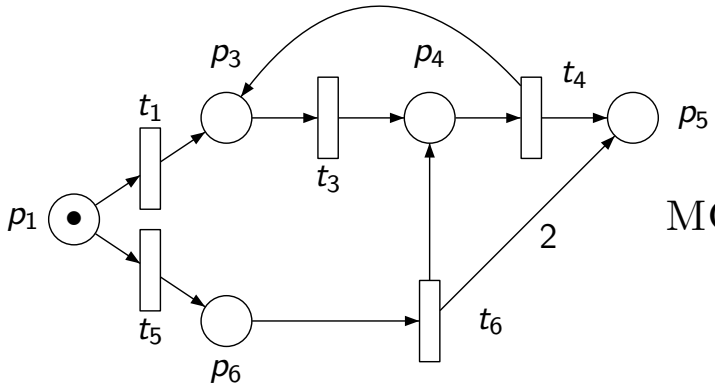

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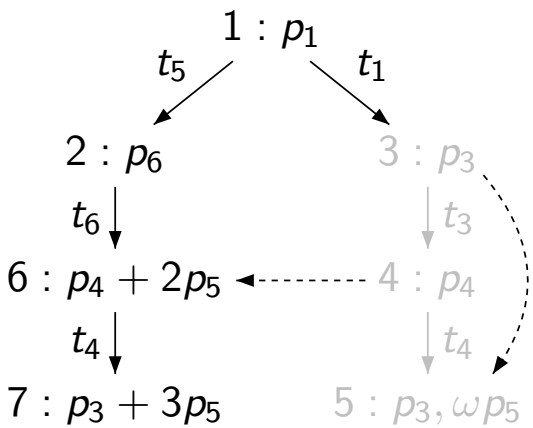

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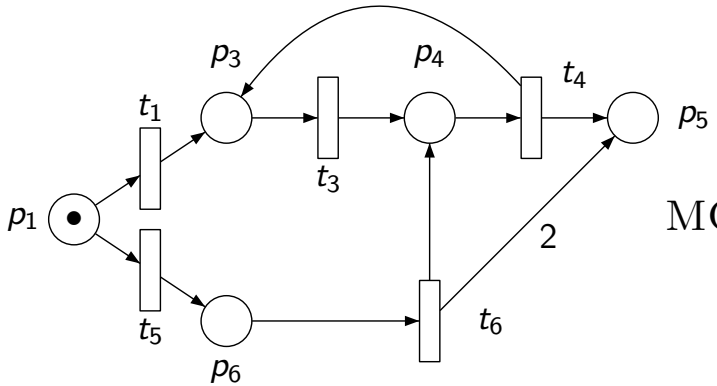

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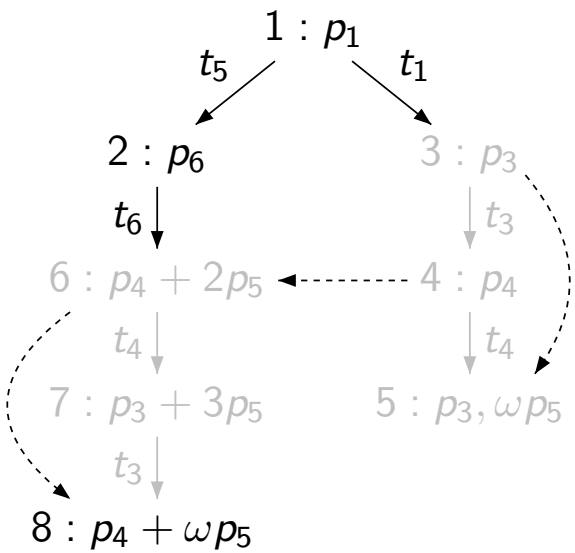

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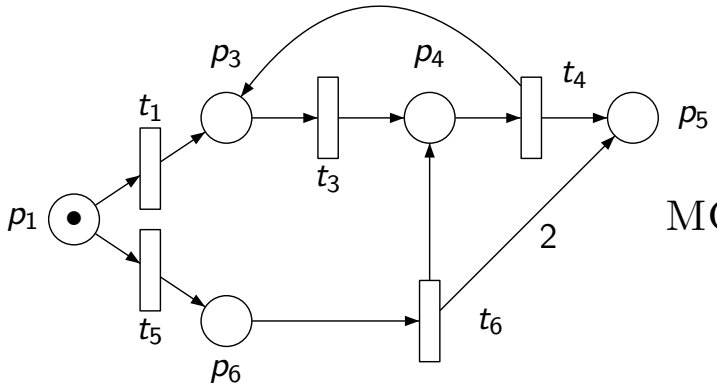

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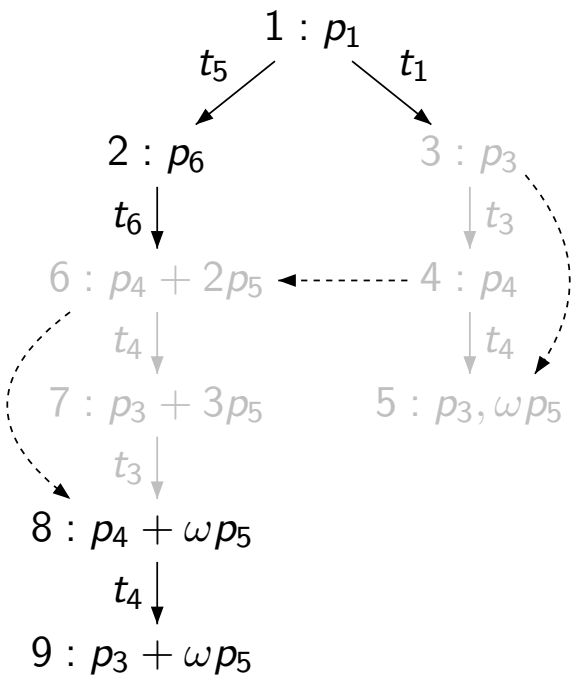

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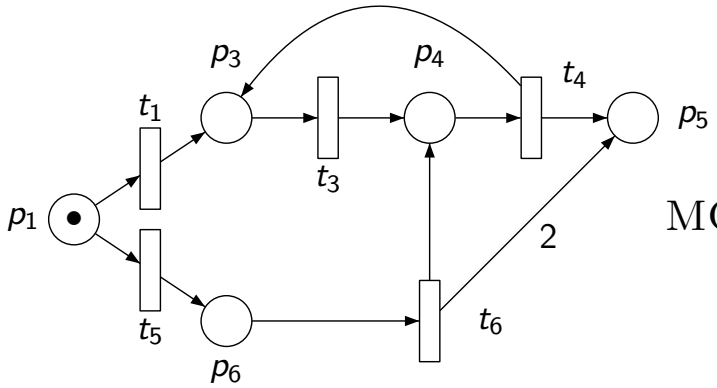

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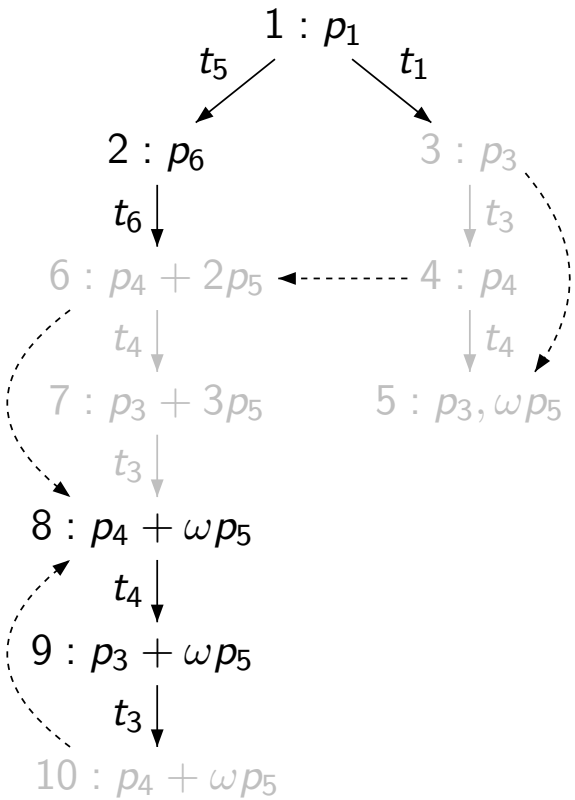

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Step 10: Node 10 is immediately declared as inactive as it is covered by node 8

In this paper

While our algorithm is a simple, elegant optimisation of the $\mathrm{K\&M}$ algorithm, the 2011 proof is horrible, 10 pages long.
In this paper:

- shorter elegant proof (less than 2 pages)
- the acceleration function is a parameter: you can plug any reasonable function, the proof still holds.
- better suited for generalisation to other WSTS.

Proof of Monotone Pruning Algorithm

Theorem

MP Algorithm terminates and is correct.

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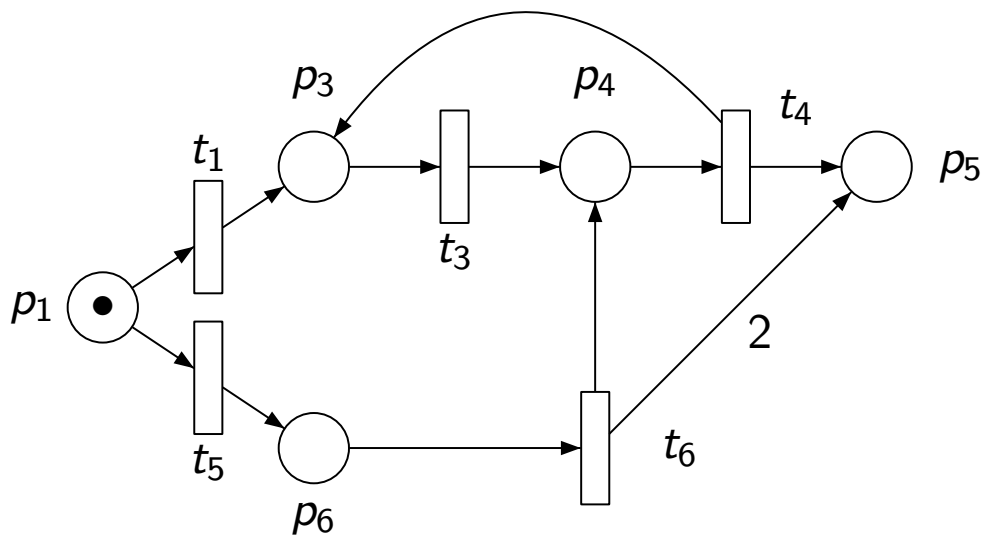
Correction:

- Soundness: each marking built by MP algorithm can be part of a coverability set
  ➜ easily follows similarly as $\mathrm{K\&M}$ Algorithm
- Completeness: prove that every reachable marking is covered by an active node
  ➜ this is the main challenge

Proof of completeness

We want to prove, for each sequence $m_0 \xrightarrow{\varrho} m$, that there exists an active node built by MP that covers m.

Consider for example the sequence $\rho = t_1\, t_3\, t_4\, t_3$:
$p_1 \xrightarrow{t_1} p_3 \xrightarrow{t_3} p_4 \xrightarrow{t_4} p_3 + p_5 \xrightarrow{t_3} p_4 + p_5$



Proof of completeness

At each step, the algorithm satisfies the following invariant.

For any reachable marking m, either m is covered by an active node or there exists an active node x and a sequence of transition ρ such that:

- $x \xrightarrow{\rho} \cdot$ covers the marking
- (x, t), where t is the first transition of ρ, is in the Waiting List
- every intermediate marking when firing ρ from x is not covered by any active node

In other words: if it is not covered now, it will be covered later.

Experiments

Test				K&M		MP		CoverProc
name	P	T	MCS	nodes	time (s)	nodes	time (s)	time (s)
BasicME	5	4	3	5	< 0.01	5	< 0.01	0.12
Kanban	16	16	1	72226	9.1	114	< 0.01	0.19
Lamport	11	9	14	83	0.02	24	< 0.01	0.17
Manufacturing	13	6	1	81	0.01	30	< 0.01	0.14
Peterson	14	12	20	609	0.2	35	0.02	0.25
Read-write	13	9	41	11139	6.33	76	.06	1.75
Mesh2x2	32	32	256	x	x	6241	18.1	330
Multipool	18	21	220	x	x	2004	4.9	365
pncsacover	31	36	80	x	x	1604	1.6	113
csm	14	13	16	x	x	102	.03	0.34
fms	22	20	24	x	x	809	0.28	2.1

- Drastic reduction of the number of states over K&M Algorithm
- A lot faster than both K&M and CoverProc Algorithms
- After running K&M Algorithm, one has to extract the MCS from the coverability set computed by K&M.

Contributions

Simpler and more general proof for the MP algorithm:

- Acceleration function as a parameter
- May allow extension to other WSTS

The MP algorithm is nice :

- Simple modification of the $\mathrm{K\&M}$ algorithm
- Any strategy of exploration is correct
- Any reasonable acceleration function can be plugged in
- Drastic improvement over $\mathrm{K\&M}$