

Local presentability of monads in double categories

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Double categories, introduced by Ehresmann [3], provide a natural 2-dimensional framework for studying contexts where two interacting types of arrows coexist. While the notion of a monad in a double category coincides with that of a monad in its underlying horizontal bicategory, the double-categorical environment provides a richer class of monad morphisms, making the double viewpoint often more appropriate than its bicategorical counterpart.

In this talk, which is based on [1], we will examine conditions on a double category under which its categories of monads and comonads are locally presentable. In particular, we extend well-established results from monoidal category theory, namely that if a monoidal category $\mathcal{V}$ is locally presentable and its tensor product is finitary in each variable then its categories of monoids and comonoids are also locally presentable [4], to the double-categorical setting. For example, we will see that if $\mathbb{D}$ is locally presentable (in the sense of [2]) and its horizontal composition and identity functors preserve filtered colimits, then the category of monads $\mathbf{Mnd}(\mathbb{D})$ is itself locally presentable.

References

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