Derived torsion theories in a non-pointed context

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In [1] we studied a non-pointed version of the notion of torsion theory, in the framework of categories equipped with a posetal monocoreflective subcategory of *zero objects* such that the coreflector inverts monomorphisms. In particular, we generalized to this context some results, proved in [2] for the pointed case, concerning the relationships of torsion theories with factorization systems and categorical Galois structures. Examples of such torsion theories can be found in the duals of elementary toposes, in varieties of universal algebras used as models for non-classical logic, and in coslices of the category of abelian groups.

Generalizing some more results obtained in [2] for the pointed case, we will show that, in a category $\mathcal{C}$ equipped with a good subcategory $\mathcal{Z}$ of zero objects, a torsion theory such that the torsion subobjects are characteristic and the torsion-free reflector is protoadditive induces a non-pointed torsion theory in the category $Arr(\mathcal{C})$ of arrows in $\mathcal{C}$, equipped with the subcategory $Arr(\mathcal{Z})$ of zero objects. We will observe that the good properties of the original torsion theory in $\mathcal{C}$ pass to the new one in $Arr(\mathcal{C})$, allowing to iterate the process and to apply these constructions to the study of higher dimensional extensions via categorical Galois theory.

References

[1] A. Cappelletti and A. Montoli, Torsion theories in a non-pointed context, *J. Pure Appl. Algebra* **230** (2026), 108261.

[2] T. Everaert and M. Gran, Protoadditive functors, derived torsion theories and homology, *J. Pure Appl. Algebra* **219** (2015), 3629–3676.