Naturally Mal'cev categories and descent

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Slices of finitely complete additive categories have long been of interest in category theory, partly because of their connection with *affine spaces*. They were first characterised by Carboni [1] in terms of algebraic properties. Later, Carboni and Janelidze [2] showed that they can equivalently be characterised as those categories $\mathcal{A}$ with finite limits and finite coproducts whose category of pointed objects $1/\mathcal{A}$ is additive, and for which the unique morphism $0 \to 1$ is an *effective codescent* morphism.

In this talk, we investigate the broader class of categories with finite limits and finite coproducts for which the category of pointed objects $1/\mathcal{A}$ is again additive, while the unique morphism $0 \to 1$ is only a *codescent* morphism. We prove that, when $0 \to 1$ is a codescent morphism, the category $1/\mathcal{A}$ is additive if and only if $\mathcal{A}$ is *naturally Mal'cev.* We further characterise these categories as distinguished subcategories of slices of additive categories. Finally, we describe their structural properties and present a number of illustrative examples.

References

[1] A. Carboni, Categories of affine spaces, *Journal of Pure and Applied Algebra* **61.3** (1989), 243–250.

[2] A. Carboni and G. Janelidze, Modularity and descent, *Journal of Pure and Applied Algebra* **99.3** (1995), 255–265.