

Relational calculus in algebra and logic

Elena Caviglia¹, Francesco Dagnino², Kishan Dayaram¹, Luca Mesiti^{1,*}

¹Department of Mathematical Sciences, Stellenbosch University ²DIBRIS, University of Genoa

*Speaker

In this talk, we will present connections and interactions between two different frameworks for relational calculus, one coming from algebra and the other one coming from logic and computer science. Both of them were established with the same aim: to develop mathematical theories using general relations between sets rather than functions. While they were developed independently and in different directions, we show how the two theories are deeply connected with each other.

The algebraic framework for relational calculus that we will consider is that of arcs, recently introduced by Z. Janelidze and D. Rodelo in the context of the theory of forms. On the logic side, we will consider the framework of relational doctrines, introduced by F. Dagnino and F. Pasquali in the context of the theory of doctrines.

Like arcs and relational doctrines, also the theory of forms and the theory of doctrines were established with the same idea: to study and advance algebra and logic respectively via a categorical fibrational approach. And both theories brought to fundamental breakthroughs in their fields. But the connections between the two theories remained undiscovered until recently, when the deep similarities have begun to be investigated on multiple fronts. This talk is inscribed in the bigger project of exploring the connections between forms and doctrines.

We will show how we employed the similarities between arcs and relational doctrines to establish new notions and results in the two worlds. In particular, we will explain how normals (in the theory of forms) play the same role of equivalence relations (in the theory of relational doctrines). The same holds for conormals and coequivalence relations. Our theory suggests that, in some sense, relational doctrines are the right framework in logic to study contexts in which not all objects are conormal.

Furthermore, we will show how we generalized the algebraic notions of regularity, exactness and Malt'sev condition from arcs to relational doctrines. And we will explain the fruitful new insights that this generalization has brought to the algebraic side. Finally, we will discuss how to complete arcs and relational doctrines under quotients and comprehensions. We will show that, interestingly, these two completions do not always distribute with each other, but they do under certain assumptions.