

Categories with indexed monoids

Leo Lobski

University College London, UK

It is well-known that equipping each object in a symmetric monoidal category with a comonoid in a way that is natural and uniform with respect to the monoidal structure is equivalent to the monoidal category being cartesian monoidal: this is Fox’s theorem (see e.g. Melliès (2009)). In this talk, we further equip each object with a monoid, and ask this choice of monoids to be uniform but not necessarily natural. Such a cartesian monoidal category with chosen monoids is called a *category with indexed monoids*. After covering some examples, we show how to construct the free category with indexed monoids on any (small) category. We then extend the notion of indexed monoids to opfibrations, and show that opfibrations with indexed monoids are equivalent to the choice of a monoid on the category of opfibrations with a fixed base. We conclude by relating this to (op)indexed monoidal categories via a result of Moeller & Vasilakopoulou (2020). The talk is based on Section 4 of Lobski & Zanasi (2026), which, in turn, is based on Chapter 7 of my PhD thesis (Lobski 2025).

References

- [1] P.-A. Melliès, Categorical semantics of linear logic, *Panoramas & Synthèses* **27** (2009), 1–197.
- [2] J. Moeller and C. Vasilakopoulou, Monoidal Grothendieck construction, *Theory and Applications of Categories* **35** (2020), 31, 1159–1207.
- [3] L. Lobski and F. Zanasi, Layered Monoidal Theories II: Fibrational Semantics, arXiv:2602.22373, 2026.
- [4] L. Lobski, Layered Monoidal Theories, arXiv:2512.12139, 2025.