Normalizers in Heyting semilattices

Danielle Kleyn[1]

[1]Stellenbosch University

The normalizer of a subgroup H of a group G can be described as the largest subgroup $N_G(H)$ of G such that H is normal in $N_G(H)$. The formula for $N_G(H)$ is given as

$$N_G(H) = \{g \in G \mid gHg^{-1} = H\}.$$

The notion of a normalizer has been formulated in any pointed category (see [1]), and has been studied in other cases, such as Lie Algebras. This talk explores normalizers of Heyting semilattices.

A Heyting semilattice is a meet-semilattice X with a top element 1 that is equipped with a binary operation $\to$ satisfying

$$x \wedge y \leq z \iff x \leq y \to z.$$

A subalgebra of a Heyting semilattice X is normal if and only if it is the kernel of some homomorphism $f : X \to Y$. A normalizer of a subalgebra S of a Heyting semilattice X (if it exists) is defined as the largest subalgebra $N_X(S)$ such that S is normal in $N_X(S)$.

In this talk, it is shown that the subvariety of Heyting semilattices generated by the chain of length 3 has normalizers, and it is also shown that this is the largest subvariety of Heyting semilattices which has normalizers. A formula for the normalizer (in the case when it does exist) is also established. Furthermore, we can deduce a result concerning the amalgamation property for normal monomorphisms, which allows us to show that the variety of Heyting semilattices generated by the chain of length 3 is action representable.

References

[1] J.R.A Gray, Normalizers, Centralizers and Action Representability in Semi-Abelian Categories, *Applied Categorical Structures* **22** (2014), 981–1007.