

Locales in (pre-)sheaf toposes vs. (pre-)sheaves of locales

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In our recent [1] we show that the forgetful functor from internal sup-lattices or frames in a presheaf topos to presheaves of sup-lattices, respectively, frames (which exists by the standard description of internal such structures in presheaves in [4]), has a left adjoint functor. This left adjoint is presented via a construction introduced in [3] but studied there in connection to a different universal property. This particular construction allows us to relate conditions on the sections and the transition maps of an internal frame to internal properties of it such as compactness, local compactness etc (much in the spirit of [2]). We review the relevant material and take the first steps towards extending these results to sup-lattices and frames that are internal to a Grothendieck topos. In particular, we show that there is a left adjoint to the forgetful functor from internal sup-lattices or frames in sheaves to sheaves of such structures.

References

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