Bialgebraic Structures from the 2-Categorical Perspective

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In his 1977 preprint [5], Ulmer introduced a semantic notion of bialgebras that unifies a broad class of algebraic and coalgebraic structures. We develop a syntactic counterpart in [2] by introducing signature pairs (Σ, σ) and bialgebraic theories T, providing a uniform language for constructing internal bialgebras in a 2-categorical setting. For every bialgebraic theory T and Σ-model M within a 2-category with PIE limits [4], we construct the object M^T of internal T-bialgebras. Our approach to bialgebras admits a general Induced Functor of Algebras Theorem extending the classical lifting of lax monoidal functors to the categories of internal monoids. Since the construction of M^T is expressed entirely in terms of PIE limits, accessibility [1, 3], local presentability, orthogonal factorization systems, regularity, and exactness lift along the construction $M \mapsto M^T$ under suitable assumptions.

References

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