

Generalized action monads and descent

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A key theorem in categorical algebra is that actions of internal groups can be captured as algebras for a monad. In this talk, we will present a similar monadicity result for actions of internal categories. We reached this result by generalizing the action monad for internal group actions to a monad on the slice category over the object of objects of the internal category. We call the monads that arise in this way *generalized action monads*. The classical action monad for a group object is recovered as a generalized action monad by viewing the group object as an internal category with object of objects given by the terminal object. As an outcome of our monadicity result, we will show that the process of associating to an internal category its category of actions is functorial, as it corresponds to a morphism of monads.

We will then present a complete characterization of which monads arise as generalized action monads associated to an internal category. To reach such characterization, we will explore some interesting properties and structures of generalized action monads.

Generalized action monads are naturally related to descent theory. In the second part of the talk, we will present some applications that we developed in this context. These applications naturally embed into the general framework for descent theory studied by Bénabou and Roubaud in [1], that associates a notion of descent to any monad. We will restrict ourselves to considering generalized action monads, which yield a notion of generalized descent that satisfies better properties than those of general monadic descent. In particular, we will explain how to construct the comparison functor using the extra structure of a generalized action monad. We will also construct a realization functor using coequalizers, in a similar way to the realization functor of classical descent along a morphism. Both classical descent along a morphism and Galois descent (studied by Marques in [3]) are simultaneously captured by our framework.

Our theory of descent embeds in that of lax descent objects, brought forward by Lucatelli Nunes [4], but it enjoys much better properties. In fact, we have monadicity of the category of descent data, while the general framework of [4] lacks monadicity.

So generalized action monads provide an intermediate well-behaved notion of generalized descent, that still enjoys good properties while being general enough to capture both descent along a morphism and Galois descent. In this sense, generalized action monads stand in a sweet spot for descent theory.

The talk will be based on our recent preprint [2].

References

- [1] J. Bénabou and J. Roubaud, Monades et descente, *Comptes Rendus de l'Académie des Sciences Paris, Série A–B*, 270 (1970), 96–98.
- [2] E. Caviglia, S. Marques and L. Mesiti Generalized action monads and descent, arXiv:2607.09595, 2026.
- [3] S. Marques, Galois descent theorem in general categories, *Research in the Mathematical Sciences* 13 (2026), Article 49. doi: 10.1007/s40687-026-00629-8
- [4] F. Lucatelli Nunes, Descent data and absolute Kan extensions, *Theory and Applications of Categories*, 37 (2021), no. 18, 530–561.