Formal toposes and monoidal toposes

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The success of the theories of sheaves and sheaf toposes [1] has encouraged the definition of similar objects in different contexts: enriched categories [2], higher categories [3], etc. But there is no overarching theory unifying these ad-hoc extensions into a general abstract framework.

In this talk I'll present a formal theory that aims to provide this framework. It is applicable in any sufficiently well-behaved 2-category $\mathcal{B}$, and in particular it specialises to Grothendieck toposes in $\mathcal{C}at$, to locales in $\mathcal{P}os$ and, to $(\infty, 1)$-toposes in the homotopy 2-category of the $(\infty, 2)$-category of $(\infty, 1)$-categories[1]. Moreover it is possible to recover several different characterisations of toposes under mild additional conditions on the 2-category $\mathcal{B}$ that are satisfied in all known examples.

This theory replaces the pullbacks appearing in the familiar characterisations of toposes with the base change functors of a new fibration generalising the codomain fibration and entirely determined by the 2-category $\mathcal{B}$. This fibration relies on the introduction of a new object, the pleronomial object, which extends one of the universal properties of slice categories.

Finally I will discuss an application of this theory to the 2-category $\mathcal{B}$ of symmetric monoidal categories and symmetric colax monoidal functors in terms of extension of scalars functors for comodule categories of the cofree comonoids.

This is part of my PhD work under the supervision of Wendy Lowen and Arne Mertens.

References

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[1]Work in progress.