Depth stitching for extended field of view

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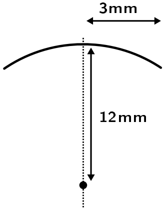
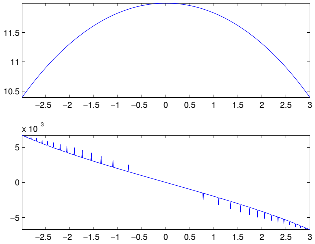
Context

Intraocular lens

- high curvature
- discontinuous surface





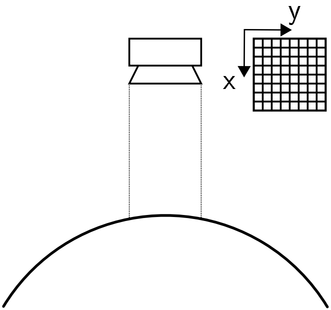
IOL characterization with an optical system

- Measurement of depth maps
- Avoid distortion due to curvature : measurement of small patches

Measurement system I

Optical system

- Confocal deflectometry for measuring depth maps
- Camera model : orthogonal projection
- narrow field of view

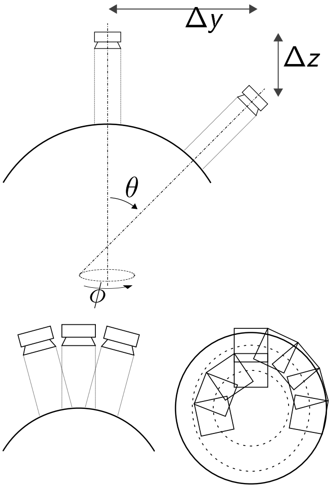


Measurement system II

Mechanical system

- Fixed lens and moving camera
- tilt θ
- azimuth ϕ
- compensation Δx, Δy, Δz
- surface normal aligned with optical axis

Positioning errors.

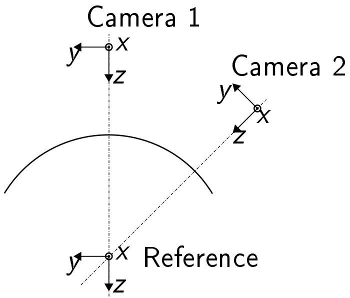


System model I

Reference frame pointing downwards.

Camera frame

- xy-plane = image plane
- z axis = viewing direction
- surface imaged from above : z-axis pointing downwards
- origin at (ϕ, θ), constant radius



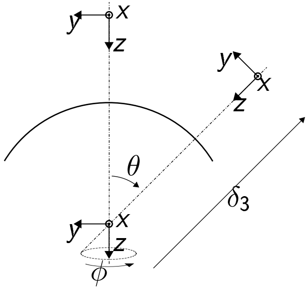
System model II

Motion representation

- 6 DOF : 3 rotations R + 3 translations T
- parameters $p = (\alpha, \beta, \gamma, \delta_1, \delta_2, \delta_3)$

Rotations : Euler angles

- Euler angles (α, β, γ) around zyz
- matrix form : $R = R^z_\gamma R^y_\beta R^z_\alpha$
- $\alpha = \phi$ (azimuth) around z
- $\beta = \theta$ (tilt) around y
- γ not needed for the position of the camera



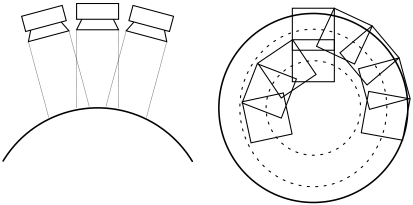
Depth stitching I

Measurements : depth maps

- (x_1, x_2) : map coordinates (image pixels)
- x_3 : depth (pixel intensities)

Set of 3D vectors $x = (x_1, x_2, x_3)$.

Several sets corresponding to overlapping patches covering the entire surface.



Depth stitching II

Stitching

- alignment
- blending

Alignment

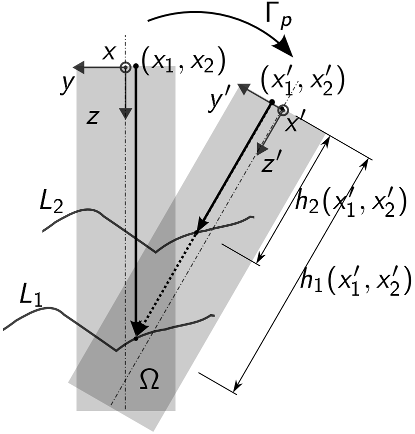
- needed : subpixel alignment to avoid artifacts when blending
- known : estimates p of the camera position for each configuration
- Δp correct positioning error

Pairwise alignment I

Align neighbouring pairs of data sets sequentially.

Transformation between camera frames

$\Gamma x = T_2 R_2 (T_1 R_1)^{-1} x$



Energy function

$x' = \Gamma x$

$E = \sum_{x'} (h_1(x_1', x_2') - h_2(x_1', x_2')))^2$

$E = \sum_{x} (h_1(\Gamma_p x) - h_2(\Gamma_p x)))^2$

Pairwise alignment II

Descent direction Δp

$E(p + \Delta p) = \sum_x (h_1(\Gamma_p x) - h_2(\Gamma_{p+\Delta p} x)))^2$

Linearization around p

$h_2(\Gamma_{p+\Delta p} x) \approx h_2(\Gamma_p x) + \frac{\partial h_2}{\partial p} \Delta p$

$E = \sum_x (h_1(\Gamma_p x) - h_2(\Gamma_p x) - \frac{\partial h_2}{\partial p} \Delta p))^2$

Pairwise alignment III

Minimization : Gauss-Newton method

- iterative search
- update rule $p_{k+1} = p_k + \alpha_k \Delta p_k$.
- at each iteration : $H\Delta p = d$

$$H = \sum_x \left(\frac{\partial h_2}{\partial p}\right)^T \left(\frac{\partial h_2}{\partial p}\right) \quad \text{(6x6)}$$

$$d = \sum_x \left(\frac{\partial h_2}{\partial p}\right)^T (h_1 - h_2) \quad \text{(6x1)}$$

- α_k by line search

Difficulties I

Energy function : more precisely

- depth map : $h : \mathbb{R}^2 \to \mathbb{R}, \quad h(x_1, x_2) = x_3$
- $\Gamma : \mathbb{R}^3 \to \mathbb{R}^3, \quad x' = (x'_1, x'_2, x'_3) = \Gamma(x_1, x_2, x_3) = \Gamma(x)$
- $h(\xi(\Gamma(x)))$ instead of $h(\Gamma(x))$
- $\xi : \mathbb{R}^3 \to \mathbb{R}^2$ extraction of the first two coordinates

$\frac{\partial h}{\partial p} = \nabla h \cdot J_{\xi(\Gamma(x))}$, with

$$J_{\xi(\Gamma(x))} = \begin{bmatrix} \frac{\partial \Gamma_1}{\partial p_1} & \dots & \frac{\partial \Gamma_1}{\partial p_6} \\ \frac{\partial \Gamma_2}{\partial p_1} & \dots & \frac{\partial \Gamma_2}{\partial p_6} \end{bmatrix}$$

but $\frac{\partial \Gamma_1}{\partial p_6} = \frac{\partial \Gamma_2}{\partial p_6} = 0$
Thus, no access to $p_6 = \delta_3$ for the optimization !

Difficulties II

Linearization around p

$$h_2(\Gamma_{p+\Delta p}x) \approx h_2(\Gamma_p x) + \nabla h_2(\Gamma_p x) \cdot J_{\Gamma_p x} \cdot \Delta p$$

Minimization : Gauss-Newton method

- iterative search
- update rule $p_{k+1} = p_k + \alpha_k \Delta p_k$.
- at each iteration : $H \Delta p = d$

$$H = \sum_x (\nabla h_2 J_\Gamma)^T (\nabla h_2 J_\Gamma) \quad (6\text{x}6)$$
$$d = \sum_x (\nabla h_2 J_\Gamma)^T (h_1 - h_2) \quad (6\text{x}1)$$

- α_k by line search

Difficulties III

Surface profile

- very smooth
- micrometric steps
- radial symmetry

Alignment along radial directions only !