The lensless endoscope:

a playground for acquisition schemes
based on compressed sensing principles

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1. Action potential reaches axon terminal and depolarizes membrane.

2. Voltage-gated Ca\textsuperscript{2+} channels open and Ca\textsuperscript{2+} flows in.

3. Ca\textsuperscript{2+} influx triggers synaptic vesicles to release neurotransmitter.

4. Neurotransmitter binds to receptors on target cell (in this case, causing positive ions to flow in).

Depolarization — more likely to fire action potential
Exploring cells and quantifying cellular processes in vivo

Current performances of molecular imaging devices for small animals

- FRI
- PET & SPECT
- FMT
- Intravital microscopy
- MR

Maximum imaging depth vs. cell size

The lensless endoscope: an ultrathin device to image cells

Wavefront shaper → Laser → Optics → Single pixel detector → Imaged (biological) sample → Fluorescence signal

Multicore optical fiber


S. Sivankutty, V. Tsvirkun, O. Vanvincq, et al., “Nonlinear imaging through a fermat’s golden spiral multicore fiber,” Optics letters, 2018
Relaxing some constraints could accelerate the acquisition.

\[ y \in \mathbb{R}^N \]

\[ M = N = n \times n \]

\[ M < N? \]
The lensless endoscope: a playground for acquisition schemes based on compressed sensing principles
Compressive sampling uses unstructured illumination patterns.

\[ y = h \otimes x + n \]

\[ y \in \mathbb{R}^N \]

\[ y = \Phi x + n \]

\[ \Phi \in \mathbb{R}^{M \times N} \]
The inverse problem is expressed as a minimization

\[ y = \Phi x + n \]

**Compressive sampling**

\[ \hat{x} = \arg \min_z \| y - \Phi z \|^2_2 \]

**Estimation problem**

\[ \hat{x} = \arg \min_z \| y - \Phi z \|^2_2 \]

\[ \hat{x} = \arg \min_z \| y - \Phi z \|^2_2 \]

Ill-posed problems!
Adding priors reduces the set of feasible solutions for \( \hat{x} \)

**Compressive sampling**

\[
y = \Phi x + n
\]

**Estimation problem**

\[
\hat{x} = \arg \min_z \| y - \Phi z \|_2^2 + \rho \Phi(z)
\]

Add priors on \( \hat{x} \)...

We assume that the wavelet representation of \( \hat{x} \), \( (\Psi^T \hat{x}) \), is sparse.
Adding priors reduces the set of feasible solutions for $\hat{x}$

Compressive sampling

$$y = \Phi x + n$$

Estimation problem

$$\hat{x} = \arg \min_z \|y - \Phi z\|_2^2 + \rho \Phi(z)$$

Add priors on $\hat{x}$...

$$\Leftrightarrow \left\| \Psi^T \hat{x} \right\|_1 \text{ is small.}$$

$$\Leftrightarrow \sum_{i=1}^r \left\| \psi_i^T S_i \hat{x} \right\|_1 \text{ is small.}$$

$r$-Redundant Discrete Wavelet Transform

$$\Psi^T = [\psi_1 \ \psi_2 \ \ldots \ \psi_r]^T, \ \psi_i^T = \psi^T S_i$$
Adding priors reduces the set of feasible solutions for $\hat{x}$

Compressive sampling

$$y = \Phi x + n$$

Estimation problem

$$\hat{x} = \arg\min_z \|y - \Phi z\|_2^2$$

$$+ \rho \sum_{i=1}^r \|\psi^T S_i z\|_1$$

Let's solve these minimizations!
Generalized Forward-Backward algorithm

\[ \hat{x} = \arg \min_z \left\{ \|y - \Phi z\|_2^2 + \rho \sum_{i=1}^r \|\psi^T S_i z\|_1 \right\} \]

**Estimation**

\[ r = 1, \gamma = 1.8/L, L \text{ is the Lipschitz constant of } \nabla f \]

\[ \hat{x}^k \leftarrow \text{prox}_{\gamma g} \left[ \hat{x}^{k-1} - \gamma \nabla f(\hat{x}^{k-1}) \right] \]

\[ \begin{align*}
\text{minimize } f \\
\text{minimize } g
\end{align*} \]

---

Generalized Forward-Backward algorithm

\[
\hat{x} = \arg \min_z \|y - \Phi z\|_2^2 + \rho \sum_{i=1}^r \|\psi^T S_i z\|_1
\]

Estimation

\( r > 1, \gamma = 1.8/L, L \) is the Lipschitz constant of \( \nabla f \)

for \( i = 1 \) to \( r \)
\[
s_i^k \leftarrow s_i^{k-1} + \text{prox}_{\gamma g_i/w_i} \left[ 2\hat{x}^{k-1} - s_i^{k-1} - \gamma/w_i \nabla f(\hat{x}^{k-1}) \right] - \hat{x}^k
\]
end
\[
\hat{x}^k = \sum_i w_i s_i^k
\]

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\[
\hat{x} = \arg\min_z \left( \| y - \Phi z \|_2^2 + \rho \sum_{i=1}^{r} \| \psi^T S_i z \|_1 \right)
\]

Estimation

\( \rho \) optimisation

\( z_0, \rho, \gamma \)

accuracy reached?

\( \rho \) update

cross-validation

\( \hat{x} \)

Choice of $\rho$ based on cross-validation

Aim: choosing $\rho$ such that $\hat{x}_\rho \approx x$, i.e., $\|\hat{x}_\rho - x\|_2$ is minimal.

Idea: if $\Phi$ has “nice” properties,

$$\|\hat{x}_\rho - x\|_2 \leq C\|\Phi \hat{x}_\rho - \Phi x\|_2 \leq C(\|\Phi \hat{x}_\rho - y\|_2 + \|n\|_2), \quad C > 1.$$
Choice of $\rho$ based on cross-validation

Synthetic data $\cdot N = 128 \times 128 \cdot$ BSNR 40 dB $\cdot$ 1 pattern $\cdot$ 20 trials $\cdot$ DWT

$M = 2^9$ and $M_{\text{test}} = 2^8$

![Graph showing the residual $\|y_{\text{test}} - \Phi_{\text{test}} \hat{x}\|_2$ vs. SNR for different $\rho$ values. The graph includes multiple curves representing different trial numbers.](image)
Choice of $\rho$ based on cross-validation

Synthetic data · $N = 128 \times 128$ · BSNR 40 dB · 1 pattern · 20 trials · DWT

$M = 2^{11}$ and $M_{\text{test}} = 2^8$
Choice of $\rho$ based on cross-validation

Synthetic data · $N = 128 \times 128$ · BSNR 40 dB · 1 pattern · 20 trials · DWT
Synthetic experiment in CS framework

Synthetic data \cdot N = 128 \times 128 \cdot BSNR 40 \text{ dB} \cdot M_{\text{test}} = 2^8 \cdot 5 \text{ trials} \cdot \text{DWT}
Synthetic experiment in CS framework

Synthetic data · $N = 128 \times 128$ · BSNR 40 dB · $M_{\text{test}} = 2^8$ · 5 trials · DWT

Raster scanning ($M = N$)

CS: $M = 2^{10}$
6.25%

CS: $M = 2^{11}$
12.5%

CS: $M = 2^{12}$
25%
Slow acquisition · Slow reconstruction · High memory
Fast acquisition?  ·  Fast transformations?  ·  Low memory?
\[ y = \Phi x + n, \quad \Phi = R_{\Omega} H \]

\[ R_{\Omega} \in \mathbb{R}^{M \times N} \quad \Omega \subset \{1, \ldots, N\} \]

\[ M_{\Omega_i} \in \mathbb{R}^{N \times N} \quad \cup_i \Omega_i = \Omega \]
\[
y = \Phi x + n, \quad \Phi = R_\Omega \sum_{i=1}^{P} M_{\Omega_i} H_i
\]
\[
R_\Omega \in \mathbb{R}^{M \times N} \quad \text{and} \quad M_{\Omega_i} \in \mathbb{R}^{N \times N}
\]
\[
\Omega \subset \{1, \ldots, N\} \quad \text{and} \quad \bigcup_i \Omega_i = \Omega
\]
How does $M$ and $P$ influence the quality of $\hat{x}$?

Synthetic data · $N = 128 \times 128$ · BSNR 40 dB · $M_{\text{test}} = 0$ · 20 trials · DWT
A fast acquisition needs few changes of patterns

Mirror galvanometer limitations: continuous trajectory & constant speed
Constant sampling frequency to collect observations
A fast acquisition needs few changes of patterns

What is the optimal distance between two repetitions of the same speckle?
Distance $d$ between two illumination patterns

Factors
- Depth
- Laser wavelength
- Single-core diameter

$\text{Distance } d$ between two illumination patterns

Factors
- Depth
- Laser wavelength
- Single-core diameter

Graph showing $\text{SNR [dB]}$ vs. $d$ [µm]

- $d$ ranging from 2 to 10 µm
- SNR ranging from 11 to 13 dB
Conclusion and perspectives

The lensless endoscope

Raster scanning

Strategies based on CS

Exploring and imaging cells in vivo

Resolution

Imaging depth

S. Guérit, S. Sivankutty, C. Scotté, et al., “Compressive sampling approach for image acquisition with lensless endoscope,” ArXiv, 2018