Path-based conditions for identifiability of nonlinear networks

Renato Vizuete and Julien M. Hendrickx
ICTEAM, UCLouvain, 1348 Louvain-la-Neuve, Belgium
Email: {renato.vizueteharo, julien.hendrickx}@uclouvain.be

1 Introduction and problem formulation

The identifiability of networks characterized by nonlinear functions at the level of edges has been analyzed in [1], [2], when the dynamic model is additive. However, in more general dynamics, additivity is not necessarily satisfied and new identifiability conditions need to be derived to guarantee the identification of these networks. Since the cost associated with the measurement of a node is usually lower than the cost of excitation, we will consider the full measurement case, where all the nodes of the network are measured and some nodes are excited.

For a network composed by n nodes, we consider that the output of each node i is given by:

$$y_i^k = u_i^{k-1} + \Phi_i(y_1^{k-m_{i,1}}, \dots, y_{M_i}^{k-m_{i,M_i}}), \text{ for } y_1, \dots, y_{M_i} \in \mathcal{N}_i, \tag{1}$$

where Φ_i is the node function associated with node i, $\mathcal{N}_i$ is the set of in-neighbors of node i, and u_i is an arbitrary external excitation signal. The topology of the network $G = (V,E)$ defines the arguments of the function Φ_i such that if $(i,j) \in E$ then $\frac{\partial \Phi_i}{\partial y_j} \neq 0$. Our objective is to identify all the node functions Φ_i.

Similarly to [1], [2], we restrict the identifiability problem to functions $\Phi : \mathbb{R}^m \to \mathbb{R}$ in a specific class. In our case, we will consider the class $\mathscr{F}_{Pol}$ of polynomial functions.

2 Directed acyclic graphs

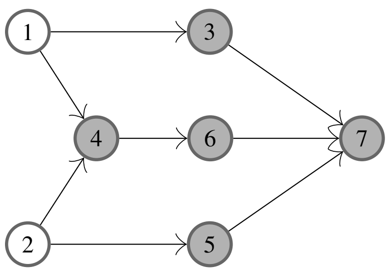

Figure 1: A DAG where the excitation of the sources 1 and 2 is sufficient for identifiability in the additive nonlinear model but is not sufficient for identifiability in the non-additive nonlinear model since it does not satisfy the vertex-disjoint path conditions of Theorem 1.

Unlike the additive model, the equivalence of partial excitation/measurement and full measurement cases does not hold for the more general model (1). However, the notion of vertex-disjoint paths used for the derivation of identifiability conditions for the partial case in [2] can still be used when the model (1) is restricted to the full measurement case.

Theorem 1 *In the full measurement case, a DAG is identifiable in the class $\mathscr{F}_{Pol}$ if and only if there are vertex-disjoint paths from excited nodes to the in-neighbors of each node.*

In the additive case, the conditions of Theorem 1 are sufficient but not necessary for identifiability. For instance, let us consider the DAG in Fig. 1 where the network is identifiable in the additive case even if there are only two excited nodes (1,2) for the three in-neighbors (4,5,6) of the node 7. This can be seen by noticing that the edge $f_{7,3}$ is a function only of u_1, the edge $f_{7,5}$ is a function only of u_2 and the edge $f_{7,6}$ contains products of u_1 and u_2, such that $f_{7,3}$ and $f_{7,5}$ are necessarily unique, and hence $f_{7,6}$ is also unique to generate the output obtained with the measurement of the node 7.

In the more general model (1), the necessity of the conditions of Theorem 1 is proved by using well known results in *algebraic varieties*. In this case, it can be shown that if the number of excitation signals is less than the number of arguments of the function Φ_i (i.e., no vertex-disjoint paths), the function Φ_i is not unique and it cannot be identified. For a more general class of functions where analytic functions are included, to the best of our knowledge, similar results in *analytic varieties* are not known, and it is a important direction for future work.

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References

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