# Local identifiability of networks with nonlinear node dynamics

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## 1 Introduction and model description

Networks of interconnected dynamical systems are widespread across various domains. Analyzing these systems and developing control strategies require understanding the interconnections, typically modeled as edges of a graph. However, identifying these systems from partial excitations and partial measurements is challenging because measured signals reflect combined dynamics.

In this work [1], we consider network systems where the dynamics is linear on the edges and nonlinear on the nodes (e.g., artificial neural networks and continuous linear threshold models). More precisely, we denote with $\mathscr{G} = (\mathscr{N}, \mathscr{E}, W)$ weakly connected directed acyclic graphs (DAGs) with set of nodes $\mathscr{N}$, set of directed links $\mathscr{E}$, and weight matrix $W$ whose entries are such that $w_{ij} \neq 0$ if and only if $(i, j) \in \mathscr{E}$. For a given nonlinear function $f$, we assume that the output of a node at time $k$ is

$$y_i^k = f\big(\sum_j w_{ij} y_j^{k-1}\big) + u_i^{k-1} \quad (1)$$

where $u_i^{k-1} \in \mathbb{R}$ is an external excitation signal. If a node is not excited, its corresponding excitation signal is set to zero. For example, for the network in Fig. 1 where node 1 and 2 are excited, the output of node 4 at time $k$ is given by $y_4^k = f(w_{23} f(w_{31} u_1^{k-3} + w_{32} u_2^{k-3}))$.

In our setting, the topology of the graph $\mathscr{E}$ and the nonlinear function $f$ are known, while the weight matrix $W$ is unknown. We are then interested in determining the identifiability of $W$ with partial excitation and partial measurement. Given a set of measured nodes $\mathscr{N}^m \subseteq \mathscr{N}$, the set of measured functions $F(\mathscr{N}^m)$ associated with $\mathscr{N}^m$ is given by:

$$F(\mathscr{N}^m) := \{F_i, i \in \mathscr{N}^m\}. \quad (2)$$

For example, for the graph $\mathscr{G}$ in Fig. 1, if we measure the node 4 at time $k$, we obtain

$$F_4(u_1^{k-3}, u_2^{k-3}) = y_4^k = f(w_{43} f(w_{31} u_1^{k-3} + w_{32} u_2^{k-3})). \quad (3)$$

We denote with $F(\mathscr{N}^m)$ the set of measured functions generated by a weight matrix $W$ consistent[1] with $\mathscr{E}$ and with $\tilde{F}(\mathscr{N}^m)$ the set generated by another matrix $\tilde{W}$ consistent with $\mathscr{E}$.

[1]A weight matrix $W$ is *consistent* with the edge set $\mathscr{E}$ if $w_{ij} \neq 0$ if and only if $(i, j)$ in $\mathscr{E}$.

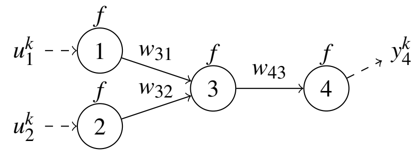


Figure 1: Example of dynamics where nodes 1 and 2 are excited and node 4 is measured.

We then say that a graph $\mathscr{G}$ is *identifiable* in a class of functions $\mathscr{F}$ if, for any given $f$ in $\mathscr{F}$, $F(\mathscr{N}^m) = \tilde{F}(\mathscr{N}^m)$ implies $W = \tilde{W}$. If a network is not identifiable, different weight matrices can generate exactly the same behavior, and therefore, recovering the weights from several experiments is impossible. On the other hand, if the network is identifiable, the weights can be distinguished from all others. Then, if the functions in $F(\mathscr{N}^m)$ can be well approximated after sufficiently long experiments, it becomes feasible to approximate the weights through excitation and measurement.

## 2 Main result

We study local identifiability when the class of functions considered is made of all analytic functions that cross the origin, i.e.,

$$\mathscr{F}_Z := \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ analytic in } \mathbb{R}, f(0) = 0\}. \quad (4)$$

Our main result is to show that, for $\mathscr{F}_Z$ be as in (4), DAGs are *generically locally identifiable* in the class $\mathscr{F}_Z$ by exciting the *sources* and measuring the *sinks*. This holds even when all other nodes remain unexcited and unmeasured and stands in sharp contrast to most findings on network identifiability requiring measurement and/or excitation of each node. The result applies in particular to feed-forward artificial neural networks with no offsets and generalize previous literature by considering a broader class of functions and topologies.

## 3 Acknowledgements

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## References

[1] M. Vanelli and J. M. Hendrickx, "Local identifiability of networks with nonlinear node dynamics", `https://arxiv.org/abs/2412.08472`, 2024