

A Two-point Random Gradient-free Method for Model-free Feedback Optimization

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1 Abstract

Feedback optimization (FO) [1] has been established as a powerful paradigm for the optimal steady-state operation of dynamical systems. By integrating optimization algorithms directly into the control loop, FO allows systems to steer toward optimal operating points. However, a significant limitation of traditional FO methods is their dependency on explicit knowledge of the plant model or gradients. In many applications, such as power electronic converters, energy markets, or industrial processes, accurate models are unavailable due to switching nonlinearities, unknown disturbances, or parameter drift. Moreover, the performance objectives (e.g., thermal stress, user satisfaction) are often only accessible via real-time measurements, whereby analytical gradient computation becomes impossible.

To overcome this, recent works have explored *model-free* approaches. Notably, one-point residual feedback methods [2] have been proposed; however, they suffer from slower convergence rates compared to first-order model-based methods. In this work, we address this limitation by proposing a novel two-point random gradient-free method for feedback optimization. Our approach adapts zeroth-order optimization techniques [3] to dynamical systems that leads to efficient model-free operation.

This work features three main contributions. *First*, we introduce a randomized algorithm that estimates descent directions using two function evaluations. Unlike static optimization, applying two-point methods to dynamical systems is non-trivial because the system state evolves between measurements. Our algorithm overcomes this by alternating between randomized *exploration steps* (perturbing inputs to probe the system and estimate the gradients) and *update steps* (adjusting controls based on estimated gradients). This design allows the controller to operate using only real-time performance measurements, without requiring state resets or model knowledge.

Second, we provide a rigorous convergence analysis. We prove that for smooth, nonconvex cost functions and the proper choice of parameters, our method drives the system to an ε -neighborhood of a stationary point. Crucially, we show that our method achieves a convergence rate of $\mathcal{O}(p\varepsilon^{-1})$, where p is the dimension of the input. This result significantly outperforms existing one-point feedback

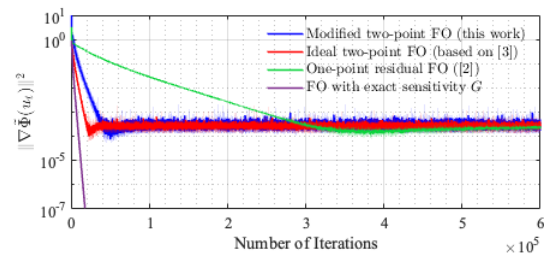


Figure 1: Comparison of convergence rates. The proposed two-point method (blue) outperforms the one-point residual method (green) and approaches the performance of the idealized baseline (red).

optimization methods (which typically scale as $\mathcal{O}(\varepsilon^{-2})$ or $\mathcal{O}(\varepsilon^{-1.5})$) and recovers the theoretical efficiency of two-point methods used in static optimization.

Third, we provide explicit guidelines for parameter selection, meaning the conditions for the stepsize η and smoothing parameter δ that guarantee stability and convergence.

Figure 1 illustrates the performance of the proposed method through numerical simulations. The plot compares the squared norm of the gradient (a measure of optimality) over time. Our method strikes a favorable balance between the fast but unimplementable “idealized” two-point baseline and the slower one-point residual feedback methods.

Future research directions include extending this framework to constrained optimization problems and exploring adaptive stepsize methods to further enhance transient performance.

References

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