Global Observability and Detectability Analysis for a Class of Nonlinear Models of Biological Processes with Bad Inputs

Alexander Schaum *, Jaime A. Moreno †, Alejandro Vargas ‡
* ISR, University of Stuttgart, Germany and Instituto de Ingeniería, UNAM, Mexico. alexander.schaum@gmx.de
† Automatización, Instituto de Ingeniería, National University of Mexico (UNAM).
Cd. Universitaria, C.P.04510 México D.F., MEXICO. JMorenoP@iingen.unam.mx
‡ Bioprocesos Ambientales, Instituto de Ingeniería, National University of Mexico (UNAM).
Cd. Universitaria, C.P.04510 México D.F., MEXICO. AVargasC@iingen.unam.mx

Abstract— **This paper proposes a new general approach to the global analysis of observability and detectability for nonlinear systems. Based on the definition of indistinguishability it is possible to derive the dynamics of the non-observable part of the system and thus to study its stability properties using methods of nonlinear systems theory. The method is first introduced in general and then applied to a class of nonlinear models for biological processes as e.g. in waste water treatment. Finally detectability conditions for the reactor model are deduced.**

Index Terms— **Bad Inputs, Bioreactors, Observability Analysis, Detectability, Indistinguishability.**

I. INTRODUCTION

This paper concerns nonlinear systems of the following general form:

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, u), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \boldsymbol{y} &= \boldsymbol{h}(\boldsymbol{x}),\end{aligned} \tag{1}$$

where $\boldsymbol{x} : \mathbb{R}_+ \mapsto \mathcal{D} \subseteq \mathbb{R}^n$, $\boldsymbol{u} : \mathbb{R}_+ \mapsto \Omega \subseteq \mathbb{R}^m$ and $\boldsymbol{y} : \mathbb{R}_+ \mapsto \mathcal{H} \subseteq \mathbb{R}^p$ with $\mathcal{D}$ open and connected. Due to the lack of knowledge of all system states, e.g for implementing a specific control law many control strategies require the design of an observer. Observability analysis is an important part of this design as all observer design methods assume some observability condition.

Most of the known methods to check the observability properties of (1) make use of the observability map [9] $\Phi = [\boldsymbol{h}(\boldsymbol{x}), \ldots, L_{\boldsymbol{f}}^{n-1}\boldsymbol{h}(\boldsymbol{x})]^T$ which is in general a vector-valued function depending on the system state $\boldsymbol{x}(t)$ as well as on the output $\boldsymbol{h}(\boldsymbol{x})$ and its first $n-1$ (or more) time derivatives, expressed as Lie derivatives along the vector-field $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$. The most common way is checking the rank of its Jacobian and making use of the *inverse function theorem* for local observability statements (for global assertions one can use the theorem of *Palais* e.g. [7]). From this approach it is hard to yield a global statement of observability. The only way to say that the observability map can distinguish all states is to check its injectivity. But for systems with higher dimensional coupled dynamics it is in general not possible to make a clear statement on the injectivity of this map. Therefore for such systems an analysis based on the observability map only serves for local observability results. Furthermore, if the map is not injective, it is difficult to determine if the system is detectable since the observability map is static and detectability includes the dynamics of the system.

The objective of this paper is to propose an alternative approach to analyze the (global) observability and detectability properties of a nonlinear system. This method is based on fundamental and well known definitions of these properties, and their relationships with the indistinguishability concept. In the following the solution of (1) will be denoted in its functional dependence, i.e. $\boldsymbol{x}(t) = \boldsymbol{x}(t; \boldsymbol{x}_0, \boldsymbol{u}(t))$ only if necessary.

II. GLOBAL OBSERVABILITY AND DETECTABILITY ANALYSIS

The following fundamental definitions of indistinguishability, observability and detectability will be used.

Definion 1: (Indistinguishability)
Two initial states $\boldsymbol{x}_0^1, \boldsymbol{x}_0^2$ are said to be indistinguishable if $\exists \boldsymbol{u}(t) \in \Omega$ such that $\boldsymbol{y}(t; \boldsymbol{x}_0^1, \boldsymbol{u}(t)) = \boldsymbol{y}(t; \boldsymbol{x}_0^2, \boldsymbol{u}(t))$, $\forall t \geq 0$ and we write $\boldsymbol{x}_0^1 I \boldsymbol{x}_0^2$. Equivalently two trajectories $\boldsymbol{x}^1(t), \boldsymbol{x}^2(t)$ are said to be indistinguishable if $\boldsymbol{h}(\boldsymbol{x}^1(t; \boldsymbol{x}_0^1, \boldsymbol{u}(t))) = \boldsymbol{h}(\boldsymbol{x}^2(t; \boldsymbol{x}_0^2, \boldsymbol{u}(t)))$, $\forall t \geq 0$ and we write $\boldsymbol{x}^1(t) I \boldsymbol{x}^2(t)$.
Effectively the indistinguishability of trajectories is caused by that of their initial states. Based on this definition observability of system (1) is defined.

Definition 2: (Observability)
System (1) is said to be globally observable if for all (initial) states $\boldsymbol{x}^1, \boldsymbol{x}^2 \in \mathcal{D}$ with $\boldsymbol{x}^1 I \boldsymbol{x}^2$ it follows $\boldsymbol{x}^1 = \boldsymbol{x}^2$.
This means that system (1) is observable if and only if it does not have any indistinguishable trajectories nor states. Similar to this definition the following definition of detectability is given in correspondence to the linear case:

Definition 3: (Detectability)
System (1) is said to be detectable if for all trajectories $\boldsymbol{x}_1(t), \boldsymbol{x}_2(t)$ with $\boldsymbol{x}_1(t) I \boldsymbol{x}_2(t)$, $\forall t \geq 0$ it follows that $\lim_{t\to\infty} \{||\boldsymbol{x}_1(t) - \boldsymbol{x}_2(t)||\} = 0$.
Note that for detectability indistinguishable trajectories are possible, but they have to be convergent. It is easy to see that observability implies detectability. Note, that the definitions for nonlinear observability are manifold and thus we gave here the definitions in a form which is adequately for the given system analysis.

IEEE Catalog Number: 05EX1097
ISBN: 0-7803-9230-2

Now assume two identical plants with the same input, i.e.

$$\Sigma : \begin{cases} \dot{\boldsymbol{x}} = & \boldsymbol{f}(\boldsymbol{x}, u), \quad \dot{\boldsymbol{\xi}} = \quad \boldsymbol{f}(\boldsymbol{\xi}, u) \\ \boldsymbol{y} = & \boldsymbol{h}(\boldsymbol{x}), \quad \boldsymbol{\gamma} = \quad \boldsymbol{h}(\boldsymbol{\xi}) \end{cases},$$

were $\boldsymbol{x}(t),\ \boldsymbol{\xi}(t) \in \mathcal{D}$ with initial states $\boldsymbol{x}(0) = \boldsymbol{x}_0$, $\boldsymbol{\xi}(0) = \boldsymbol{\xi}_0$, respectively. Note that the second plant is an exact copy of the first one, driven with the same input signal. Now define the difference between the two states according to distinct initial conditions $\boldsymbol{\epsilon} := \boldsymbol{x} - \boldsymbol{\xi}$ and the difference between the corresponding outputs $\boldsymbol{\delta} := \boldsymbol{y} - \boldsymbol{\gamma}$. With these definitions one can formulate the difference or error dynamics

$$\Sigma^* : \begin{cases} \dot{\boldsymbol{x}} = & \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \\ \dot{\boldsymbol{\epsilon}} = & \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) - \boldsymbol{f}(\boldsymbol{x} - \boldsymbol{\epsilon}, \boldsymbol{u}) \\ \boldsymbol{\delta} = & \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{h}(\boldsymbol{x} - \boldsymbol{\epsilon}). \end{cases} \quad (2)$$

For indistingushable trajectories of system (1) the output $\boldsymbol{\delta} \equiv 0$ of (2) vanishes, and so the Differential-Algebraic-System (DA-system)

$$\tilde{\Sigma} : \begin{cases} \dot{\boldsymbol{x}} = & \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \\ \dot{\boldsymbol{\epsilon}} = & \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) - \boldsymbol{f}(\boldsymbol{x} - \boldsymbol{\epsilon}, \boldsymbol{u}) \\ 0 = & \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{h}(\boldsymbol{x} - \boldsymbol{\epsilon}). \end{cases} \quad (3)$$

represents the *zero*-dynamics of Σ^* and describes exactly all indistinguishable trajectories of (1), i.e. its indistinguishable dynamics. Therefore the observability and detectability of the plant can be studied analyzing $\tilde{\Sigma}$, as stated in the following proposition, which proof is directly derived from Definitions.

Proposition 1: System (1) is observable in the sense of Definition 2 if $\forall \boldsymbol{x}_0, \boldsymbol{\epsilon}_0, \boldsymbol{u}(t)$ the only solution of $\tilde{\Sigma}$ is $\boldsymbol{\epsilon} \equiv 0$. It is detectable in the sense of Definition 3 if $\forall \boldsymbol{x}_0, \boldsymbol{\epsilon}_0, \boldsymbol{u}(t)$ in $\tilde{\Sigma}$ it follows that $\lim_{t\to\infty} \boldsymbol{\epsilon}(t) = 0$.

Note that detectability doesn't imply that there exists an $\boldsymbol{\epsilon}(t) \neq 0$ such that $\boldsymbol{\delta} \equiv 0$ and therefore observability of (1) implies its detectability as mentioned above. As it is well known, DA-systems can have strange behavior and thus it may be difficult to analyze $\tilde{\Sigma}$. Nevertheless, the here proposed method is the only known that allows to study the dynamical properties of the indistinguishable dynamics in a direct manner. Sometimes it is possible to reduce the DA-System to an explicit differential system.

III. Application to a bioreactor model

Now the proposed method is used to analyze a bioreactor model describing e.g. a process applied in the treatment of waste water. Note that for such processes most models are similar to the one studied here and thus this analysis is valid for a great class of nonlinear systems occurring in biological process engineering. In the following X represents the biomass used to degrade the toxic substrate S. As the biomass and the substrate are hard to measure, the dissolved oxygen concentration O is considered in order to obtain an easily measurable output of the system. The input $u(t)$ is

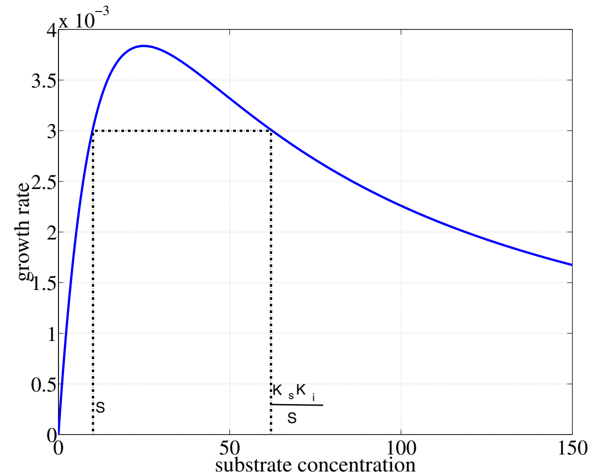

Fig. 1. Specific biomass growth rate $\mu(S)$

given by the dilution rate. The systems equations then read:

$$\begin{aligned} \dot{X} =& \mu(S)X - (K_d + u)X \\ \dot{S} =& - C_1\mu(S)X + u(S_{in} - S) \\ \dot{O} =& - (C_2\mu(S) + b)X + \\ & + u(O_{in} - O) + K_{la}(O_s - O) \\ y =& h(\boldsymbol{\eta}) = O, \end{aligned} \quad (4)$$

with the initial condition $[X_0\ S_0\ O_0]^T$. Note that the system is positive, i.e. $\mathcal{D} = \mathbb{R}^3_+$. Further all parameters are positive. Thus by defining the positive system state vector $\boldsymbol{\eta}(t) := [X(t)\ S(t)\ O(t)]^T$, (4) can shortly be rewritten as

$$\begin{aligned} \dot{\boldsymbol{\eta}} =& \boldsymbol{f}(\boldsymbol{\eta}, u), \quad \boldsymbol{\eta}(0) = \boldsymbol{\eta}_0 \\ y =& [0\ 0\ 1]^T \boldsymbol{\eta}. \end{aligned} \quad (5)$$

The specific biomass growth rate μ depends on the substrate concentration S in a non monotonic way according to a *Haldane* kinetics. In the case of the considered bioreactor used in the waste water treatment its S-dependence is modeled as

$$\mu(S) = \frac{\mu_0 S}{\frac{S^2}{Ki} + S + K_s} \quad (6)$$

and is thus noninjective. It reaches its maximum $\mu_{max} < \mu_0$ in $S_{max} = \sqrt{K_s K_i}$. For further studies and deduction of this model using mass balance see [3], [5], [4] and references therein. In [4] it has been shown that this system is stable in the sense of Lyapunov and therefore all system trajectories are bounded. Further it already has been studied with respect to observability using classical methods. The main result was that because of the non injectivity of $\mu(S)$ the observability map $\Phi = [h, L_f h, L_f^2 h]^T$ becomes non injective too, except in S_{max} and thus can't be used to distinguish the states. One can derive certain cases where the observability is lost using Φ, but it is very hard to derive a complete representation of the dynamics on the indistinguishable manifold. Such a global analysis is necessary for analyzing the detectability in the sense of Definition 3, i.e. the convergence of all indistinguishable trajectories, as well as to obtain a complete

IEEE Catalog Number: 05EX1097
ISBN: 0-7803-9230-2

knowledge of all cases causing indistinguishability. This could not be obtained using Φ.

To obtain a complete representation of the indistinguishable dynamics we follow the proposed way and introduce a copy of the system with the system state $\boldsymbol{\zeta}(t) = [\xi(t)\ \sigma(t)\ \omega(t)]^T \in \mathcal{D}$ as

$$\begin{aligned}\dot{\boldsymbol{\zeta}} =& \boldsymbol{f}(\boldsymbol{\zeta}, u), \quad \boldsymbol{\zeta}(0) = \boldsymbol{\zeta}_0 \\ \rho =& [0\ 0\ 1]\boldsymbol{\zeta},\end{aligned} \tag{7}$$

with the initial state $\boldsymbol{\zeta}_0 = [\xi_0\ \sigma_0\ \omega_0]^T$. Further we introduce the state error $\boldsymbol{\epsilon} = \boldsymbol{\zeta} - \boldsymbol{\eta}$ and suppose that the two outputs are dynamically identical, i.e. $y(t) = \rho(t),\ \forall t \in \mathbb{R}_+, \ \Rightarrow \epsilon_3 = 0$. Thus we have the following DAE-representation of the indistinguishability dynamics of (4):

$$\begin{aligned}\dot{\boldsymbol{\eta}} =& \boldsymbol{f}(\boldsymbol{\eta}, u^*) \\ \dot{\boldsymbol{\epsilon}} =& \boldsymbol{f}(\boldsymbol{\eta} + \boldsymbol{\epsilon}, u^*) - \boldsymbol{f}(\boldsymbol{\eta}, u^*) \\ 0 =& \epsilon_3,\end{aligned} \tag{8}$$

using $u^*(t)$ defined as the corresponding necessary input to satisfy the algebraic part under the assumption of the satisfaction by the initial conditions. The algebraic constraint is fulfilled if $O(t)$ and $\omega(t)$ are dynamically identical, i.e. $O(t) = \omega(t),\ \forall t \in \mathbb{R}_+$ and also $O^{(n)}(t) = \omega^{(n)}(t),\ \forall t \in \mathbb{R}_+,\ n \in \mathbb{N}$. This yields the following identity necessary for fulfilling the algebraic constraint

$$-C_2\Big(\mu(S+\epsilon_2)(X+\epsilon_1) - \mu(S)X\Big) - b\epsilon_1 \equiv 0. \tag{9}$$

The corresponding necessary input to reach this equivalence under this assumption is determined as a rational function

$$u^* := \frac{Z(X, S, \epsilon_1, \epsilon_2)}{N(X, S, \epsilon_1, \epsilon_2)}, \tag{10}$$

where $Z(X, S, \epsilon_1, \epsilon_2)$ and $N(X, S, \epsilon_1, \epsilon_2)$ are polynomials in $X, S, \epsilon_1, \epsilon_2$. Using (9) the following DA-system is derived, which lives in $\mathbb{R}^5$ as the restriction to ϵ_3 already has been applied:

$$\begin{aligned}\dot{\boldsymbol{\eta}} =& \boldsymbol{f}(\boldsymbol{\eta}, u^*) \\ \dot{\epsilon_1} =& -\big(bC_2^{-1} + K_d + u^*\big)\epsilon_1 \\ \dot{\epsilon_2} =& \frac{C_1}{C_2}b\epsilon_1 - u^*\epsilon_2 \\ 0 =& \epsilon_3.\end{aligned} \tag{11}$$

From (9) one derives a relationship between ϵ_1 and ϵ_2:

$$\epsilon_1 = X\frac{\mu(S) - \mu(S+\epsilon_2)}{\mu(S+\epsilon_2) + bC_2^{-1}}. \tag{12}$$

Thus the dynamics lives on a 4-dimensional submanifold $\Psi \subset \mathbb{R}^5$. From (12) one directly derives, that if $X \to 0$ then $\epsilon_1 \to 0$. Further that if $\epsilon_2 \to 0$ then also $\epsilon_1 \to 0$, when X is bounded. Analyzing (11) it follows that the ϵ_1-dynamics is exponentially stable because of $u(t) \geq 0,\ \forall\, t \in \mathbb{R}_+$ and the positivity of all system parameters and thus $\big|\epsilon_1(t)\big| \leq \big|\epsilon_{10}\big| \exp\big(-(bC_2^{-1}+K_d)t\big)$, defining $\epsilon_{10} := \epsilon_1(0)$. Its analytical solution can be written as

$$\epsilon_1(t) = \epsilon_{10}\exp\Big(-(bC_2^{-1} + K_d)t - \int_0^t u^*(\tau)d\tau\Big). \tag{13}$$

With this particular solution of (11) one already knows the analytic solution for $\epsilon_2(t)$, which reads, using $\epsilon_{20} := \epsilon_2(0)$:

$$\begin{aligned}\epsilon_2(t) =& \epsilon_{20}\exp\Big(-\int_0^t u^*(\tau)d\tau\Big) + \\ &+ bC_1C_2^{-1}\epsilon_{10}\int_0^t \exp\Big[-\int_0^{t-\tau} u^*(\tau)d\tau - \\ &- (bC_2^{-1} + K_d)\tau - \int_0^\tau u^*(\theta)d\theta\Big]d\tau.\end{aligned} \tag{14}$$

On the other hand one can see that if $\epsilon_1 \to 0$ then it follows from (12) that $\mu(S+\epsilon_2) \to \mu(S)$ or, equivalently, either $\epsilon_2 \to \frac{K_sK_i}{S} - S$ or $\epsilon_2(t) \to 0$. Further one notices that if $\epsilon_1(t) = 0,\ \forall t \in \mathbb{R}_+$, i.e. $X(t) = \xi(t),\ \forall t \in \mathbb{R}_+$ it follows

$$\mu(S(t) + \epsilon_2(t)) \equiv \mu(S(t)). \tag{15}$$

Thus this case can be interpreted as follows: Consider a reduced system representation of (11) supposing the dissolved oxygen concentration to be not considered and further $X(t)$ and $\xi(t)$ as outputs of the systems (4) and (7) respectively. Then system (11) reduces to the the dynamics of $X(t),\ S(t)$ and $\epsilon_2(t)$, as $\epsilon_1 \equiv 0$. Note that the bad trajectories calculated based on this assumption are valid in both systems because of their equivalence under (15). The corresponding input in this case reduces to

$$\begin{aligned}\tilde{u}^* =& \mu(S)X\frac{\mu'(S+\epsilon_2) - \mu'(S)}{\mu'(S+\epsilon_2)(S_{in} - S - \epsilon_2) - \mu'(S)(S_{in} - S)} \\ =& \mu(S)X\frac{2S + \epsilon_2}{S_{in}(2S+\epsilon_2) - 2K_sK_i}.\end{aligned} \tag{16}$$

Using this result and the equivalence (15) the dynamics can be reduced to

$$\begin{aligned}\dot{X} =& X\big(\mu(S) - K_d - \tilde{u}^*\big) \\ \dot{\epsilon_2} =& -\tilde{u}^*\epsilon_2. \\ S =& \frac{1}{2}\big(-\epsilon_2 + \sqrt{\epsilon_2^2 + 4K_SK_i}\big).\end{aligned} \tag{17}$$

This system now describes the indistinguishability dynamics of (11) for the case of $\epsilon_1(t) = 0$ and thus represents the asymptotic dynamics on the submanifold $\Xi \subset \Psi$ which is attractive in the sense that the extended state vector $[X\ S\ \epsilon_1\ \epsilon_2]^T \to \Xi$ as $\epsilon_1 \to 0$. Note, that by using the algebraic constraint for S, the constitutive dynamics of (17) turns into an autonomous system living in the plane. Based on this interpretation we formulate the following proposition

Proposition 2: Consider the dynamics (17) on the differentiable manifold Ξ. Further suppose $S_{in} > S_{max}$ constant, then the dynamics of $\epsilon_2(t)$ is asymptotically stable $\forall\ [X_0\ S_0\ \epsilon_{20}]^T \in \Xi$.

IEEE Catalog Number: 05EX1097
ISBN: 0-7803-9230-2

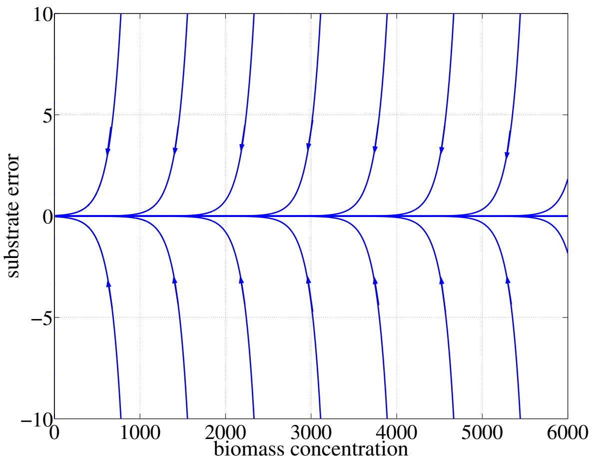

Fig. 2. Phase portrait of the indistinguishability dynamics

Proof: From (17) one can directly see that the analytical solution of $\epsilon_2(t)$ is

$$\epsilon_2(t) = \epsilon_{20} \exp\big(-\int_0^t \tilde{u}^*(\tau)d\tau\big). \qquad (18)$$

Thus if one can assure the input to be integrally unbounded, i.e. $\int_0^t u(\tau)d\tau \rightarrow \infty$ as $t \rightarrow \infty$, its asymptotical stability is warranted. As one can see from the algebraic constraint on S, it is not possible for S to converge to zero on the indistinguishable manifold Ξ. Thus $\mu(S(t)) > 0 \; \forall \; t \in \mathbb{R}_+$ holds. Further the only zero of (16) is $\epsilon_2 = -2S$ which contradicts (15). The positiveness of the input signal (16) is assured if $S_{in} > S_{max}$ and thus under this constraint $\left|\epsilon_2(t)\right| \leq \left|\epsilon_{20}\right| \; \forall \; t \in \mathbb{R}_+$, i.e. ϵ_2 is bounded. Now to proof the asymptotical stability of ϵ_2, i.e. its attractiveness and stability in the sense of *Lyapunov*, we make use of the well-known theorem of *Poincaré-Bendixson*. Therefore we first remind that $\tilde{u}^* > 0 \Leftrightarrow X \neq 0$. According to the theorem mentioned there exists only three possibilities for each trajectory in the plane: *(i)*: it converges asymptotically to an equilibrium point, *(ii)*: it diverges, i.e. $\left|\left| \cdot \right|\right| \rightarrow \infty$ or *(iii)*: it converges to a limit-cycle. The dynamics (17) has at most two equilibrium points located on the X-axis. Thus if the trajectory converges then $\epsilon_2(t) \rightarrow 0$, i.e. $\epsilon_2 \equiv 0$ is attractive. As ϵ_2 is non-increasing it is further stable. If $\left|\left|[X(t) \;\; \epsilon_2(t)]^T\right|\right| \rightarrow \infty$, then $\exists \; \kappa > 0 \; : \; \tilde{u}^*(t) > \kappa \; \forall \; t \in \mathbb{R}_+$, i.e. the input is integrally unbounded and thus $\lim_{t\rightarrow\infty} \epsilon_2(t) = 0$. The case of a limit cycle cannot occur as $\epsilon_2(t)$ is non-increasing. □

Thus under the above assumptions ($y = X$, $\rho = \xi$, $\epsilon_1 \equiv 0$, $S_{in} > S_{max}$) system (4) is detectable. Figure 2 illustrates the phase diagram of the indistinguishability dynamics in this case.

It should be mentioned that there exist initial values $\boldsymbol{\eta}_0 \in \Psi$ for which the input would have to be piecewise negative and thus the according trajectories can not be maintained in the indistinguishable manifold for all time. This is another advantage of the proposed method, that one can analyze the definiteness of possible indistinguishability. Completing, it should be mentioned that detectability admits infinitely slow convergence of trajectories and that the problem of the existence of indistinguishable trajectories for the observation thus does not become much better.

Since it is impossible to distinguish different trajectories causing the same output signal, while the input is the same, with any observer, detectability is a necessary condition for the existence of an observer. So in this case one can try to obtain an observation law enabling to estimate the distinguishable trajectories. However, one cannot yield convergence of all observer trajectories to the system trajectories, but nevertheless one could be able to estimate those which are distinguishable, while having the knowledge, that all indistinguishable will converge as it was shown.

IV. Conclusions

This paper has presented a natural way to investigating the observability and detectability of nonlinear systems, based on the indistingushability concept. This method is based on the dynamical description of the indistingushable trajectories of the system, the so called *indistinguishable dynamics*, and its convergence properties. The method is applied to the study of the detectability properties of a biological reactor with inhibitory kinetics, that is frequently used in e.g. the treatment of waste water. It could be shown that there exist indistinguishable trajectories and according bad input functions and thus the system is not observable. The corresponding identities as well as the analytical solutions for the error dynamics were derived. Further detectability conditions have been deduced by classical stability analysis.

V. Acknowledgement

Thanks to PAPIIT-UNAM (Project IN111905-2) for its financial support.

This paper includes results of the EOLI project that is supported by the INCO program of the European Community, Contract number ICA4-CT-2002-10012. The scientific responsibility rests with the authors.

References

[1] Moreno J., Dochain D., *Global Observability and Detecability Analysis for Uncertain Reaction Systems.* IFAC World Congress, Prague, *2005*

[2] Ibarra-Rojas S., Moreno J. Espinoza-Perez G., *Global observability analysis of sensorless induction motors*, Automatica 40, p.1079-1085, *2004*

[3] Henze M. et al, *Activated sludge model* IAWPRC Scientific and Technical Reports No.1, *1986*

[4] Moreno J., Vargas A., *Nonlinear Observer Design and Observability of an Aerobic Bioreactor using Respirometry*, Proc. of the ECC99, Karlsruhe, Germany, *1999*

[5] Moreno J., Buitron G. *Optimation of an Aerobic Bioreactor for the Industrial Wastewater Treatment*, Computacion y Sistemas. Numero Especial pp. 74–82, *2002*

[6] Hermann R., Krener A.J., *Nonlinear Controlability and Observability*, IEEE Trans. on automatic control, vol. AC-22, No.5, *1977*

[7] Sandberg I. W., *Global Inverse Function Theorems*, IEEE Trans. on Circuits and Systems, Vol. CAS-27, No. 11, *1980*

[8] Sandberg I. W., *Global Implicit Function Theorems*, IEEE Trans. on Circuits and Systems, Vol. CAS-28, No. 2, *1981*

[9] Zeitz M. *Canonical forms for nonlinear systems.* In Proc. of IFAC Nonlinear Control Systems Design, pages 33–38, Capri, Italy, *1989*.

IEEE Catalog Number: 05EX1097
ISBN: 0-7803-9230-2