ATTAINABILITY AND SUBOPTIMAL MINIMAL TIME CONTROL OF A CLASS OF BIOLOGICAL SEQUENCING BATCH REACTORS

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Abstract: In this paper, geometric tools are used for the attainability study of biological Sequencing Batch Reactors (SBR). The objective is to find the set of initial conditions from which one can reach a final state target with an admissible control. The possible solutions can be determined easily once the reachability set has been characterized. In a second step, an optimal time control problem is considered. It consists in finding the switching instants between the different phases of the bioreaction. Pontryagin's Maximum principle is used to solve a part of this problem. A suboptimal solution is proposed allowing at most two commutations. A new problem is then considered where the criteria is parameterized by the switching concentrations. A numerical solution is finally proposed to solve this new minimal time problem.

Keywords: Suboptimal control, Reactor control modeling, Reachability, Numerical optimuzation

1. INTRODUCTION

Nitrogen removal in batch reactor for wastewater treatment is realized in two successive steps: denitrification and nitrification. (Henze *et al.*, 1987). Usually, the reactors are a priori designed for a known range of concentrations. The reactor volume and the phases duration are fixed. However, if there are significant variations of the influent concentrations, no guarantee on the effluent purification is given and the possibility of carrying out both reactions in the batch is not assured. In this kind of process, the reactions are related and the final concentrations have to reach a terminal target. This leads to a controllability analyses. The possible solutions can be analyzed and the trajectories are optimized. To handle this problem we proceeded as follows :

- The batch process is modeled using standard mass balance principles. The dynamical behavior of the process is studied and all the relations between the state variables are derived to reduce the model and to simplify the reachability study.
- With terminal state constraints representing a target set, admissible solutions are analyzed to estimate the necessary number of switching.
- Optimal time control problem is solved to reduce the phases duration and to find the switching instants. Initial guess of optimal

trajectory is determined using Pontryagin's maximum principle and a numerical algorithm is used to get a suboptimal control law.

This work was motivated by a practical optimization problem : to design a low cost industrial batch reactor for carbon and nitrogen removal. Reducing the total cycle time is equivalent to increasing the volume treated per day or decreasing the reactor volume. Small and effective reactors can be proposed for industrial use.

2. BIOLOGICAL REACTIONS MODELLING

Let us consider an activated sludge bioreactor. To eliminate both carbon and nitrogen, two operating modes are necessary.

- aerobic mode : With aeration, two groups of microorganisms X_1 and X_2 consume the carbon and the nitrogen, respectively, ensuring two independent reactions: carbon removal (reaction 1) and nitrogen removal (reaction 2) where nitrogen is converted into nitrates.

$$S_1 + O_2 \xrightarrow{r1} X_1 \tag{1}$$
$$S_2 + O_2 \xrightarrow{r2} X_2 + S_3 \tag{2}$$

- anoxic mode : without oxygen the denitrification takes place. The X_1 biomass replaces the oxygen by the nitrates S_3 to consume the carbon with respect to the third reaction

$$S_1 + S_2 \xrightarrow{r3} X_1 \tag{3}$$

With respect to these reactions two models are derived. Biological Reactions are modeled using standard laws of biological kinetics (see for instance (Bastin and Dochain, 1990)):

- aerobic model :

$$\dot{X}_1 = r_1(S_1, X_1) \tag{4}$$
$$\dot{X}_2 = r_2(S_2, X_2) \tag{5}$$
$$\dot{S}_1 = -k_{11} r_1(S_1, X_1) \tag{6}$$
$$\dot{S}_2 = -k_{22} r_2(S_2, X_2) \tag{7}$$
$$\dot{S}_3 = k_{32} r_2(S_2, X_2)) \tag{8}$$

- anoxic model :

$$\dot{X}_1 = r_3(S_1, S_3, X_1) \tag{9}$$
$$\dot{X}_2 = 0 \tag{10}$$
$$\dot{S}_1 = -k_{13} r_3(S_1, S_3, X_1) \tag{11}$$
$$\dot{S}_2 = 0 \tag{12}$$
$$\dot{S}_3 = -k_{33} r_3(S_1, S_3, X_1) \tag{13}$$

where $r_j(.)$ is the biomass growth given by $\mu_j\left(S_1, .., S_n\right) X_i$, μ_j is the growth rate modeled with a positive map which vanishes if and only if one of the S_i vanishes, (X_i is strictly positive and k_{ij} are the stoichiometric yield coefficients related to the i^{th} substrat of the j^{th} reaction.

In the aerobic phase, O_2 is controlled and fixed at a constant value.

Furthermore, yield coefficients k_{11} and k_{13} in reactions (1) and (3) are identical.

3. REACHABILITY ANALYSIS

At the end of the batch process, substrate concentrations have to be lower than the normative constraints. The control problem to be solved is to find the switching sequences to ensure the three reactions (1-3) alternatively take place.

Definition 1. Let $u \in U = \{0, 1\}$ be the control variable. It corresponds to the switching signal : $u = 1$ for aerobic mode and $u = 0$ for anoxic mode.

By associating the two models in the following matrix form :

$$\begin{bmatrix} \dot{X}_1 \\ \dot{S}_1 \\ \dot{X}_2 \\ \dot{S}_2 \\ \dot{S}_3 \end{bmatrix} = \left[\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -k_{11} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -k_{33} \end{bmatrix} + u \begin{bmatrix} 1 & 0 & -1 \\ -k_{11} & 0 & k_{11} \\ 0 & 1 & 0 \\ 0 & -k_{22} & 0 \\ 0 & k_{32} & k_{33} \end{bmatrix} \right] \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \tag{14}$$

we easily derive the following relationships :

$$\dot{S}_1 + k_{11} \dot{X}_1 = 0 \tag{15}$$
$$\dot{S}_2 + k_{22} \dot{X}_2 = 0 \tag{16}$$

By integration we get :

$$M_1 = S_1 + k_{11} X_1 = S_1(0) + k_{11} X_1(0) \tag{17}$$
$$M_2 = S_2 + k_{22} X_2 = S_2(0) + k_{22} X_2(0) \tag{18}$$

where M_1 and M_2 depend only on the initial conditions and remain constant during the reaction whatever the control. This is the consequence of the mass conservation : the mass of substrate which is degraded is transferred to the biomass (Szederkenyi *et al.*, 2002)(Moreno, 1999). Thus, there exists a linear relationship between S_1 and X_1 (respectively S_2 and X_2) :

$$X_1 = -\frac{S_1 - M_1}{k_{11}} \tag{19}$$
$$X_2 = -\frac{S_2 - M_2}{k_{22}} \tag{20}$$

3.1 Model simplification

In this paper, Monod kinetics are considered :

$$r_1(S_1, X_1) = \mu_{1max} \frac{S_1}{k_{S1} + S_1} X_1 \quad (21)$$

$$r_2(S_2, X_2) = \mu_{2max} \frac{S_2}{k_{S2} + S_2} X_2 \quad (22)$$

$$r_3(S_1, S_3, X_1) = \mu_{1\max} \rho \frac{S_1}{k_{S1} + S_1} \frac{S_3}{k_{S3} + S_3} X_1 \quad (23)$$

One can replace expressions of X_1 and X_2 in (21) - (23) to obtain the reduced model :

$$\begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \end{bmatrix} = \begin{bmatrix} f(S_1) \\ g(S_2) \\ -\alpha g(S_2) \end{bmatrix} u + \begin{bmatrix} f(S_1)h(S_3) \\ 0 \\ \beta f(S_1)h(S_3) \end{bmatrix} (1-u) \quad (24)$$

where $\alpha = \frac{k_{32}}{k_{22}}$ and $\beta = \frac{k_{33}}{k_{11}}$ and $\rho < 1$
$f(S_1) = \mu_{1\max}(S_1 - M_1) \frac{S_1}{k_{S1}+S_1}$, $h(S_3) = \rho \frac{S_3}{k_{S3}+S_3}$
and $g(S_2) = \mu_{2\max}(S_2 - M_2) \frac{S_2}{k_{S2}+S_2}$

A compact form of the model is given by :

$$\begin{cases} \dot{Z} = F(Z) + G(Z)u \\ Z_0 = [S_1(0),\ S_2(0),\ S_3(0)] \end{cases} \quad (25)$$

where $Z^t = [S_1, S_2, S_3]$ is the state vector, F and G are analytic field vectors in $\mathbb{R}^3$ easily identified from equation (24).

3.2 Dynamic behavior

Proposition 2. Let $S_1(t, S_1(0), u)$, $S_2(t, S_2(0), u)$ and $S_3(t,(S_1(0), S_2(0)), u)$ be the solutions of the differential equations (24)where u is a constant control.

- $S_1(.)$ and $S_2(.)$ are two decreasing maps tending towards zero.
- $S_3(t, Z_0, 1)$ decreases and $S_3(t, Z_0, 0)$ increases.

Proof

- $M_1 = S_1 + k_{11}X_1 > S_1$ (res $M_2 = S_2 + k_{22}X_2 > S_2$) so for all $S1$ (res $S2$), $f(S_1) < 0$ (res $g(S_2) < 0$). Since $h(S_3) > 0$ and $u \in \{0, 1\}$ one can deduce that $\frac{dS_1}{dt} < 0\ \forall S_1 > 0$ (res $\frac{dS_2}{dt} < 0\ \forall S_2 > 0$) moreover $\left.\frac{dS_1}{dt}\right|_{S_1=0} = 0$ (res $\left.\frac{dS_1}{dt}\right|_{S_1=0} = 0$) so using Lyapunov theorem for asymptotic convergence (Clarke and Ledayaev, 1998) we deduce that $S_1(.)$ and $S_2(.)$ decrease and tend asymptotically toward zero. $\square$
- For the same reason, if $u = 1$, $\frac{dS_3}{dt} > 0$ then $S_3(t, Z_0, 1)$ increases, and if $u = 0$, $\frac{dS_3}{dt} < 0$ then $S_3(t, Z_0, 0)$ decreases. $\square$

Proposition 3. Since u is a piecewise constant mapping, the solution of system (25) verifies the following properties :

- $Z(t, Z_0, 1) \subset \Sigma(Z_0)$
- $Z(t, Z_0, 0) \subset \Delta(Z_0)$

where

$$\Sigma(Z_0) = \{Z \in \mathbb{R}^{3+}/S_1 < S_1(0), S_2 < S_2(0), S_3 - S_3(0) = -\alpha(S_2 - S_2(0))\}$$

$$\Delta(Z_0) = \{Z \in \mathbb{R}^3/S_1 < S_1(0), S_2 = S_2(0), S_3 - S_3(0) = \beta(S_3 - S_3(0))\}$$

Proof If $u = 1$, $\dot{S}_3 = -\alpha\dot{S}_2$ then $S_3 - S_3(0) = -\alpha(S_2 - S_2(0))$ moreover $S_1 < S_1(0)$ and $S_2 < S_2(0)$ so $Z(t, Z_0, 1) \in \Sigma(Z_0)$
If $u = 0$, $\dot{S}_2 = 0$ and $\dot{S}_3 = \beta\dot{S}_1$ then $S_2 = S_2(0)$ and $S_3 - S_3(0) = \beta(S_1 - S_1(0))$ moreover $S_2 < S_2(t)$ so $Z(t, Z_0, 0) \in \Delta(Z_0)$ $\square$

Definition 4. (Accessibility and Reachability). Consider the system (25) where $Z(t, Z_0, u)$ is its maximal solution. The set of accessible points at time $T > 0$ is $A^+(Z_0, T) = \cup_{u\in U} Z(t, Z_0, u)$ and the set of accessibility is given as $A^+(Z_0) = \cup_{T>0} A^+(Z_0, T)$ in a similar way, we note $A^-(Z_0, T)$ the set of point from which Z_0 can be reached at time T and $A^-(Z)$ the set of reachable points.

Now, consider a family of targets C, open sets in $\mathbb{R}^3$, given by : $C(Z_N) = \{Z \in \mathbb{R}^{3+}, Z_N^t = [S_{1N}, S_{2N}, S_{3N}]/S_1 < S_{1N}, S_2 < S_{2N}, S_3 < S_{3N} - \alpha S_2\}$, $S_{iN, i=1..3}$ are the normative rejection norms. The Reachability set $\Omega(Z_N)$ is the set of points from which the target is reachable using the dynamics of the system (25). It is given by :
$\Omega(Z_N) = \{A^-(Z_0)/Z(t, Z_0, u) \in C(Z_N)\}$. We define also the sets:
$\Omega_A(Z_N) = \{Z \in \mathbb{R}^{3+}, S_2 < \alpha^{-1}S_{3N}, S_3 < \beta S_1 - \alpha S_2 + S_{3N}\}$
$\Omega_B(Z_N) = \{Z \in \mathbb{R}^{3+}, S_2 > \alpha^{-1}S_{3N}, S_3 < S_1 - \alpha S_2 + \beta(S_3 - S_{3N})\}$
$\Omega_C(Z_N) = \Omega_A(Z_N) \cup \Omega_B(Z_N)$

In a previous work (Mazouni *et al.*, 2004), it has been shown that from some initial conditions we cannot reach the target and the set of reachability for this problem is given by $\Omega(Z_N) = \Omega_C(Z_N)$. This set is constructed using all the possible concatenations between piecewise trajectories $Z(t, Z_0, 1)$ and $Z(t, Z_0, 0)$ from which the target can be reached. For more details, refer to (Mazouni *et al.*, 2004). We also proved that with one anoxic phase the trajectory can reach the target from any point in this reachable set.

In the following, we consider only initial conditions in this reachable set.

4. MINIMAL TIME CONTROL PROBLEM

4.1 Problem and statement

In this section we consider the minimal time problem in which we try to find an optimal sequence of switching between aerobic and anoxic modes to minimize the total cycle time. Optimal switching instants have to be determined. In order to solve the problem using the Pontryagyn's maximum principle, we consider the extended problem where the control variable takes values in the closed convex set [0, 1]. The possible solutions are analyzed to check possible bang-bang solutions. Our solution is sub-optimal is the sense that we look for at most two commutations.

4.2 Pontryagin's Maximum Principle (PMP)

Consider a control-affine system in $\mathbb{R}^3$ of the form:

$$\dot{Z}(t) = F(Z(t)) + G(Z(t))u(t) \tag{26}$$

where F and G are two analytical field vectors in $\mathbb{R}^3$, u a bounded map defined on $\mathbb{R}^+$ and takes value in $U = [u_{\min}, u_{\max}]$. For $Z_0 \in \mathbb{R}^3$, $Z(t, Z_0, u(.))$ is the solution of the differential equation with the initial condition Z_0 at $t = 0$ and control $u(.)$.
Let C be a regular sub manifold in $\mathbb{R}^3$ and $T_Z C$ the tangent space of C at the point Z.
We note $Z^*(t, Z_0, u^*)$ the minimum time trajectory connecting the initial point Z_0 at the target C in the time t^*. The triplet $(Z^*(t), \lambda^*(t), u^*(t))$ verifies:

$$\dot{Z}^*(t) = \frac{\partial H^*}{\partial \lambda}\left(Z^*(t), \lambda^*(t), u^*(t)\right) \tag{27}$$

$$\dot{\lambda}^*(t) = -\frac{\partial H^*}{\partial Z}\left(Z^*(t), \lambda^*(t), u^*(t)\right) \tag{28}$$

$$H(Z^*(t), \lambda^*(t), u^*(t)) = \min_{u \in U} H\left(Z^*(t), \lambda^*(t), u(t)\right) \tag{29}$$

with the boundary condition

$$\lambda^*(t^*) \perp T_{(Z^*(t^*))} C, \tag{30}$$

where

$$H\left(Z(t), \lambda(t), u(t)\right) = \lambda^t(t)\left(F(Z(t)) + G(Z(t))u(t)\right) + \lambda_0 \tag{31}$$

The adjoint vector $\lambda(.)$ verifies $\lambda(t) \not\equiv 0$ at any time and λ_0 is constant positive or null.

$u^*(.)$ is computed as follows(Srinivasan *et al.*, 2003) :

$$u^*(t) = \begin{cases} u_{\min} & if \ \lambda^t(t)G(Z(t)) > 0 \\ u_s & if \ \lambda^t(t)G(Z(t)) = 0 \\ u_{\max} & if \ \lambda^t(t)G(Z(t)) < 0 \end{cases} \tag{32}$$

with the singular control u_s computed by solving the following equations at time t :

$$\begin{aligned} \frac{d}{dt}\left(\lambda^t(t)G(Z(t))\right) &= 0 \\ &\vdots \\ \frac{d^k}{dt^k}\left(\lambda^t(t)G(Z(t))\right) &= 0 \end{aligned} \tag{33}$$

k is chosen such that the control variable u appears explicitly in the k^{th} derivative of the switching function :

$$\lambda^t(t)G(Z(t)) \tag{34}$$

We first study particular cases without any switch and then analyze cases with one anoxic phase.

4.3 Particular case: Solution without switching

Consider again the target $C(Z_N)$ defined previously. The trajectory $Z(t, Z_0, u)$ can reach one of the three sides P_A, P_B or P_C of the target (Cf figure 1). The optimal solutions that reach $P_A \cup P_B$ verify (according to the equation (30)) the following transversality conditions : $\lambda_3(tf) = 0$, $\lambda_1(tf) \geq 0$ and $\lambda_2(tf) \geq 0$. The switching function becomes

$$\Phi(t_f) = \lambda_1(t_f)(f(S_1(t_f))(1 - h(S_3(t_f))) + \lambda_2(t_f)g(S_2(t_f)) \tag{35}$$

Since $\lambda(t) \not\equiv 0$, $f(S_1) < 0$, $g(S_2) < 0$ and $0 < h(S_3) < 1$ we have $\Phi(t_f) < 0$ so $u(t_f) = 1$ according to (32).
with $u = 1$ in a non empty interval $[t,\ t_f]$ while the adjoint vectors are given by

$$\dot{\lambda}_1(t) = -\lambda_1(t)\left(\frac{\partial f(S_1(t))}{\partial S_1}\right) \tag{36}$$

$$\dot{\lambda}_2(t) = -(\lambda_2 + \alpha\lambda_3(t))\left(\frac{\partial g(S_2(t))}{\partial S_2}\right) \tag{37}$$

$$\dot{\lambda}_3(t) = 0 \tag{38}$$

with the boundary conditions $\lambda_3(tf) = 0$, $\lambda_1(tf) \geq 0$ and $\lambda_2(tf) \geq 0$. The solutions of the differential equations (36-38) verify $\lambda_3(t) = 0$, $\lambda_1(t) \geq 0$ and $\lambda_2(t) \geq 0$, so the switching function is negative and does not change its sign in the interval $[t,\ t_f]$ whatever is $t \in [t,\ t_f]$

Corollary 5. All trajectories $Z(t, Z_0, 1)$ solution of (24) that reach the target $C(Z_N)$ i.e. $Z(t, Z_0, 1) \cap C(Z_N) \neq \phi$ verify the PMP conditions. Thus, it is an optimal trajectory and the optimal control is $u^* = 1$

Proposition 6. The set of initial conditions for which the solution of (24) verifies $Z(t, Z_0, 1) \cap C(Z_N) \neq \phi$ is given by :
$\Omega_1(Z_N) = \{Z \in \mathbb{R}^{3+} / S_3 \leq -\alpha S_2 + S_{3N}\}$ referring to the Corollary (5), the optimal control for this set is $u^* = 1$

Proof Lets $Z_0 \in \Omega_1(Z_N)$ so $S_3(0) \leq -\alpha S_2(0) + S_{3N}$. In addition, for $u = 1$, $\dot{S}_3 + \alpha\dot{S}_2 = 0$ so $S_3 - S_3(0) + \alpha(S_2 - S_2(0)) = 0$. When $t \to \infty$ we have $S_1 \to 0$, $S_2 \to 0$ and $S_3^\infty + \alpha S_2^\infty = S_3(0) + \alpha S_2(0) \le S_{3N}$ so $Z^\infty \in C(Z_N)$. $\square$

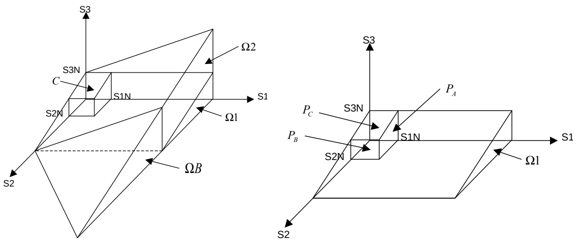

Fig. 1. Reachability set
$\Omega_A(Z_N) = \Omega_1(Z_N) \cup \Omega_2(Z_N)$, $\Omega_C(Z_N) = \Omega_A(Z_N) \cup \Omega_B(Z_N)$

4.4 General case: solution with one anoxic phase

The reachability analysis showed that for each point in $\Omega(Z_N)$ there exists at most one anoxic phase to reach optimally the target. Let us solve the optimal time problem with one anoxic phase i.e. with at most two switches aerobic-anoxic-aerobic. To do so we define the following control :

$$t \in [0\ t_1]\,,\ u = 1 \tag{39}$$

$$t \in [t_1\ t_2]\,,\ u = 0 \tag{40}$$

$$t \in [t_2\ t_f]\,,\ u = 1 \tag{41}$$

to cover all the possible cases, we can have $t_0 = t_1$ and/or $t_1 = t_2$ and /or $t_2 = t_f$. The maximal trajectory is given by: $Z(t \le t_1, Z(t_0), 1) \cup Z(t \le t_2, Z(t_1), 0) \cup Z(t \le t_f, Z(t_2), 1)$

If $Z_0 \in \Omega_1(Z_N)$ we apply corollary (5) so $t_0 = t_1 = t_2$. t_f is given when the trajectory $Z(t, Z(t_2), 1)$ reach the target. If $Z_0 \notin \Omega_1(Z_N)$, t_2 is given when the trajectory $Z(t, Z(t_1), 0)$ reaches $\Omega_1(Z_N)$. If $Z(t, Z(t_1), 0)$ reaches $C(Z_N)$ then $t_f = t_2$. The unknown parameter is t_1 that we parameterize in the following.

Hypothesis 7. The trajectory reaches the side P_A of the target $C(Z_N)$. So at $t = t_f$, we have $S_2(t_f) = S_{2N}$.

4.5 Anoxic phase duration

In anoxic phases, $u = 0$. Thus S_2 is constant. $S_1(t)$ and $S_3(t)$ are linearly related referring to the relationships obtained from the first integral $\frac{dS_3}{dt} = \beta \frac{dS_1}{dt}$. In order to parameterize the anoxic phase time, we introduce the following variable : $l = S_1(t_1) - S_1 = \frac{1}{\beta}(S_3(t_1) - S_3)$ such that $S_1 = S_1(t_1) - l$ and $S_3 = S_3(t_1) - \beta l$.

Let d be the variation of S_1 in anoxic phase which is proportional to the variation of S_3. It is given by : $d = S_1(t_1) - S_1(t_2) = \frac{1}{\beta}(S_3(t_1) - S_3(t_2))$. The anoxic phase time is then given by :

$$J(S_1^1, S_3^1, d) = \int_0^d \frac{1}{f(S_1^1 - l)} \frac{1}{h(S_3^1 - \beta l)} dl = \int_0^d L(l, S_1^1, S_3^1) dl \tag{42}$$

where $(S_1^1,\ S_2^1,\ S_3^1)$ is the solution of the dynamical system (24) for $u = 1$ at $t = t_1$.

Corollary 8. $L(l, S_1^1, S_3^1)$ is a non-negative and decreasing map. We can deduce that if $d1 < d2$ then $J(S_1^1, S_3^1, d1) < J(S_1^1, S_3^1, d2)$. The minimal value of d to reach the target is given by : $d = \frac{1}{\beta}\left(S_3(0) + \alpha S_2(0) - S_{3N}\right)$. It depends neither on the switching time nor on the dynamic of the system. It only depends on the initial conditions and the target.

Proof By definition $d = \frac{1}{\beta}(S_3(t_1) - S_3(t_2))$. The optimal solution $Z(t, Z(t,1), 0)$ crosses Ω_1 at t_2, then $S_3(t_2) = S_{3N} - \alpha S_2(t_2) = S_{3N} - \alpha S_2(t_1)$ so that $d = \frac{1}{\beta}(S_3(t_1) + \alpha S_2(t_1) - S_{3N}$. Moreover t_1 is the final time of the first aerobic phase. In aerobic phase the linear relationships is verified: $S_3(t_1) + \alpha S_2(t_1) = S_3(t_0) + \alpha S_2(t_0)$ because $\frac{dS_3}{dt} = \alpha \frac{dS_2}{dt}$. Thus $d = \frac{1}{\beta}\left(S_3(0) + \alpha S_2(0) - S_{3N}\right)$. $\square$

4.6 Reaction time

In the case when $Z(t, Z(t_2), 1)$ reaches P_B i.e. $S_2(tf) = S_{2N}$ the reaction time is given by :

$$t = \int_{t_0}^{t_1} dt + \int_{t_1}^{t_2} dt + \int_{t_2}^{t_f} dt = \int_{S_2(t_0)}^{S_2(t_1)} \frac{ds}{g(s)} + \int_{t_1}^{t_2} dt + \int_{S_2(t_2)}^{S_2(t_f)} \frac{ds}{g(s)} \tag{43}$$

Since S_2 is constant when $u = 0$ one has $S_2(t_1) = S_2(t_2)$. The equation (43) becomes:

$$t = \int_{S_{20}}^{S_{2N}} \frac{ds}{g(s)} + \int_{t_1}^{t_2} dt = cts + \int_{t_1}^{t_2} dt \tag{44}$$

Remark 9. In this case minimizing the total time is equivalent to minimize the anoxic phase time.

5. NUMERICAL OPTIMISATION AND RESULTS

In the following section we propose a numerical algorithm to find the optimal switching concentrations. The next algorithm is used
Algorithm :

I: Solve the dynamical system with $u = 1$ to determine the set of points S_1^1, S_3^1
II: Deduce the subset where the anoxic phase can be applied i.e $S_1 > d$ and $S_3 > \frac{d}{\beta}$

III: Using an exploration algorithm find (S_1^{1*}, S_3^{1*}) that minimize $J(S_1^1, S_3^1, d)$. (ex: decent method using the gradient)

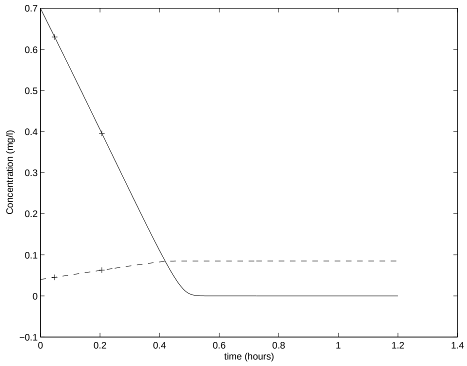

Fig. 2. Set of variations

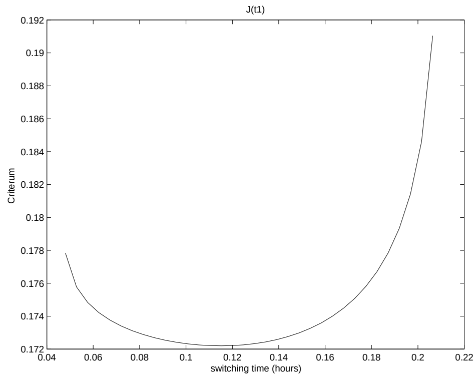

Fig. 3. Criterium to be minimize

6. INTERPRETATION

The problem studied in this paper is motivated by a practical problem. The nitrogen removal kinetics is very slow compared to the carbon removal. Thus, the required aerobic time corresponds essentially to the nitrogen removal. Since nitrogen is not eliminated in anoxic phase, the aerobic phase duration cannot be minimized. We can only reduce the total cycle time by reducing the anoxic phase time. In standard cases the anoxic phase takes place at the beginning of the cycle. The reaction rate depends on the two substrat concentrations S_1 and S_3. At $t_0 = 0$ the maximum of S_1 is available. In aerobic mode S_1 is eliminated and S_3 is produced. The suboptimal control strategy consists in maximizing the anoxic reaction rate by increasing S_3 in aerobic phase, (of course, S_1 decreases in this case). The anoxic phase is applied when the maximal possible rate of the anoxic reaction is reached, so that the total time of this phase is reduced, as it is shown in the latest figures (2 and 3).

7. CONCLUSION

This paper describes a new approach for finding a control law that minimize the total cycle time for carbon and nitrogen removal in SBR processes. In these processes, both aerobic and anoxic phases are required. The dynamical behavior of the model is analyzed and it is shown that at most two switches allow one to reach the target. In this case, the anoxic phase takes place between two aerobic phases. Alterning process sequences can reduce the total time. To find the optimal switching time, a suboptimal time control problem is formulated. The possible solutions and the optimum trajectory are analyzed using Pontryagin's Maximum principle. This leads to find the second switching surface defined by the state variables. Using this information, the problem is translated in a new problem where the criteria is parameterized by the switching concentrations. The new problem is solved using a gradient algorithm to find the first switch. With this approach the total cycle time is optimized.

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