A REFINEMENT OF DYNAMIC PRINCIPAL COMPONENT ANALYSIS FOR FAULT DETECTION

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Abstract: For process monitoring the Dynamic Principal Component Analysis (DPCA) models multivariate historical data under the assumption of stationarity, however, false alarms result for non-stationary new observations during the monitoring task. In order to reduce the false alarms rate, this paper extends the DPCA based monitoring for non-stationary data of linear dynamic systems, including an on-line means estimator and standardizing new observations according to the estimated means. The effectiveness of the proposed methodology is evaluated for faults detection in a control valve benchmark, assuming as measured signals the control signal, stem displacement and flow.

Keywords: Fault Detection, Statistical Analysis, Dynamic Principal Component Analysis, Time Series Analysis, Non-Stationary Signals.

1. INTRODUCTION

The on-line process monitoring for fault detection and isolation, FDI, is an important task to ensure plant safety and product quality. The fault diagnosis is a sequential procedure which involves the symptoms generation through a comparison between the characteristics obtained from a model of the process and those actual characteristics generated by the process, and the corresponding diagnosis from the symptoms analysis. One of the most consolidated FDI techniques of the last twenty years is the analytical approach, which is based on explicit modeling, this is, models obtained from primary physical principles, some of the analytical approaches are very well revised e.g. (Patton *et al.*, 2000). In the case of process or devices like actuators or sensors whose analytical models are not available or are difficult to obtain the techniques based on data-driven can help to overcome the problem of modeling, these techniques are based on implicit modeling through multivariate statistical methods, some of this methods are resumed in (Chiang *et al.*, 2001) and (Venkatasubramanian *et al.*, 2003).

Principal Component Analysis (PCA) is a multivariate statistical method which is able to model the linear correlation structure of a multivariate process from its nominal historical data. PCA transforms by a parallel correlation analysis, a set of multivariate observations to a lower dimension orthogonal space, retaining the most variability of the original data (Jackson, 1991). Because of the simplification and the orthogonal property obtained with PCA, this has been used with success for fault diagnosis issues as in (Kresta *et al.*, 1991) and (Raich and Çinar, 1996).

It is important to note that like other multivariate statistical methods, PCA works under three assumptions: the data of each variable have a normal distribution; there exist no auto-correlation among observations; and the variables are sta-

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tionary, this is, the variables should keep constant mean and standard deviation over time. In the case of data with non normal distribution it is possible to carry out an appropriate transformation like square root or logarithm (Montgomery, 2001), in order to improve the distribution of data. In the case of dynamic systems exists auto-correlation among observations in each variable, so to include the dynamic effects in the PCA based monitoring, it is necessary to incorporate time lags of the time series during the modeling stage, this extension is called dynamic principal component analysis, DPCA (Ku *et al.*, 1995). Some of the applications of DPCA for process monitoring are (Russell *et al.*, 2000) and (Chen and Liu, 2002).

However, since PCA and DPCA assume stationarity during modeling process, high rate of false alarms are generated in the diagnosis stage if the test data are non-stationary. A non-stationary condition has many possible causes e.g. due to components aging, to faults, to changes in the operating point of the plant, etc. Aging and faults are those that a supervision system has to diagnose in order to carry out the appropriate action, however, changes in the operating point have another context and the fault detection scheme has to be robust over normal changes in the operating point while remain sensitive to faults.

The above described problem motivated this work in which the stationary condition is overcome through signals means estimation in order to reduce the false alarms rate. The means estimation of the input and output signals is based in single moving average (SMA) and the associated relations between variables for normal operating condition. Fault detection based on DPCA will be briefly reviewed. Next, the extension of DPCA based fault detection for changes in the operating point (non-stationary conditions) will be described. Finally the proposed methodology will be evaluated for faults detection in a flow control valve.

2. FAULT DETECTION VIA DPCA

The fault detection task is divided in two stages, the first one is to obtain a nominal data based process statistical model and the last is the fault detection as such which consists in a classification procedure of an on line generated symptom.

2.1 *Nominal Statistical Model*

Let the matrix $\mathbf{X}^p$ be a set of nominal historical data composed of n_t observations from p variables of a process, described as

$$\mathbf{X}^p = \left[\, \mathbf{X}_1 \ \mathbf{X}_2 \ \cdots \ \mathbf{X}_p \,\right]_{(n_t \times p)} \tag{1}$$

If data set $\mathbf{X}^p$ is generated from a dynamic system, then each colum $\mathbf{X}_h$ represents an auto-correlated time series. In DPCA the serial correlation is included constructing the named *trajectory matrix* applying 'time lags' of order w on each time series of the matrix $\mathbf{X}^p$. This is

$$\mathbf{X}_h^w = \begin{bmatrix} X_h(1) & X_h(2) & \cdots & X_h(w) \\ X_h(2) & X_h(3) & \cdots & X_h(w+1) \\ \vdots & \vdots & \ddots & \vdots \\ X_h(n_t - w + 1) & X_h(n_t - w + 2) & \cdots & X_h(n_t) \end{bmatrix}_{(n \times w)}$$

$$\mathbf{X}^m = \left[\, \mathbf{X}_1^w \ \mathbf{X}_2^w \ \cdots \ \mathbf{X}_p^w \,\right]_{(n \times m)} \tag{2}$$

where $n = n_t - w + 1$ and $m = pw$. The w value is selected on the base of the number of correlated lags of the variables.

To avoid some particular variables dominate the modeling process, it is convenient to carry out a data standardization in matrix $\mathbf{X}^m$ in relation to its means and standard deviations, from which a zero mean and unit variance standardized data matrix $\widetilde{\mathbf{X}}^m$ is obtained.

The means vector and standard deviations vector of the trajectory matrix $\mathbf{X}^m$ are

$$\mu^m = \left[\, \mu_1^w \ \mu_2^w \ \cdots \ \mu_p^w \,\right]_{(1 \times m)} \tag{3}$$

$$\sigma^m = \left[\, \sigma_1^w \ \sigma_2^w \ \cdots \ \sigma_p^w \,\right]_{(1 \times m)} \tag{4}$$

thus, the data standardization is obtained through

$$\widetilde{X}^m(i,j) = \frac{X^m(i,j) - \mu^m(j)}{\sigma^m(j)} \tag{5}$$

for $i = 1, \ldots, n$ and $j = 1, \ldots, m$.

The principal components statistical model $\mathbf{Y}^l$ of dimension $n \times l$ is defined as a linear transformation of the original variables involved in $\widetilde{\mathbf{X}}^m$, such principal components extracted are uncorrelated vectors. The matrix of principal components is obtained through the following transformation

$$\mathbf{Y}^l = \widetilde{\mathbf{X}}^m \mathbf{V}_t \tag{6}$$

where the transformation matrix $\mathbf{V}_t \in \Re^{m \times l}$ is composed of an appropriate selection of l eigenvectors associated to the correlation matrix $\mathbf{R}$ of the trajectory matrix $\mathbf{X}^m$. Note that $\mathbf{R}$ is composed of the set of auto-correlation and cross-correlation coefficients for the w lags.

For each l-variate observation in the principal components model $\mathbf{Y}^l$ calculate the behavior symptom described by the univariate statistic $T^2_{\mathbf{Y}_i^l}$ called Hotelling parameter, this is

$$T^2_{\mathbf{Y}_i^l} = \left(\mathbf{Y}_i^l\right) \mathbf{S}_{\mathbf{Y}^l}^{-1} \left(\mathbf{Y}_i^l\right)^T \tag{7}$$

where $\mathbf{S}_{\mathbf{Y}^l}^{-1}$ is the covariance matrix of $\mathbf{Y}^l$. Finally, a threshold of normal condition from the probability density function of the set of parameters $T^2_{\mathbf{Y}_i^l}$ is calculated. (Tracy *et al.*, 1992) propose, among others, for a β distribution of the data set $T^2_{\mathbf{Y}^l_i}$ the threshold UCL as

$$UCL = \frac{(n-1)^2\left(\frac{l}{n-l-1}\right)F\left(\frac{\alpha}{2};l,n-l-1\right)}{n\left(1+\left(\frac{l}{n-l-1}\right)F\left(\frac{\alpha}{2};l,n-l-1\right)\right)} \quad (8)$$

where n and l are the dimensions of $\mathbf{Y}^l$ and α is a level of significance.

2.2 *Fault Detection*

In the fault detection stage, a m-dimensional observation vector $\mathbf{Xa}^m$ is taken on line, which is standardized with respect to the means (3) and standard deviations (4); transformed through the matrix $\mathbf{V}_t$ in the principal components subspace $\mathbf{Ya}^l$; and mapped through the matrix $\mathbf{S}^{-1}_{\mathbf{Y}^l}$ in the univariate parameter $T^2_{\mathbf{Ya}^l}$; if the resulting value deviates from the UCL, then an indication of the presence of a fault is generated.

DPCA detects a deviation of vector $\mathbf{Xa}^m$ from the nominal reference in terms of its mean and its standard deviation, this is the main property of DPCA which can be managed to solve fault detection issues. However, it is important to note that the modeling process is based in the data set $\mathbf{X}^p$ which was obtained in a particular operating point of the system, so any change in the nominal values of the signals is interpreted by DPCA as a fault, even when the process is healthy, this misinterpretation is because of the time variant (non-stationary) behavior of components in (3).

To overcome this difficulty, during the detection stage, the on line estimation of the statistical set (3) is suggested assuming healthy conditions and its use for the standardization procedure. This estimation can be achieved if simple relations among variables of the system in normal conditions are obtained. The objective is to standardize the actual observation vector $\mathbf{Xa}^m$, which can be around any operating point, in relation to the estimated means, obtaining for a healthy observation $\mathbf{Xa}^m$ the condition of zero mean and unit variance.

3. DPCA WITH MEAN PARAMETER ESTIMATION

3.1 *Nominal Statistical Model*

Lets consider for simplicity the case of a SISO linear system, operating in normal condition around an operating point, which sets of input data $\mathbf{X}_1$ and output data $\mathbf{X}_2$ are described by the matrix

$$\mathbf{X}^p = \mathbf{X}^2 = \left[\, \mathbf{X}_1 \;\, \mathbf{X}_2 \,\right]$$

$$\mathbf{X}^m = \left[\, \mathbf{X}^w_1 \;\, \mathbf{X}^w_2 \,\right] \quad (9)$$

Following the standard DPCA procedure described in section 2.1, the principal components model $\mathbf{Y}^l$ can be obtained from the historical data $\mathbf{X}^2$ as well as the threshold from the probability density function of the set of $T^2_{\mathbf{Y}^l_i}$.

3.2 *Estimating the relations between input and output variables*

Given that in a dynamic system the output is a function of the input, then under normal operating conditions the lost of the stationarity in signals X_1 and X_2 is a consequence of variations in the level changes of X_1, so a Moving Average model of order q, MA(q), is proposed for the output mean estimation, this is

$$\begin{aligned} X_2(k) &= \phi_0 X_1(k) + \cdots + \phi_q X_1(k-q) \\ &= \sum_{j=0}^{q} \phi_j X_1(k-j) \end{aligned} \quad (10)$$

The $q+1$ parameters ϕ of the model MA(q) are obtained as follows. Multiplying (10) by $X_1(k-s)$ and taking the expected values it results

$$E\left[X_1(k-s)X_2(k)\right] = \phi_0 E\left[X_1(k-s)X_1(k)\right] + \cdots + \phi_q E\left[X_1(k-s)X_1(k-q)\right] \quad (11)$$

where $E\left[X_1(k_1)X_2(k_2)\right] = R_{X_1X_2}(k_1,k_2)$ is the cross-correlation between $X_1(k_1)$ and $X_2(k_2)$, and $E\left[X_1(k_1)X_1(k_2)\right] = R_{X_1X_1}(k_1,k_2)$ is the autocorrelation between $X_1(k_1)$ and $X_1(k_2)$, thus, rewriting (11)

$$R_{X_1X_2}(s) = \phi_0 R_{X_1X_1}(s) + \phi_1 R_{X_1X_1}(s-1) + \cdots + \phi_q R_{X_1X_1}(s-q) \quad (12)$$

Evaluating (12) for $s = 0, \ldots, q$ a set of linear equations for $\phi_0, \ldots, \phi_q$ is obtained in terms of $R_{X_1X_2}(.)$ and $R_{X_1X_1}(.)$. For the estimation of parameters ϕ is necessary to carry out estimations of the cross-correlation and auto-correlation coefficients at the required lags. Estimated cross-correlation and auto-correlation coefficients are given in the correlation matrix $\mathbf{R}$ used in the nominal statistical modeling, this fact is important since it is possible to exploit the correlation structure among variables summarized in $\mathbf{R}$ in order to obtain relations among variables for means estimation.

So, the output mean estimated is given by

$$\widehat{X}_2(k) = \sum_{j=0}^{q} \widehat{\phi}_j X_1(k-j)$$

$$E\left[\widehat{X}_2(k)\right] = \widehat{\mu}_{X_2}(k) = \sum_{j=0}^{q} \widehat{\phi}_j E\left[X_1(k-j)\right] \quad (13)$$

From (13), the mean in the output variable $\widehat{X}_2$ depends on the $q+1$ means of X_1 which can be calculated applying SMA of order $q+1$ on the input variable X_1.

It is important to note that the estimators are designed just to calculate on-line the means of the variables, thus this approach doesn't require precise analytical models like the explicit model based fault detection algorithms.

3.3 *Fault Detection*

According to the proposed extension to the standard DPCA based fault detection algorithm, the procedure to evaluate and classify an actual observation $\mathbf{X}a^m$ can be summarized as follows:

(1) Estimate through the relation given in (10) the w output values $\widehat{X}_2(k)$ ... $\widehat{X}_2(k+w-1)$ from actual input data $Xa_1(k)$ ... $Xa_1(k+w-1)$ and construct the following vector

$$\widehat{\mathbf{X}}a^m = \left[\, Xa_1(k) \ \cdots \ Xa_1(k+w-1) \ \cdots \right. \\ \left. \cdots \ \widehat{X}_2(k) \ \cdots \ \widehat{X}_2(k+w-1) \,\right]_{(1\times m)} \tag{14}$$

(2) Estimate through SMA the mean vector $\widehat{\mu}_1^w$ for the input variable, and through (13) the mean vector $\widehat{\mu}_2^w$ for the output variable

$$\widehat{\mu}^m = \left[\, \widehat{\mu}_1^w \ \widehat{\mu}_2^w \,\right]_{(1\times m)} \tag{15}$$

$\widehat{\mu}^m$ will be used for the standardization procedure.

(3) Generate, from real data of the input and output signals of the process, the vector of actual observations with time lags of order w

$$\mathbf{X}a^m = \left[\, Xa_1(k) \ \cdots \ Xa_1(k+w-1) \ \cdots \right. \\ \left. \cdots \ Xa_2(k) \ \cdots \ Xa_2(k+w-1) \,\right]_{(1\times m)} \tag{16}$$

(4) Standardize the m terms in (16) using the estimated means vector (15) and the standard deviations in (4), this is

$$\widetilde{X}a^m(j) = \frac{Xa^m(j) - \widehat{\mu}^m(j)}{\sigma^m(j)}$$

for $j = 1, \ldots, m$. $\widetilde{X}a^m(j)$ will have approximately unit variance and zero mean under normal operating condition even before level changes in the input signal.

(5) Transform the vector $\widetilde{\mathbf{X}}\mathbf{a}^m$ in the principal component subspace $\mathbf{Ya}^l$ through $\mathbf{V}_t$

$$\mathbf{Ya}^l = \widetilde{\mathbf{X}}\mathbf{a}^m \mathbf{V}_t$$

(6) Map $\mathbf{Ya}^l$ in the behavior symptom $T^2_{\mathbf{Ya}^l}$ through

$$T^2_{\mathbf{Ya}^l} = \left(\mathbf{Ya}^l\right) \mathbf{S}^{-1}_{\mathbf{Y}^l} \left(\mathbf{Ya}^l\right)^T$$

(7) If the resulting value deviates from the normal condition threshold a fault is present in the system.

The key of the proposed methodology is in the standardization procedure with respect to the online estimated means where as is pointed out in step 4 for a healthy observation $\mathbf{Xa}^m$ its standardization will have approximately unit variance and zero mean even before level changes. The last statement is valid since any healthy actual observation vector $\mathbf{Xa}^m$ around to a set of means different to that considered in the modeling stage, in fact, is affine to each one of the observations in $\mathbf{X}^m$ and as a consequence the correlation $\mathbf{R}$ between the m variables in $\mathbf{Xa}^m$ remains unchanged (Kailath *et al.*, 2000).

3.4 *Motivating Example*

Lets consider the case of a second order linear system (17), from which 400 nominal data, collected in the matrix $\mathbf{X}^2 = \left[\, \mathbf{X}_1 \ \mathbf{X}_2 \,\right]$, have been measured at a sample time of 0.1s.

$$\frac{X_2(s)}{X_1(s)} = \frac{16}{s^2 + 2.4s + 16} \tag{17}$$

Following the procedure described in sections 2.1 and 3.2 the DPCA based principal components model $\mathbf{Y}^l$ results of dimensions 301×5, so for an $\alpha = 0.1$ the resulting threshold is $UCL = 10.95$. By the other hand the following relation between the input and output variable for mean estimation was estimated

$$E\left[\widehat{X}_2(k)\right] = -0.35E\left[X_1(k)\right] + 0.45E\left[X_1(k-1)\right] + \\ +0.25E\left[X_1(k-2)\right] + 1.08E\left[X_1(k-3)\right] - \\ -0.47E\left[X_1(k-4)\right] \tag{18}$$

The current estimated mean of the output variable $\widehat{X}_2$ will be obtained evaluating (18) with the last 5 estimated means of the input variable X_1.

To evaluate the performance of the proposed algorithm, before changes in the operating point, the system has been simulated for step changes in X_1 of magnitude +0.3 and -0.3, additional to the operating point considered in the statistical modeling stage, in the intervals $(50s \le t \le 150s)$ and $(300s \le t \le 400s)$, respectively.

In Fig. 1 the upper plot shows the input and output variables subject to positive and negative step changes, while the lower plot shows the corresponding behavior symptom, generated by the standard DPCA based fault detection algorithm, compared with the UCL threshold where a high rate of false alarms is observed principally during the step changes intervals. In Fig. 2 the upper plot shows the output X_2 and estimated output

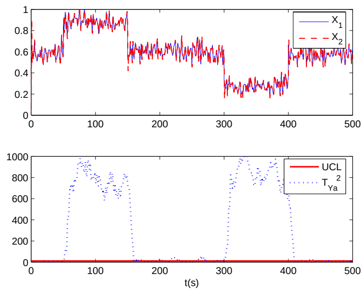

Fig. 1. Response of the standard DPCA algorithm (assuming constant means) under step changes in U

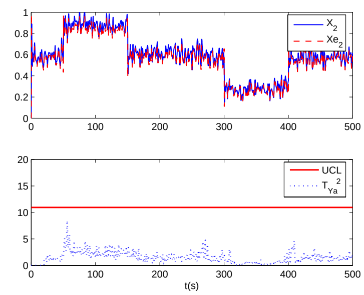

Fig. 2. Response of the modified DPCA algorithm (estimating means) under step changes in U

Xe_2, while the lower plot shows the corresponding behavior symptom, generated by the proposed fault detection algorithm, compared with the UCL threshold where a significative reduction in the rate of false alarms is observed which demonstrates the ability of the proposed algorithm to manage non-stationary signals.

In the following section the proposed fault detection algorithm is applied to detect faults in a control valve considering simple relations for the means estimation.

4. EXAMPLE OF A FLOW CONTROL VALVE

4.1 *Valve Simulator*

The simulator reported in the DAMADICS (Development and Application of Methods for Actuator Diagnosis in Industrial Control Systems) benchmark (Bartyś and Syfert, 2002) is used. This benchmark consists of the valve model and other modules which comprise the DABLib (DAMADICS Actuator Benchmark Library).

The fault detection task is carried out exploiting the variety of internal signals measured in the flow control valve: control signal CV, as input variable, and the stem displacement X, and flow F, as output variables, will be used by the proposed algorithm to detect mechanical faults in the valve. Taking a set of 200 nominal observations measured every 0.5s and following the procedure described in section 2.1 the DPCA based principal components model $\mathbf{Y}^l$ results of dimensions 151×10, so for an $\alpha = 0.1$ the resulting threshold is $UCL = 17.79$.

4.2 *Estimating the relations among variables*

For $\widehat{X}$ means estimation from estimated means of CV the following relation has been obtained

$$E\left[\widehat{X}(k)\right] = 0.03E\left[CV\left(k\right)\right] + 0.27E\left[CV\left(k-1\right)\right] + \\ +0.31E\left[CV\left(k-2\right)\right] + 0.02E\left[CV\left(k-3\right)\right] + \\ +0.44E\left[CV\left(k-4\right)\right]$$

By the other side, between $\widehat{X}$ and $\widehat{F}$ a static non-linear relation was fitted to the type equal percentage valve flow characteristic, this is

$$P(\widehat{X}) = -7.7\widehat{X}(k)^3 + 19.9\widehat{X}(k)^2 - 17.8\widehat{X}(k) + 5.6$$

$$\widehat{F}(k) = \underset{[0,1]}{sat}\left(P(\widehat{X})\right)$$

4.3 *Detection Results*

The fault detection algorithm is evaluated considering the following cases:

(1) Normal operation of the valve with step changes in the input signal.
(2) Fault condition, *f1-Valve Clogging* of magnitude 0.5 with step changes in the input signal.

The first evaluation has as goal to test the performance of the algorithm before step changes in the control signal CV when the valve is operating in normal conditions. Positive and negative step changes of 10% of the operating point has been proposed in the following sequence: positive step from $(100s \leq t \leq 300s)$ and negative step from $(500s \leq t \leq 700s)$.

As it can be seen from Fig. 3 the simple DPCA algorithm generates false alarms during the transient response of the step changes in CV even when the valve is working healthy; this phenomenon is due to the non-stationary behavior of time series CV, X and F.

The response of the proposed algorithm, with online means estimation, is given in Fig 4 which shows the reduction of the false alarm rate.

The second part of the validation shows the response of the proposed algorithm under fault $f1$ of magnitude 0.5. The input signal is changed at $200s$ and the fault appears at time $500s$.

As it is seen in Fig. 5, the symptom value has a value close to zero if there is a change in the

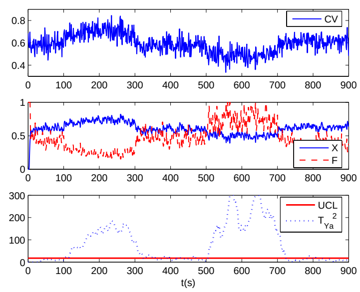

Fig. 3. Response of the standard DPCA algorithm before step changes in *CV*

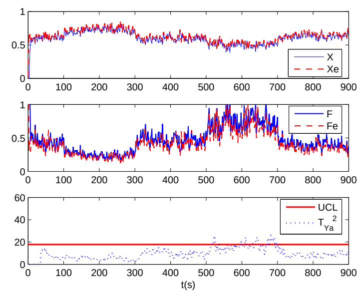

Fig. 4. Actual and estimated values of variables X and F; and response of the proposed algorithm before step changes in *CV*

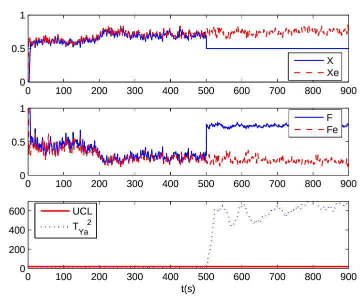

Fig. 5. Detection of fault *f1-Valve Clogging* of magnitude 0.5 before step change in *CV*

input signal and only exceed the threshold when the fault is present.

5. CONCLUSIONS

Here, a modification to the DPCA algorithm for fault detection has been proposed, in which an appropriate standardization with respect to on-line estimated statistical parameters is carried out if simple healthy relations between variables can be obtained. It has been exploited the information of the correlation structure of the correlation matrix not only for DPCA based statistical modeling but also to obtain simple relations among variables for mean estimation purposes. This idea allows to deal with non-stationary signals and to reduce significatively the rate of false alarms. It was shown through a series of tests the effectiveness of the proposed fault detection algorithm to distinguish between normal changes in signals and the variations due to the presence of faults.

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