

## PRACTICAL OPTIMAL CONTROL FOR FED-BATCH BIOREACTORS

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**Abstract:** Optimal operation of fed-batch bioreactors is an important practical issue. Since control actions are saturation-limited, the optimal control consists usually of both singular and bang-bang arcs. However, its realization requires good model knowledge and also measurement of all state variables, requirements hardly satisfied in real applications. In this paper a method is proposed, for a class of bioreactors, to robustly optimize the operation when few measurements are available and the model is uncertain. Such control law is justified and its properties analyzed. The class of processes addressed includes a biomass growth bioreactor and an experimental wastewater treatment plant for degrading toxicants. *Copyright © 2004 IFAC*

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### 1. INTRODUCTION

Many important industrial fermentation processes, for the production of antibiotics and enzymes or for the treatment of wastewater, are carried out using bioreactors operated in fed-batch mode. Since it is reasonable to improve their performance, optimal control theory has been used to determine the best control policy (Smets et al., 2002; Sarkar and Modak, 2003; Moreno, 1999). Such strategy can be described as a feedback control law, and it is usually necessary to know perfectly the model of the plant, and to measure the whole state to implement it. In many applications these two conditions are very restrictive: a perfect model and parameter knowledge is very often unrealistic, and in biotechnology and wastewater treatment it is either impossible or very expensive to measure all state variables. In order to cope with the first problem different robust approaches have been proposed in the literature. Most often different adaptive algorithms identify the parameters of the (otherwise assumed well known) mathematical model, and adapt accordingly the control strategy (Bastin and Dochain, 1990; Van Impe and Bastin, 1995; Van Impe, 1998). Adaptive Extremum-Seeking strategies have been also proposed (Marcos et al., 2004; Titica et al., 2003). Although the

methodology is most appropriate for continuous reactors, where an optimal steady state operation is searched, under suitable conditions it can also operate correctly for fed-batch processes (Betancur et al., 2004b).

In this work a different approach to deal with the lack of measurements and the uncertainty in the model, while optimizing operation, of a class of bioreactors will be proposed. The main idea is based on the following observations. Usually, the exact optimal control, when the realistic assumption of limited input variables is done, can be decomposed in bang-bang and singular arcs. When this solution can be implemented via feedback the information required for the bang-bang part is very low, and its implementation is very robust to model uncertainties. More problematic is the determination and implementation of the singular arc. It requires basically a good knowledge of the model and parameters of the plant, and it is usually very sensitive to uncertainties. Our method proposes to replace the sensitive and smooth singular control by a bang-bang one that maintains the system trajectory around the singular surface. The produced error can (theoretically) be made as small as desired (Hermes and LaSalle, 1969; Moreno, 1999). The advantage of this replacement is that it is usually very robust against

uncertainties and, for its implantation, a reduced quantity of information is required (Moreno, 1999). The robustness of this implementation is linked to the well-known properties of Sliding Mode Control (Khalil, 2002; Slotine and Li, 1991). The requirement of low quantity of information is related to the fact that it is only necessary to determine the singular surface in the state space. This surface is usually associated to some events on internal variables. If such events could be software-sensed using just the measurable variables, then a practical solution is feasible. Moreover, if the switching or singular surfaces are robustly related to these variables, the approach can be made robust against model changes or uncertainties.

In the following section the class of problems for which these ideas are developed will be introduced. In Section 3 the previously outlined strategy will be carried out for these problems. In Section 4 some experimental results will be presented.

## 2. PROBLEM FORMULATION

The optimization problem for a class of simple microbial growth process in a fed-batch bioreactor is considered, using the following dynamical model (Smets et al., 2002):

$$\frac{dX}{dt} = \mu X - \frac{F}{V} X \quad (1)$$

$$\frac{dS}{dt} = -(k_1 \mu + m) X + \frac{F}{V} (S_i - S) \quad (2)$$

$$\frac{dV}{dt} = F \quad (3)$$

where states  $X$  (g/L) and  $S$  (g/L) hold for biomass and substrate concentrations, respectively,  $\mu$  ( $h^{-1}$ ) is the specific growth rate,  $S_i$  (g/L) denotes the concentration of the substrate in the inflow,  $k_1$  is a yield coefficient,  $m$  is a maintenance constant,  $F$  (L), the control variable, is the inflow and  $V$  (L) is the volume of the liquid medium in the tank.  $\mathbf{z}^T = [X, S, V]$  represents the state vector, that is restricted to the set

$$\Omega = \{ \mathbf{z} \in \mathfrak{R}^3 \mid 0 < X; 0 \leq S \leq S_{\max}; 0 < V_o \leq V \leq V_{\max} \}. \quad (4)$$

The input is restricted to  $0 = F_{\min} \leq F \leq F_{\max}$ . For any system variable  $\phi$  let's denote its value at instant  $t$  as  $\phi(t, t_0, \mathbf{z}_0, F)$  for input  $F$ , being  $\mathbf{z}_0$  the state at  $t_0 < t$ .

Inhibitory specific growth rates will be considered, although everything is also valid for  $\mu$  monotonic. They are described by a non-monotonic function of the substrate concentration, like the Haldane law:

$$\mu(S) = \frac{\mu_0 S}{K_S + S + S^2 / K_I} \quad (5)$$

where  $\mu_0$  is a parameter related to the maximum value of the specific growth rate  $\mu^*$ ,  $K_S$  denotes the saturation and  $K_I$  the inhibition constant. In general, it will be assumed that  $\mu(S)$  grows monotonically for  $S \in [0, S^*]$  and decreases for  $S \in [S^*, \infty]$ . Such a model is typically used to describe the substrate inhibition effect in fed-batch bioreactors, as shown in Figure 1.

In industrial fermentations (e.g. production of baker's yeast) the aim is to maximize the total biomass at the end of the process (Smets et al., 2002), but when  $m > 0$  this amounts to minimize the reaction time. For Waste Water Treatment Plants (WWTP)  $m$  is usually considered to be zero and the objective is to minimize the time necessary to degrade the substrate (Moreno, 1999). The measured variables depend strongly on the application. In general, it is difficult and expensive to measure biomass and substrate concentrations, but it is easy to measure volume, gaseous products and dissolved oxygen concentrations. For the biotechnological application it will be assumed that the gas production  $y = k_2 \mu X$  is measurable, and then, using the volume  $V$ , it is possible to calculate  $y_B$ , which will be used for the control implementation:

$$y_B = yV = k_2 \mu X V = k_2 \mu B \quad (6)$$

where  $B = XV$  (g) is the total Biomass.

For the WWTP it is assumed that Dissolved Oxygen concentration  $O$  (g/L) and volume level  $V$  (L) are measured. Using its dynamics

$$\frac{dO}{dt} = -(k_3 \mu + b) X + K_1 a (O_s - O) - \frac{F}{V} O \quad (7)$$

and (2), the variable  $y_W$  can be calculated:

$$y_W = (k_3 \mu + b) B = K_1 a V (O_s - O) - F O + V \frac{dO}{dt} \quad (8)$$

For the WWTP the variable  $S_i$  is a (possibly) time-varying perturbation term difficult to measure, but it is a known constant for the biotechnological process. For both systems the functional description of the specific growth is not well known so (5) is only an approximation. The objective is to design an optimal control law that uses the measured variables, respectively  $y_B$  and  $y_W$ , and that is robust against uncertain model and uncertain parameters.

## 3. CONTROL STRATEGY

The design of a control law that satisfies the imposed requirements will be explained in three steps.

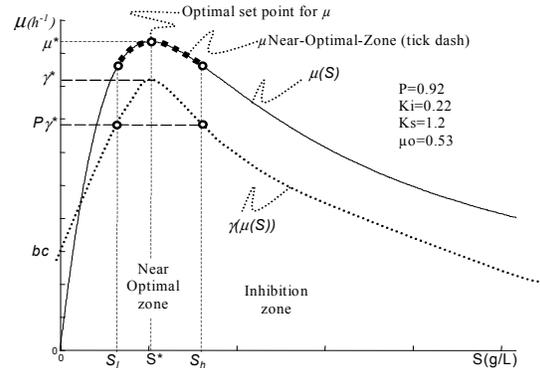


Figure 1. Haldane's type biomass specific growth rate. The dotted line depicts a measurable  $\gamma = (a\mu + b)c$

### 3.1. Exact optimal control law

For both systems the optimal control law for one batch consists of two bang-bang arcs, and an intermediate singular arc. The batch finishes when  $V=V_{max}$  and  $S \leq S_{end}$ , where  $S_{end}$  is some (small) rest substrate value. The optimal control law, to minimize reaction time, is given by (Smets et al., 2002; Moreno, 1999)

$$F_{opt} = \begin{cases} 0 & \text{if } V = V_{max} \text{ or } S > S^* \\ F_{sin} & \text{if } S = S^* \\ F_{max} & \text{if } S < S^* \end{cases} \quad (9)$$

$$\text{where } F_{sin} = \frac{k_1(\mu + m) VX}{S_i - S} \quad (10)$$

is the control inflow along the singular arc, and  $S^*$  is the value where  $\mu$  reaches its maximum  $\mu^*$ . Define the function  $T(F, \mathbf{z}_0)$  to represent the time necessary to bring the initial state  $\mathbf{z}_0$  to the target set  $Z_f = \{(X, S, V) / S \leq S_{min} \wedge V \geq V_{max}\}$  using  $F$  as input (the other model parameters are fixed).  $T_{opt}(\mathbf{z}_0) = T(F_{opt}, \mathbf{z}_0)$  corresponds to the optimal path. Note that in the state space  $\Omega$  the surfaces defined by  $S=S^*$  and  $V=V_{max}$  are, respectively, the singular and a switching surface. Note that implementation of feedback law (9) requires, in principle, good knowledge of the model of the plant and the measurement of all state variables.

### 3.2. Approximated optimal control law

A well-known result (Hermes and LaSalle, 1969) states that any trajectory of a nonlinear system can be arbitrarily well approximated by one generated using a bang-bang control law. So, it is possible to approximate the time optimal trajectory generated by the control law (9) with a bang-bang one. This corresponds in this case to the approximation of the trajectory along the singular arc with a bang-bang one (Moreno, 1999). Since the performance criterion does not depend on the input function, and it is continuous with respect to the trajectories, then the optimal index is arbitrarily nearly reached by the approximated trajectory. For the problem at hand this has also been directly demonstrated by (Moreno, 1999).

Approximate control laws can be generated in different forms. They differ basically in the information required to implement them. Some examples will be discussed in the following paragraphs. The following assumption will be made:

*Assumption 1:* In the state space  $\Omega$  it is satisfied that

$$\frac{dS}{dt} = -(k_1\mu + m)X + \frac{F_{max}}{V}(S_i - S) > \varepsilon > 0 \quad (11)$$

Note that inequality (11) has to be satisfied for  $S \leq S^*$  for the control law (9) to be feasible, and so it is a natural condition for the problem.

#### 3.2.1. Suboptimal control law using $S$

If  $S$  is measured and  $S^*$  is known, then selecting values  $0 < S_l < S^* < S_h$  (see Figure 1) the following control law approximates arbitrarily well the optimal trajectory (Moreno, 1999)

$$F = \begin{cases} 0 & \text{if } V = V_{max} \text{ or } S \geq S_h \\ F_{max} & \text{if } S \leq S_l \end{cases} \quad (12)$$

In the interval  $S_l < S < S_h$  the control function can, in fact, take any value but, in order to obtain a bang-bang cycle, the previous one is kept until the other extreme is reached (this is a hysteresis effect, see Figure 2). Moreover, when  $S \leq S_{end}$  and  $V \geq V_{max}$  then the reaction phase is finished and a new cycle may be started.

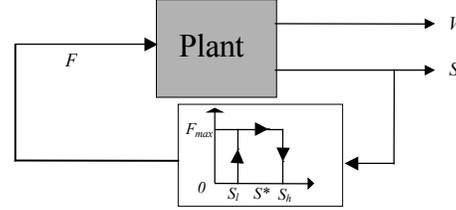


Figure 2. Control scheme for  $S$  measurable,  $S^*$  known. Control of Volume not shown for simplicity

The important feature of this control law is that, since

$$\frac{dS}{dt} = -(k_1\mu + m)X < 0, \forall \mathbf{z} \in \Omega / \{S = 0\} \quad (13)$$

and because of Assumption 1, every system trajectory tends to the set  $S_l \leq S \leq S_h$ , stays there until the reactor is full, and then reaches the final set  $Z_f$ , finishing the reaction phase. Moreover, if Assumption 1 is satisfied, substrate  $S(t, t_o, S_l, F_{max})$  increases monotonically until it reaches  $S_h$ , and  $S(t, t_o, S_h, 0)$  decreases monotonically until  $S_l$ . It is important to note that the trajectory is confined to the set  $S_l \leq S \leq S_h$ , independently of and without knowledge of the parameters and/or the exact form of the growth rate and of the value of the input substrate concentration  $S_i$ .

*Theorem 1:* (Moreno, 1999) Suppose that system (1-3) is given,  $S$  is measured and  $S^*$  is known. Then replacing the optimal control law (9) by the approximate control law (12) the time to reach the target, from any initial condition  $\mathbf{z}_0 \in \Omega$ , is  $T(F, \mathbf{z}_0) = T_{opt}(\mathbf{z}_0) + \Delta(F, S_l, S_h, \mathbf{z}_0)$  with  $\Delta(F, S_l, S_h, \mathbf{z}_0)$  continuous and finite and  $\Delta \rightarrow 0$  as  $S_h, S_l \rightarrow S^*$ . i.e. the target will be reached in finite time, and the time along the approximate trajectory can be made as near to the optimal one as desired. This is true for any form of the growth rate (positive if  $S$  positive), for unknown  $S_i$  and for any positive value of parameters  $k_1, m$ .

Note that the approximate control law is robust against model and parameter uncertainties and/or changes. Moreover, it requires little information from the system: only  $S$  needs to be measured and  $S^*$  and the instant when  $V=V_{max}$  need to be known.

#### 3.2.2. Robust Suboptimal control law using $f(\mu)$

If instead of  $S$  an appropriate but unknown function of  $\mu$  is available, i.e.  $\gamma = f(\mu)$ , then it is possible to derive a control law that approximates arbitrarily well the optimal one. It will be assumed that  $f(\mu)$  is a strictly monotonic increasing and continuous function of  $\mu$ , as for example  $f(\mu) = (a\mu + b)c$ ; for  $a, b, c$  unknown constants,  $a, c > 0$ . The maximum  $\gamma^* = \max_{S \geq 0} f(\mu(S)) = f(\mu^*)$

is then well defined. It should be noted that it is not necessary to know the exact shape of  $\mu$  or  $f(\mu)$ . If Assumption 1 is satisfied, and because of the monotone increasing (decreasing) behavior of  $S(t)$  when it goes from  $S_l$  to  $S_h$  (from  $S_h$  to  $S_l$ , respectively), it is clear that  $\gamma(t)$  increases from  $\gamma_l = f(\mu(S_l))$  ( $\gamma_h = f(\mu(S_h))$ ) until maximum  $\gamma^*$  and then decreases until  $\gamma_h$  ( $\gamma_l$  respectively). If  $\gamma^*$  is assumed constant but unknown, this leads to scheme in Figure 3. There, an Events Software Sensor (ESS) estimates a maximum of  $f(\mu)$  in real time, as  $\gamma_k^*(t) = \max(f(\mu(S(\tau))))$  for  $\tau \in (t_k, t)$ , were  $t_k$  is the instant when the last  $k^{th}$  state change in the Event Driven Optimal Controller (EDOC) took place.

Comparing the actual values of  $\gamma$  and  $\gamma_k^*$  allows to determine if  $S=S_l$  or  $S=S_h$  have been reached, even if  $S_l$ ,  $S_l$  and  $S_h$  remain unknown. In particular, consider  $S_l < S^*$  and  $S_h > S^*$  such that  $f(\mu(S_l)) = f(\mu(S_h)) = P\gamma^*$ , where  $0 < P < 1$  is a near-optimality control parameter. Making  $P$  close to one renders  $S_l$ ,  $S_h$  close to  $S^*$ . Figure 1 shows the relation between  $S_l$ ,  $S_h$ ,  $\gamma^*$  and  $P$  for the case  $\gamma = f(\mu) = (a\mu + b)c$ . The ESS also determines easily if  $S > S^*$  or not, i.e. if the system is in the inhibition zone.

Table 1. ESS events for fed-batch processes

Tag	Trigger	Estimation	Meaning
$e_{1,0}$	$d\gamma/dt > 0$	$S < S^*$	Not Inhibited
$e_{2,0}$	$d\gamma/dt < 0$	$S > S^*$	Inhibited
$e_{2,1}$	$\gamma \leq P\gamma_k^*$	$S = S_h$	MustWait
$e_{1,2}$	$\gamma \leq P\gamma_k^*$	$S = S_l$	MustFill
$e_3$	$V \geq V_{max}$	(measured)	TankFull
$e_4$	$\gamma < \gamma_{end}$	$S < S_{end}$	EndReaction

Table 1 shows the events estimated by the ESS. Using such events the EDOC shuts down or powers up the influent pump  $F$  as appropriate. The finite state EDOC is depicted in Figure 4. The initial state at  $k=0$  for  $t=t_0=0$  is always  $\sigma_0$ . If inhibited, the system will jump instantaneously to state  $\sigma_2$ . After the initial bang-bang arc, the cycling between  $\sigma_1$  and  $\sigma_2$  will approximate the singular arc. Once the tank is filled,  $\sigma_3$  will complete the last bang-bang arc. The reaction finishes when  $\sigma_4$  is reached. After this the rest of the batch sequence (settling, draw...) may be completed and afterwards a whole new cycle may be started. A result similar to Theorem 1 applies in this case.

*Theorem 2:* Suppose that system (1-3) is given, that a continuous and strictly monotone increasing function  $f(\mu)$  of  $\mu$  is available, and Assumption 1 is satisfied. Assume, furthermore, that  $\gamma^*$  is constant for each batch cycle, but unknown. Then replacing the optimal control law (9) by the EDOC law the time to reach the target from any initial condition  $\mathbf{z} \in \Omega$  is

$$T_{EDOC}(F, \mathbf{z}_0) = T_{opt}(\mathbf{z}_0) + \Delta(P, \mathbf{z}_0) + \delta \quad (14)$$

with  $\Delta(P, \mathbf{z}_0)$  continuous and finite and  $\Delta \rightarrow 0$  as  $P \rightarrow 1$ . i.e. the target will be reached in finite time, as near to the optimal one as desired, up to a small value  $\delta > 0$ . This is true for any form of the growth rate (positive if  $S$  positive), of the function  $f(\mu)$ , for unknown  $S_l$  and for any values of the parameters  $k_l > 0, m \geq 0$ .

*Proof (sketch):* Similar to the proof of Theorem 16 in (Moreno,1999) and Theorem 3. The important observation here is that limits  $P\gamma^*$  given in terms of  $f(\mu)$  correspond (in an unknown form) to limits  $S_l < S < S_h$  in  $S$  (because of the continuity of  $\mu$  and  $f(\mu)$  with respect to  $S$ ). The value  $\delta > 0$  is introduced by the initial test (Figure 4) when  $S_0 > S^*$ . Its value depends on the time it takes the ESS to determine the first event. ■

*Robustness:* Analyzing the event triggers in Table 1 for transitions between  $\sigma_1$  and  $\sigma_2$ , i.e.  $e_{1,2}$  and  $e_{2,1}$ , it follows that, even if for some reason (i.e. noise or perturbation) the system enters a wrong state, after some time the conditions for a transition to the correct state will always appear. This is so because even if the detected  $\gamma_k^*$  is not the maximum i.e.  $\gamma^*$ , at some time a  $\gamma = P\gamma_k^*$  will be found. Note that the approximate control law is robust against model and parameter uncertainties and changes. Moreover, it requires low information from the system: only  $\gamma = f(\mu)$  and the instant when  $V = V_{max}$  have to be known, and the finishing value  $\gamma_{end}$  given. It is important to note that the trajectory is confined to set  $S_l \leq S \leq S_h$  independently of and without knowledge of the parameters, the exact form of the growth rate, of  $f(\mu)$  and of the value of  $S_l$ .

### 3.3. Robust general near-optimal control law

In the previous paragraph a suboptimal control law was given. If estimations in Table 1 are exact, it can be made as near to optimality as desired by selecting appropriately the parameter  $P$ . However, this property depends strongly on the availability of an exact value for  $\gamma$ . If there is some measurement noise or perturbation, if the parameters vary in time or as function of some other state variables, then Theorem 2 is no longer valid as estimations in Table 1 might have errors. In this Section it will be shown that, under reasonable circumstances, the estimation errors are tolerably low and so the approximated control still behaves correctly and robustly. The tradeoff is that optimality cannot be arbitrarily well approximated.

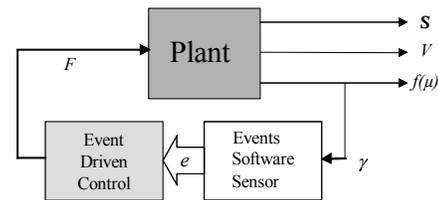


Figure 3. Event Driven Optimal Control scheme

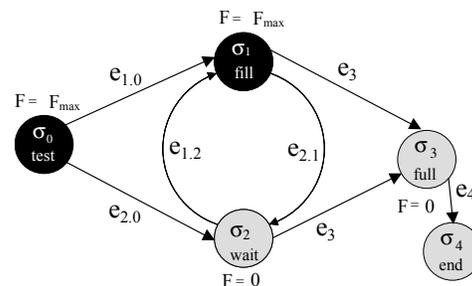


Figure 4. Finite state transitions for EDOC

Note that the practically possible measurements for the two case systems, (6) and (8), are of the form  $\eta = f(\mu)B = \gamma B$ . Since the total biomass  $B = XV$  satisfies  $dB/dt = \mu B$ , it is clear that it is not a constant but a monotone increasing function of time, and  $\eta$  does not have the required form for the EDOC. Let us analyze its behavior when instead of  $\gamma$  the measured  $\eta$  is used.

Along a trajectory of the plant it is easy to see that

$$\dot{\eta} = (\dot{\gamma} + \mu f(\mu))B = (f' \mu' \dot{S} + \mu f(\mu))B \quad (15)$$

Where  $\dot{\varphi}$  represents the derivative of  $\varphi$  with respect to its argument. When  $dS/dt > 0$ , which is always satisfied if  $F = F_{max}$  under Assumption 1,  $sign(d\eta/dt) = sign(\mu')$ . When  $dS/dt < 0$ , i.e. when  $F = 0$ ,  $sign(d\eta/dt) = -sign(\mu')$ . Note from (15) that the behavior of  $\eta$  does not reflect that of  $\gamma$ , and the EDOC would work wrongly. However, if  $\mu f \ll f' dS/dt$  one would expect that the behavior of  $\eta$  approximates that of  $\gamma$ , and the EDOC would work correctly.

*Theorem 3:* Suppose that system (1-3) is given, with continuously differentiable  $\mu$ . Suppose that  $\eta = f(\mu)B$ , with  $B$  the total biomass and  $f(\mu)$  a continuously differentiable and strictly monotone increasing function of  $\mu$ , is available and Assumption 1 is satisfied. Assume, furthermore, that  $\gamma^* = \max f(\mu)$  is constant for each batch cycle, but unknown. Suppose that there exists some  $S_{max} > S_2 > S^*$  such that for  $\Omega \cap [S_2, S_{max}]$

$$\left( \left[ -(k_1 \mu + m)X + \frac{F_{max}}{V}(S_i - S) \right] f' \mu' + \mu f(\mu) \right) > 0 \quad (16)$$

and that there exists  $0 < S_1 < S^*$  such that for  $\Omega \cap (0, S_{max}]$

$$-(k_1 \mu + m)X f' \mu' + \mu f(\mu) < 0 \quad (17)$$

Denote as  $B_{max} = \max_{z \in \Omega} B(T_{opt}, 0, z_0)$  the maximal quantity of biomass that can be obtained in the reactor, and as  $B_{min}$  the minimal possible quantity of biomass in the reactor. It is said that  $P \in (0, 1)$  is feasible if there exist  $0 < S_3 < S_1$  and  $S_2 < S_4 < S_{max}$  such that  $Pf(\mu(S_1))B_{min} = f(\mu(S_3))B_{max}$ ,  $Pf(\mu(S_2))B_{min} = f(\mu(S_4))B_{max}$  are satisfied. Suppose that there exists a feasible  $P_f$ . Then replacing  $\gamma$  by  $\eta$  in EDOC law, the time to reach the target from any initial condition  $z \in \Omega_1$  and for any  $P \in [P_f, 1)$  is

$$T_{ROB}(z_0) = T_{EDOC}(z_0) + \Xi(P, z_0) \quad (18)$$

with  $\Xi(P, z_0)$  continuous and finite, i.e. the target will be reached in finite time. However, when  $P \rightarrow 1$  it is not always true that  $\Xi(P, z_0)$  tends to zero.

*Proof:* Note that if  $P_f$  is feasible, so is every  $P \in [P_f, 1)$ . From (15) and Assumption (16) it follows that when  $z_0 \in \Omega_1$  is such that  $S_0 \in [S^*, S_2]$  (and EDOC is in  $\sigma_0$ ) then  $sign(d\eta/dt) = sign(\mu')$ , and the transitions are the same when using  $\eta$  instead of  $\gamma$ . If  $S_0 \in [S^*, S_2]$ , then both transitions  $e_{1,0}$  or  $e_{2,0}$  are possible.

Now suppose that the EDOC is in  $\sigma_1$  and  $S \leq S^*$ . Then from (15) it follows that  $sign(d\eta/dt) > 0$ , and  $\gamma_k^*(t_k) = \eta(t_k)$ . Then  $\gamma$ ,  $\eta$  and  $S$  increase until  $S = S^*$ , at  $t = t_k + T^*$ , where  $\gamma_k^*(t) = \eta(t_k + T^*)$ . At this point  $\gamma$  begins to decrease, although  $\eta$  can increase further. However,

because of (16),  $\eta$  is decreasing at the latest when  $S = S_2$ . Furthermore, if  $e_3$  is not reached before, the state transition  $e_{2,1}$  is satisfied at the latest when  $S = S_4$ .

Now suppose that the EDOC is in  $\sigma_2$  and  $S \geq S^*$ . Then  $sign(dS/dt) < 0$ ,  $sign(d\eta/dt) < 0$ , and  $\gamma_k^*(t_k) = \eta(t_k)$ . Then  $\gamma$ ,  $\eta$  increase and  $S$  decreases until  $S = S^*$ , at  $t = t_k + T$ , where  $\gamma_k^*(t) = \eta(t_k + T^*)$ . Although at this point  $\gamma$  begins to decrease,  $\eta$  can still increase further. However, because of (17),  $\eta$  is decreasing at the latest when  $S = S_i$ . Furthermore, if  $e_3$  is not reached before, the state transition  $e_{1,2}$  is satisfied at the latest when  $S = S_3$ .

Note that once  $\sigma_1$  or  $\sigma_2$  have been reached, then the substrate  $S$  stays in the set  $[S_3, S_4]$  until  $\sigma_3$  is reached. If the EDOC is in  $\sigma_1$  and  $S \in [S^*, S_2]$  it is clear that at the latest when  $S = S_4$  the state transition  $e_{2,1}$  gets satisfied.

Note that when  $P \rightarrow 1$  then  $[S_3, S_4] \rightarrow [S_i, S_2]$ , but it is not true that  $\Xi(P, z_0)$  tends to zero. ■

Some remarks are in order:

- (16) and (17) are satisfied if, for example, the relative change  $|f' \mu' / f \mu|$  is big outside a vicinity of  $S^*$ , and if  $F_{max}$  and/or  $S_i$  are big. Note also that as  $(S_2 - S_1) \rightarrow 0$  the results of EDOC are recovered.
- For WWTPs the change in  $B$  during one cycle is very small, since the amount of toxics that can be treated is typically small compared to  $B$ . This means that (16) and (17) are typically satisfied, that  $S_2 - S_1$  is very small, and therefore the loss of optimality is small. Successful experimental results are available in (Betancur et al., 2004a). For biotechnology process, instead, the change in  $B$  is usually large. Even for such a case simulation results show the effectiveness of the EDOC (Betancur et al., 2004b).
- The form of  $\mu$  and  $f$  influence how big is the minimum loss of optimality.
- No noise analysis has been carried out because of lack of space. However, it can be done in a similar fashion as the proof of Theorem 3.
- In practice, derivatives for Equation (7) and Table 1 are not available. They are calculated using numerical real-time methods. This introduces some time delays and distortion in the derivative signal. The same is true for signals generated using real practical sensors devices. A way to deal with such time delays is given in (Betancur et al., 2004a). Distortions could be treated theoretically in the same way as noise.

#### 4. EXPERIMENTAL RESULTS

A 7L laboratory scale bioreactor acclimated with sludge taken from a municipal WWTP was used to degrade 4-chloro-phenol (4CP). The usual influent toxicant concentration for traditional sequencing batch processing is  $S_i = 350 \text{ mg 4CP/L}$ . Applying more than twice such quantity would greatly inhibit and stress biomass, increasing the needed treating time nonlinearly. Applying higher toxicant concentrations might even inhibit and/or disable the bioreactor permanently. By using the EDOC strategy, instead, the biomass was never stressed or inhibited. A linear increase relation of treating time with respect to  $S_i$  was observed in a series of experiments for increasing  $S_i$ , even when making it as high as  $7000 \text{ mg 4CP/L}$ .

Theoretically, treating time was near 95% of the optimal-time, in all series, for a programmed  $P = 0.9$

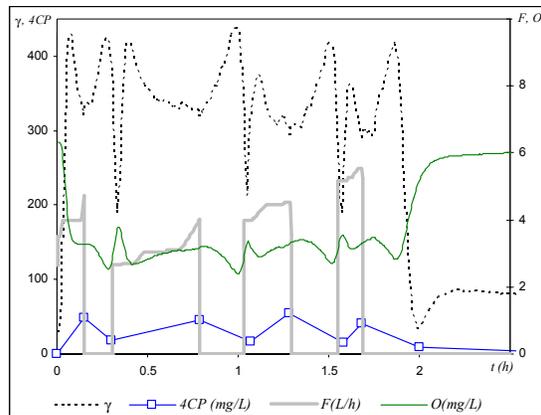


Figure 5. Experimental kinetics using EDOC in WWTP case 14/07/2003;  $P = 0.9$ ; influent toxicant concentration  $S_i = 634\text{mg}4\text{CP/L}$

Figure 5 shows one experimental kinetic for the 4CP degradation case, using  $y_w$  in Equation (8) for EDOC implementation. Toxicant substrate concentration  $S$  inside the bioreactor (see 4CP in Fig. 5, square marks) was measured off-line from manually taken samples and was not used for control purposes. Up to  $S=200\text{mg/L}$  it is considered normal and safe for the biomass. A model identification exercise later revealed a 95% confidence interval of  $\pm 7.4\%$  for  $S^* = 13.99 \text{ mg}4\text{CP/L}$ . Figure 5 shows that  $S$  was kept oscillating around  $S^*$ , in an acceptably low concentration range, by properly turning on and off the influent pump (Fig. 5, continuous thick line). Such behaviour shows the effectiveness of EDOC strategy.

Biomass was  $B=1.4\text{g}$  exhibiting an increase of less than 2% during the reaction. Its value was not used by the controller. Values of  $S$ ,  $S_i$  and  $S^*$  were not used either. Another perturbation comes from the online sensor used to measure Dissolved Oxygen (Fig.5, continuous thin line). It introduced appreciable second order delay effects, and some noise, to state variable  $O$ . It follows that some delays and signal distortion are to be expected when calculating  $\gamma=y_w$  in Equation 8 (Fig. 5, dotted line) for using it in EDOC. But thanks to EDOC robustness the system did cope smoothly with all this perturbations and uncertainties.

## 5. CONCLUSIONS

A methodology for the robust and practical implementation of optimal control strategies for a class of nonlinear processes has been introduced. When the control law is composed of bang-bang and singular arcs the basic idea is to replace the singular arc with a bang-bang control. This makes the control robust and requires a reduced quantity of information. This general idea is developed here for a class of fed-batch bioreactors.

The use of measurable variables giving minimal indirect information is shown to be effective for software-sensing events related to the crossing of the

singular surface of the process. This allows the controller to generate bang-bang cycles to approximate the singular arcs of the optimal solution.

Studies of two different practical cases were carried out successfully: one in the biotechnology area and other, including experimental results, in the wastewater treatment area.

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The scientific responsibility rests with the authors.

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