Fault Detection Using Dynamic Principal Component Analysis and Statistical Parameters Estimation

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Abstract—One of the most popular multivariate statistical methods used for signals based process monitoring and data compression is the Dynamic Principal Component Analysis. This method computes the orthogonal principal directions assuming stationarity in the time series of the process, however, if observations are not stationary, false alarms could be generated during the fault detection and isolation task. To reduce the false alarms rate, this paper extends the dynamic principal component analysis for the case on non stationary data. This is achieved including in the monitoring procedure an on-line mean estimator and standardizing the time series data of the process according to the values generated by the estimator. As study case the detection of faults in a flow control valve has been used, in which it is assumed that the control signal, stem displacement and flow are measured signals. Simulator data are used to adjust the procedure and show the improvement of the novel dynamical principal component analysis methodology.

Index Terms—Dynamic Principal Component Analysis, Signal Based Fault Detection, Classification in Diagnostics, No Stationary Time Series, Feature Extraction.

I. INTRODUCTION

The on-line process monitoring for fault detection and isolation, FDI is an important task to ensure plant safety and maintaining product quality. Both, model based [1] and signal based methods for FDI [2] have been proposed and they have been applied in diverse diagnosis problems during the last twenty years. While model based methods can be used to detect and isolate specific faults in the system, only if a good model is available, signal based methods attempt to extract maximum information from historical data generated by sensors and actuators and apply classification methods according to specific features. In particular, pattern recognition approaches considered as signal based methods require an implicit nominal behavior of process signals which is compared on-line with the signals taken from the process and if they don’t match with the nominal, a fault symptom is obtained. A variety of approaches can be used to obtain the implicit nominal model and the fault symptom [3].

On the other hand, the multivariate statistical tool called Principal Component Analysis (PCA) is a signal based method and has been recognized as a powerful tool for fault diagnosis issues when many data of process variables are available. The key of this method is its habilility to model the nominal behavior of a process through the extraction of the main features of the process variables from historical data when the process is operating in normal conditions. Thus, PCA transforms by a parallel correlation analysis, a set of multivariate observations to a lower dimension space, retaining the most important variables of the original data [4]. Since, PCA has shown its potential in the case of FDI for static process, the procedure has been extended to the case of data coming from a dynamic system taking into account not only the parallel correlation between variables but also the series correlation between observations and is called dynamic principal component analysis, DPCA [5]. However, since PCA and DPCA assume stationarity in time series during the process of modeling, high rate of false alarms are generated if the statistical property of stationarity is not hold in the test data during the diagnosis stage.

The above described problem has not been pointed out before and solved. This fact motivated this work in which one overcomes the stationary condition assuming signals with known time variant means obtained on line by single moving average estimation. One shows that the estimation of the statistical parameters of the input and output signals assuming normal condition reduces the false alarms rate in the DPCA based fault detection algorithm without demanding a precise system description. As a case of study it is considered a flow control valve which implicit model is obtained by simulation from the control input signal, stem displacement signal and flow signal. Three faults conditions in the valve are introduced to evaluate the effectiveness of the proposed methodology.

II. FAULT DETECTION VIA DPCA

The fault detection task can be divided in two general stages, the first one is associated with the procedure of defining a data based reference pattern...
(nominal statistical model) of the process under monitoring assuming normal operating conditions; and the last is the fault detection as such which consists in a classification procedure of an on line generated symptom.

Let \( \mathbf{X} \) be a set of historical data composed of \( n_t \) observations from \( p \) variables of a process, in which some of the \( p \) time series are inputs and the rest outputs, described by the matrix

\[
\mathbf{X} = [ X_1 \ X_2 \ \cdots \ X_p ]_{(n_t \times p)}
\]  

(1)

in which it is assumed that each time series \( X_i \) is stationary, i.e. its means and standard deviations are constant for every time interval of observation. If the data of the set \( \mathbf{X} \) is generated from a dynamic system, there exist a time dependence in each time series where current values depend on past values. So to include the serial correlation of data, it is constructed the so called trajectory matrix applying a 'time lag shift' of order \( w \) on each of the \( p \) columns of the matrix \( \mathbf{X} \). This means

\[
\mathbf{X}_w^i = 
\begin{bmatrix}
X_i(1) & X_i(2) & \cdots & X_i(w+1) \\
X_i(2) & X_i(3) & \cdots & X_i(w+2) \\
\vdots & \vdots & \ddots & \vdots \\
X_i(n_t-w+1) & X_i(n_t-w+2) & \cdots & X_i(n_t)
\end{bmatrix}
\] 

\[ (n_t \times w) \]

(2)

where \( n = n_t - w + 1 \), \( m = pw \) and the value \( w \) is selected on the base on the number of correlated lags of the variables. One option is to calculate \( w \) in the same way as is defined the number of lags to use in an auto-correlation function of a signal as it is suggested in [6], \( w = \frac{m}{T} \).

In a multivariate observation some of the variables could have different range of values, therefore, before the application of PCA it is convenient to carry on a data standardization in matrix \( \mathbf{X}_w^i \) with respect to its means and standard deviations in order to obtain a standardized data matrix \( \mathbf{X}_w^i \) with zero mean and unit variance.

The means vector and standard deviations vector of the trajectory matrix \( \mathbf{X}_w^i \) are

\[
\mu = [ \mu_1(.) \ \mu_2(.) \ \cdots \ \mu_m(.) ]_{(1 \times m)}
\] 

(3)

\[
\sigma = [ \sigma_1(.) \ \sigma_2(.) \ \cdots \ \sigma_m(.) ]_{(1 \times m)}
\] 

(4)

Assuming that data are generated from a stationary process, it is satisfied the following equality

\[
\mu_1(.) = \mu_2(.) = \cdots = \mu_m(.) \]

\[
\sigma_1(.) = \sigma_2(.) = \cdots = \sigma_m(.)
\]

Thus, the data standardization is obtained through

\[
x_{s_w}^{ij} = \frac{x_{ij}^{w} - \mu_j}{\sigma_j}
\] 

(5)

for \( i = 1, \ldots, n \) and \( j = 1, \ldots, m \), which constitute the entries of the standardized matrix \( \mathbf{X}_w^i \). This data matrix is the start point to decompose the multivariate data and to get its principal components.

The principal components statistical model \( \mathbf{Y} \) of dimension \( n \times l \) is defined as a linear transformation of the original variables involved in \( \mathbf{X}_w^i \), such principal components extracted are uncorrelated vectors. Specifically, the matrix of principal components is obtained through the following transformation

\[
\mathbf{Y} = \mathbf{X}_w^i \mathbf{V}_t
\] 

(6)

where the transformation matrix \( \mathbf{V}_t \in \mathbb{R}^{m \times l} \) is composed of an appropriate selection of \( l \) eigenvectors associated to the correlation matrix \( \mathbf{R} \) of the standardized trajectory matrix \( \mathbf{X}_w^i \).

For each observation in the principal components model \( \mathbf{Y} \) it is possible to define a kind of behavior symptom described by the univariate statistic \( T_y^2 \) called Hotelling parameter, this is

\[
T_y^2 = y \mathbf{S}^{-1} \mathbf{y}^T
\] 

(7)

where \( \mathbf{S} \) is the covariance matrix of \( \mathbf{Y} \). Finally, it is defined a threshold of normal condition from the probability density function (pdf) of the set of parameters \( T_y^2 \). In [7] it is very well described the procedure to obtain this threshold named UCL (Upper Control Limit) in the area of Statistical Quality Control.

By the other hand during the fault detection stage, it is taken on line an actual \( m \)-dimensional observation vector \( \mathbf{x}_w^i \) which is standardized with respect to the means (3) and standard deviations (4), transformed through the matrix \( \mathbf{V}_t \) in the principal component subspace, and mapped in the univariate parameter \( T_y^2 \); if the resulting statistic deviates from the threshold it is an indication of the presence of a fault.

DPCA allows straightforward to detect a deviation of vector \( \mathbf{x}_w^i \) from the historical reference data in terms of its mean and its standard deviation. This is the main property of DPCA which can be managed to solve fault detection issues. It is important to note that if the trajectory matrix \( \mathbf{X}_w^i \) was obtained with data around one operation point of the system, any change in the nominal values of the signals is interpreted by DPCA as a fault, even when the process is healthy, this is because the components of (3) are time variant and as a consequence the stationarity property is not satisfied.

To overcome this difficulty one suggests the on line estimation of the statistical set (3) assuming healthy conditions and the use of it for the standardization procedure. This estimation can be achieved if simple relations in normal conditions between system input...
and its output are known. Thus, the objective is to standardize the data around any normal condition of the process keeping the condition of zero mean and unit variance.

III. DPCA WITH MEAN PARAMETER ESTIMATION

Lets consider for simplicity in the presentation the case of a SISO system operating in normal condition around an operating point of which sets of input data \( X_1 \) and output data \( X_2 \) are described by the matrix

\[
X = \begin{bmatrix} X_1 & X_2 \end{bmatrix}
\]  

(8)

Following the standard DPCA procedure described in section II, the principal components model \( Y \) can be obtained from the historical data \( X \) as well as the threshold from the pdf of \( T^2 \).

For the fault detection task assume as known a linear relationship in normal condition for the input and output variable given by \( x_2 = L(x_1) \). Then one can evaluate, on line, from the input variable \( x_1(t) \) and for a time window its mean \( \mu_{x_1}(t) \) and successively estimate the mean of the output variable \( \mu_{x_2}(t) \) assuming that the process is in normal condition and using the linear relation between variables. Thus, to evaluate and classify an actual observation \( x_{a} \) this has to be previously standardized with respect to the estimated means. The procedure described before is what constitutes the extension to the standard DPCA based fault detection algorithm and which allows to manage no-stationary processes. The implementation of this extension of DPCA for the case of non stationary process can be summarized as follows:

1) Estimate through \( L(.) \) the \( w \)-output components \( \hat{x}_2(k), \ldots, \hat{x}_2(k+w-1) \) from the input actual data \( x_{1a}(k), \ldots, x_{1a}(k+w-1) \) and construct the following vector

\[
\hat{x}_w = \begin{bmatrix} x_{1a}(k) & \cdots & x_{1a}(k+w-1) & \cdots \\
\cdots & \hat{x}_2(k) & \cdots & \hat{x}_2(k+w-1) \end{bmatrix}_{(1 \times m)}
\]  

(9)

2) Using the well known single moving average on \( \hat{x}_w \), determine the actual estimated means vector

\[
\hat{\mu} = \begin{bmatrix} \hat{\mu}_1(.) & \hat{\mu}_2(.) & \cdots & \hat{\mu}_m(.) \end{bmatrix}_{(1 \times m)}
\]  

(10)

which will be used for the standardization.

3) Generate, from real data of the input and output signals of the process, the vector of actual observations with time lag shifts of order \( w \)

\[
x_{a} = \begin{bmatrix} x_{1a}(k) & \cdots & x_{1a}(k+w-1) & \cdots \\
\cdots & x_{2a}(k) & \cdots & x_{2a}(k+w-1) \end{bmatrix}_{(1 \times m)}
\]  

(11)

Fig. 1. Actuation System with Fault Diagnosis Module

4) Standardize the \( m \) terms in (11) using the estimated means vector (10) and the standard deviations in (4), this is

\[
x_{as} = \frac{x_{a} - \hat{\mu}}{\sigma}
\]

for \( j = 1, \ldots, m \). \( x_{as} \) will have approximately unit variance and zero mean under normal operating condition even before level changes in the input signal.

5) Transform the vector \( x_{as} \) in the principal component subspace \( y_a \) through \( V_t \)

\[
y_a = x_{as} V_t
\]

6) Map \( y_a \) in the univariate parameter \( T^2 \) through

\[
T^2_{y_a} = y_a S^{-1}_{Y} y_a
\]

if the resulting statistic deviates from the normal condition threshold a fault is present in the system.

In the following section this procedure is applied to detect faults in a valve in which only a very simple input output model is considered for the estimation of the mean value of the output.

IV. EXAMPLE OF A FLOW CONTROL VALVE

A. Scheme for Fault Diagnosis

The fault detection task is carried out, in general, exploiting the variety of internal signals that can be measured in a system. In particular Fig. I shows the block diagram of a valve considered as an actuator in a FDI distributed system, with its FDI block, the control of flow integrated with a position sensor, the signal conversion, the modulator and the final control element. In this case, the fault diagnosis module is designed in such a way that it gets at every sampling period (0.5 s) the control signal \( CV \), as input variable, and the stem displacement \( X \), and flow \( Q \), as output variables. On the base of these signals, one takes the advantage of the correlation between these three time series of the variables to detect and isolate mechanical faults in the valve.
B. Valve Simulator

To generate the time series required to adjust the algorithm the simulator reported in the DAMADICS (Development and Application of Methods for Actuator Diagnosis in Industrial Control Systems) benchmark [8] is used. This benchmark consists of the valve model in Simulink, and other modules which comprise the DABLib (DAMADICS Actuator Benchmark Library).

C. Detection Results

The fault detection algorithm is adjusted considering the trajectory matrix $X$ with one input variable $CV$, and two outputs $X$ and $Q$. Just three principal components are used for the characterization of the valve. The procedure has been evaluated considering the following cases:

1) Normal operation of the valve with step changes in the input signal.
2) Fault condition, $f_1$-Valve Clogging of magnitude 0.75 with step changes in the input signal.
3) Fault condition, $f_2$-Valve Plug or Valve Seat Sedimentation of magnitude 0.75, with step changes in the input signal.
4) Fault condition, $f_3$-Valve Plug or Valve Seat Erosion of magnitude 0.75, with step changes in the input signal.

The first evaluation has as goal to test the performance of the algorithm with respect to the step changes in the control signal $CV$ when the valve is operating in normal conditions. One has simulated positive and negative step changes non higher than the maximum step size permitted for a valve ($\approx 10\%$, [9]) the sequence is as follows: positive step change from $(18s \leq t \leq 43s)$ and negative step change from $(93s \leq t \leq 143s)$.

As it can be seen from Fig. 2 the simple DPCA algorithm generates false alarms during the transient response of the step changes in $CV$ even when the valve is working healthy; this phenomenon is due to the absence of the stationary condition in the time series of $CV$, $X$ and $Q$.

As it is seen in Figs. 4, 5 and 6, the three symptom values have a value close to zero if there is a change in the input signal and only exceed the threshold when the faults are present.

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D. Isolation Issue

For the fault isolation task one suggest to use a cluster analysis approach. The cluster analysis classifies a set of multivariate observations in a number of mutually exclusive groups based in a kind of similarity among the observations. It is important to note that the classification for the purpose of fault isolation will only be possible if there exist different signatures for each of the faults which one is interesting to isolate.

This section shows preliminary analysis of the isolation issues based in a graphical description of the signatures for each one of the faults under study. Fig. 7 shows the three principal components space for each set of faults considered. From the graphs one observes four well defined clusters. This means, exist different signatures, one for the normal condition, and the rest for each one of the faults under study. If the valve is operating in normal conditions, the actual data stay in the normal
cluster, however, if a fault appears, the data trajectories moves from the normal condition cluster to a specific region of fault depending on each fault. This analysis indicates that it is possible to isolate in a very simple way three faults using only two outputs of the valve.

At the present, one is working in the implementation of an automatic algorithm to solve the isolation task. The idea is to assign a specific region for each considered fault \( f_i \) in the \( l \)-dimensional space of the principal components with a similarity measure between point to point or point to set.

V. CONCLUSIONS

Even when the DPCA can overcome the restriction of applying PCA on time correlated variables, however, due to the dynamic changes in the input and output signals in normal conditions, the stationarity assumption is not satisfied. Here it has been proposed a modification to the DPCA algorithm for fault detection, in which it is carried out an appropriate standardization with regard to the statistical parameters on-line of input and output signals. This idea allows to deal with non-stationary signals and to reduce significantly the rate of false alarms. By the other side it is taken advantage of the multivariate statistical analysis approach, where it is possible to complement the methodology with cluster analysis which is another multivariate tool to isolate three faults in the valve. It was shown through a series of tests the effectiveness of the fault detection modified algorithm to make a distinction between normal changes in signals and the variations due to the presence of a fault.

References


