OPTIMAL SENSOR LOCATION BASED ON THE OBSERVABILITY MEASURES FOR AN ANAEROBIC WASTEWATER TREATMENT PROCESS

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Abstract: In this paper the choice of the optimal sensor location, especially of the gas product in an anaerobic wastewater treatment process is discussed. First, we have applied the observability measures theory considered in Waldraff, *et al.* (1998) on the distributed parameter model of an anaerobic wastewater treatment process (Schoefs, *et al.*, 2003), which is discretized by using the orthogonal collocation method. Then a state observer has been implemented to illustrate the results. These investigations lead to recommend appropriate sensor locations.

Keywords: anaerobic wastewater treatment process (WWTP), parameter distributed model, observability, gas sensor location.

1. INTRODUCTION

A usual problem of monitoring and control of anaerobic wastewater treatment is the lack of sensors and/or analyzers, which could in a reliable way provide on-line information about the degradation process. In order to overcome this difficulty, model-based state estimation methods (e.g. Kalman or Luenberger observers) can be used to estimate variables that are not available from on-line measurements. The used sensors play a crucial role to warrant a successful implementation of the state estimation. In this paper, we have chosen an example where only the produced gas is measurable on-line (Bernard, *et al.*, 2001).

The observability of a distributed parameter system is not only affected by the choice of sensor but also by the sensor location. It is important to know the appropriate positions in order to obtain the best information about the process dynamics along the reactor. Here, we discuss the choice of optimal sensor positions with a predefined number of sensors.

This paper is organized as follows. We introduce first the distributed parameter model described by partial differential equations (PDE's) as proposed in (Schoefs, *et al.*, 2003). The orthogonal collocation is used to solve it, and different observability measures (Waldraff, *et al.* (1998)), i.e. the observability matrix and Gramian methods as well as the Popov-Belevitch-Hautus one, are considered. The results of this study will be illustrated in numerical simulations by implementing a state observer, the extended Luenberger observer, here. Finally, the appropriate sensor positions will be recommended.

2. DYNAMICS MODEL OF THE WWTP

Let us consider an anaerobic WWT process operated in a fixed-bed reactor (Bernard, *et al.*, 2001). Anaerobic digestion is basically composed of two phases, acidogenesis and methanization. The axial dispersion in the reactor (i.e. tubular reactor) is taken into account (Schoefs, *et al.*, 2003). The dynamics of the concentrations of the acidogenic and methanogenic biomasses X_1 and X_2, respectively, are described by the following equations:

$$\frac{\partial X_1}{\partial t} = (\mu_1 - \alpha D)X_1 \qquad (1)$$

$$\frac{\partial X_2}{\partial t} = (\mu_2 - \alpha D)X_2 \qquad (2)$$

where α and D are the bacteria fraction in the liquid phase and the effluent dilution rate, respectively. Here, the biomass kinetics, μ_1 and μ_2, follow *Monod-Contois* and *Haldane-Contois* (to account for possible volatile fatty acid (VFA) accumulation) models, respectively:

$$\mu_1 = \frac{\mu_{1\max} S_1}{K_{C1} X_1 + S_1} \qquad (3)$$

$$\mu_2 = \frac{\mu_{2S} \cdot S_2}{K_{C2} X_2 + S_2 + \frac{S_2{}^2}{K_{I2}}} \quad (4)$$

with $\mu_{1\max}$, μ_{2s}, K_{C1}, K_{C2} and K_{I2}, the biokinetic parameters. The time evolution of the concentrations of alkalinity Z, organic substrate S_1, VFA S_2, and inorganic carbon C are represented by the following equations, $\forall z \in [0, H]$,

$$\frac{\partial Z}{\partial t} = -u_l \frac{\partial Z}{\partial z} + \varepsilon_l E_z \frac{\partial^2 Z}{\partial z^2} \quad (5)$$

$$\frac{\partial S_1}{\partial t} = -u_l \frac{\partial S_1}{\partial z} + \varepsilon_l E_z \frac{\partial^2 S_1}{\partial z^2} - k_1 \mu_1 X_1 \quad (6)$$

$$\frac{\partial S_2}{\partial t} = -u_l \frac{\partial S_2}{\partial z} + \varepsilon_l E_z \frac{\partial^2 S_2}{\partial z^2} + k_2 \mu_1 X_1 - k_3 \mu_2 X_2 \quad (7)$$

$$\frac{\partial C}{\partial t} = -u_l \frac{\partial C}{\partial z} + \varepsilon_l E_z \frac{\partial^2 C}{\partial z^2} - q_c + k_4 \mu_1 X_1 + k_5 \mu_2 X_2 \quad (8)$$

where q_c is the CO_2 concentration flow rate:

$$q_c(z,t) = k_L a(C + S_2 - Z - K_H P_C) \quad (9)$$

For the above expression, we can express the CO_2 molar flow rate accumulated at height z as follows:

$$\begin{aligned} Q_C(z,t) &= \int_v q_c dV \\ &= k_L a A \int_0^z (C + S_2 - Z - K_H P_C) d\zeta \end{aligned} \quad (10)$$

with the CO_2 partial pressure, P_C, equal to:

$$P_C(z) = \frac{\phi(z) - \sqrt{\phi(z)^2 - 4K_H P_T(z)\left(C(z) + S_2(z) - Z(z) + \frac{Q_C(z-\Delta z)}{k_L a \cdot A \cdot \Delta z}\right)}}{2K_H} \quad (11)$$

$$\phi(z) = C(z) + S_2(z) - Z(z) - K_H P_T(z) + \frac{Q_M(z) + Q_c(z-\Delta z)}{k_L a \cdot A \cdot \Delta z} \quad (12)$$

Indeed, $P_T(z)$ is the total pressure at the point z:

$$P_T(z) = P_{atm} + \rho g(H - z) \quad (13)$$

and Q_M is the CH_4 molar flow rate, such as:

$$Q_M(z,t) = k_6 A \int_0^z \mu_2 X_2 d\zeta \quad (14)$$

k_i, $i \in [1,6]$, are the yield coefficients, $k_L a$ and K_H are the liquid-gas transfer coefficient and the Henry's constant, respectively.

The following boundary conditions have also to be added, $\forall t \in \mathbb{R}^+$, $\forall \xi^* = [S_1, S_2, C, Z]$,

$$\begin{cases} \xi^*(z=0,t) = \dfrac{\xi^*_{in}(t) + R \cdot \xi^*_{out}(t)}{R+1} \\ \dfrac{\partial \xi^*}{\partial z}(z=H,t) = 0 \end{cases} \quad (15)$$

where $R = Q_{rec}(D \cdot V_{eff})^{-1}$ is the recycle rate with Q_{rec} and V_{eff}, the effluent recycle flow rate and the effective tank volume, respectively.

3. OBSERVABILITY OF AN APPROXIMATE FINITE-DIMENSIONAL MODEL

Sensor location studies typically refer to quantitative tests based observability tests for a discretized model of the PDE equations of the process (Waldraff, *et al.*, 1998). That's why a discretization scheme, e.g. finite difference or orthogonal collocation, has to be applied to have a finite-dimensional approximation. The observability measure of a finite-dimensional system (Waldraff, *et al.*, 1998) is related to the condition number, or more specifically to the singular values of the system observability matrix, denoted ϑ, which can be defined different ways (presented in the sequel).

Let us consider the following system representation:

$$\begin{cases} \dot{x} = Ax \\ y = Gx \end{cases} \quad (16)$$

where $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n \times n}$ and $G \in \mathbb{R}^{m \times n}$, n is the number of state variables and m is the number of measurements.

3.1. Observability measure based on the observability matrix singular value

This method is based on the condition number of the following observability matrix:

$$\vartheta = \begin{bmatrix} G & GA & \ldots & GA^{n-1} \end{bmatrix}^T \quad (17)$$

i.e., the ratio between its maximal, denoted $\sigma_{\max}(\vartheta)$, and its minimal singular values, $\sigma_{\min}(\vartheta)$.

The observability index is measured by the inverse of the condition number, i.e. $\sigma_{\min}(\vartheta)/\sigma_{\max}(\vartheta)$.

3.2. Observability measure based on the observability Gramian singular value

The Gramian observability matrix is given by:

$$\vartheta_G = \int_0^\infty e^{A^T t} G^T G e^{At} dt \quad (18)$$

The system is completely observable, if and only if:

$$\text{Rank}(\vartheta_G) = n \quad (19)$$

As the previous method, the observability measure of the considered system can also be related to the ratio between $\sigma_{\min}(\vartheta_G)$ and $\sigma_{\max}(\vartheta_G)$.

3.3. Observability measure based on the Popov-Belevitch-Hautus singular value

The observability matrix of Popov-Belevitch-Hautus (PHB) can be written as follows:

$$\vartheta_{PHB}(\lambda_i) = \begin{pmatrix} \lambda_i \mathrm{I} - A \\ G \end{pmatrix} \; ; \forall \lambda_i,\ i=1\ldots n \quad (20)$$

where λ_i is the i^{th} eigenvalue of A. The system is observable if and only if:

$$\text{Rank}(\vartheta_{PHB}(\lambda_i)) = n \quad (21)$$

An equivalent form for observability evaluation of eq. (20) is $\min_{\lambda_i} \dfrac{\sigma_{\min}(\vartheta_{PHB}(\lambda_i))}{\sigma_{\max}(\vartheta_{PHB}(\lambda_i))}$.

PHB rank test proposes also another observability measure. If E is the smallest perturbation to the matrix A so as to make the pair (A+E, G) unobservable, the minimal distance to the set of all unobservable pairs ($A+E$, G) is then given by the solution of the following minimization problem:

$$\|E\|_2 = \min_{s\in \mathbb{C}} \sigma_{\min}\begin{pmatrix} sI - A \\ G \end{pmatrix} \quad (22)$$

The minimization problem (22) is generally non-convex, i.e. difficult to solve. However, the algorithm presented in (Boley, 1990) allows to have upper and lower bounds on $\|E\|_2$. Therefore, the lower bound can be considered as an observability measure.

4. ORTHOGONAL COLLOCATION APPROACH

4.1. Transformation into a lumped system

The orthogonal collocation allows to transform the PDE model into ODE's by expressing the state variables $x(z,t)$ as a finite weighted sum of the state variables at a number n+1 of locations called the collocation points:

$$x(z,t) = \sum_{j=0}^{n} \lambda_j(z) x_j(t)\,;\ x\in \{S_1, S_2, C, Z, X_1, X_2\} \quad (23)$$

where $x_j(t) = x(z_j,t)$ is the value of x at the collocation point z_j, $j\in[0,n]$, with $0 < z_1 < z_2 < \ldots < z_n = H$. $\lambda_j(z_i)$ are the orthogonal functions (e.g. the Lagrange polynomials):

$$\lambda_j(z_i) = \begin{cases} 1 & \text{for } i = j \\ 0 & i \neq j \end{cases} \text{ or } \lambda_j(z_i) = \lambda_{ij} = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{z - z_j}{z_i - z_j} \quad (24)$$

By applying (23), equations (6), (7), (8) and (5) become, $\forall z_i \in [0, H]$,

$$\frac{dS_{1i}}{dt} = \sum_{j=0}^{n} \left(-u_l \dot{L}_{ij} + \varepsilon_l E_z \ddot{L}_{ij}\right) S_{1j} - k_1 \mu_{1i} X_{1i} \quad (25)$$

$$\frac{dS_{2i}}{dt} = \sum_{j=0}^{n} \left(-u_l \dot{L}_{ij} + \varepsilon_l E_z \ddot{L}_j\right) S_{2j} + k_2 \mu_{1i} X_{1i} - k_3 \mu_{2i} X_{2i} \quad (26)$$

$$\frac{dC_i}{dt} = \sum_{j=0}^{n} \left(-u_l \dot{L}_{ij} + \varepsilon_l E_z \ddot{L}_{ij}\right) C_j + k_L a\left(K_H P_{Ci} + Z_i - C_i - S_{2i}\right) + k_4 \mu_{1i} X_{1i} + k_5 \mu_{2i} X_{2i} \quad (27)$$

$$\frac{dZ_i}{dt} = \sum_{j=0}^{n} \left(-u_l \dot{L}_{ij} + \varepsilon_l E_z \ddot{L}_{ij}\right) Z_j \quad (28)$$

Similarly, the biomass equations at the collocation points are written as follows:

$$\frac{dX_{1i}}{dt} = \left(\frac{\mu_{1\max} S_{1i}}{K_{C1} X_{1i} + S_{1i}} - \alpha D\right) X_{1i} \quad (29)$$

$$\frac{dX_{2i}}{dt} = \left(\frac{\mu_{2S} \cdot S_{2i}}{K_{C2} X_{2i} + S_{2i} + \frac{S_{2i}^2}{K_{I2}}} - \alpha D\right) X_{2i} \quad (30)$$

Hence, an ODE nonlinear system can be considered here:

$$\frac{d\xi}{dt} = f(\xi) + A\xi + B\xi_0 \quad (31)$$

where $f(\xi)$ represents the nonlinear term of the model, and

$$\xi = \left[S_{1,1}..S_{1,n}\ \ S_{2,1}..S_{2,n}\ \ C_1..C_n\ \ Z_1..Z_n\ \ X_{1,1}..X_{1,n}\ \ X_{2,1}..X_{2,n}\right]^T \quad (32)$$

ξ_0 is the input vector, such as:

$$\xi_0 = \begin{bmatrix} S_{1,0} & S_{2,0} & C_0 & Z_0 \end{bmatrix}^T \quad (33)$$

with the following boundary conditions:

$$\xi_0(t) = \frac{R(t)}{1+R(t)} \xi_n(t) + \frac{1}{1+R(t)} \xi_{in}(t) \quad (34)$$

where $\xi_n = [S_{1,n}\ \ S_{2,n}\ \ C_n\ \ Z_n]^T$, $\xi_{in} = [S_{1in}\ \ S_{2in}\ \ C_{in}\ \ Z_{in}]^T$. Thus, the matrices A and B are:

$$A = \begin{bmatrix} -u_l\Lambda_1 + \varepsilon_l E_z \Lambda_2 & & & & 0 & 0 \\ & -u_l\Lambda_1 + \varepsilon_l E_z \Lambda_2 & & & \vdots & \vdots \\ & & -u_l\Lambda_1 + \varepsilon_l E_z \Lambda_2 & & & \\ & & & -u_l\Lambda_1 + \varepsilon_l E_z \Lambda_2 & & \\ 0 & \cdots & & & 0 & 0 \\ 0 & \cdots & & & 0 & 0 \end{bmatrix} \quad (35)$$

$$B = \begin{bmatrix} -u_l\Lambda_{10} + \varepsilon_l E_z \Lambda_{20} & & & \\ & -u_l\Lambda_{10} + \varepsilon_l E_z \Lambda_{20} & & \\ & & -u_l\Lambda_{10} + \varepsilon_l E_z \Lambda_{20} & \\ & & & -u_l\Lambda_{10} + \varepsilon_l E_z \Lambda_{20} \\ 0 & \cdots & & 0 \\ 0 & \cdots & & 0 \end{bmatrix} \quad (36)$$

with

$$\begin{cases} \Lambda_1 = \left[\dot{L}_{ij}\right]_{i\in[1,n],\, j\in[1,n]} \\ \Lambda_2 = \left[\ddot{L}_{ij}\right]_{i\in[1,n],\, j\in[1,n]} \end{cases} \text{ and } \begin{cases} \Lambda_{10} = \left[\dot{L}_{i0}\right]_{i\in[1,n]} \\ \Lambda_{20} = \left[\ddot{L}_{i0}\right]_{i\in[1,n]} \end{cases};$$

where $\dot{L}$ and $\ddot{L}$ are the first and second derivatives of L (i.e. Lagrange polynomial) according to z (i.e. axial position), respectively.

4.2. Tangent linearization around an operating point

Let us now consider a tangent linearization around an operating point, defined by the nominal values of the state vectors and the input vectors, $\bar{\xi}$ and $\bar{u}$, respectively. If $\delta\xi = \xi - \bar{\xi}$ and $\delta u = u - \bar{u}$, a linear system can be derived, as follows:

$$\frac{d(\delta\xi)}{dt} = \bar{A}(\delta\xi) + \bar{B}(\delta u) \quad (37)$$

where $\bar{A}$ is the Jacobian matrix of the system, which also represents the dynamics of the system around the operating point.

Referring to the model (31) with $u = \xi_{in}$, one can write:

$$\frac{d(\delta\xi)}{dt} = \left(A + \frac{f(\xi)}{\partial\xi}\right)\cdot(\delta\xi) + \frac{R}{1+R} B\cdot(\delta\xi_n) + \frac{1}{1+R} B\cdot(\delta u) \quad (38)$$

If the measured variable(s) satisfy this expression:

$$y = h(\delta\xi) = G\cdot(\delta\xi) \quad (39)$$

then, the study of the system observability can be carried out on the observability matrix, denoted ϑ, which is given by:

$$\vartheta = \begin{pmatrix} G \\ G\left(A + \frac{f(\xi)}{\partial\xi}\right) \\ \vdots \\ G\left(A + \frac{f(\xi)}{\partial\xi}\right)^{n-1} \end{pmatrix} \quad (40)$$

As the Lagrange polynomial L is considered here, at a given instant, one can interpolate the solution at all points from the solutions at collocation points according to:

$$\delta\xi(z) = \sum_{j=0}^{n} L_j(z)\cdot \delta\xi_j \quad (41)$$

Note that

$$\delta\xi(z_0)=\frac{R}{1+R}\delta\xi(z_n)+\frac{1}{1+R}\delta u \quad (42)$$

$$\delta\xi(z_i)=\sum_{j=1}^{n-1}L_j(z)\cdot\delta\xi_j+\left[L_n(z_i)+\frac{L_0(z_i)R}{1+R}\right]\delta\xi_n+\frac{L_0(z_i)}{1+R}\delta u \quad (43)$$

If one locates a sensor at point z_i, the matrix G in equation (39) can be written as follows:

$$G=[0 \;\ldots\; 0 \;\; G_m \;\; 0 \;\ldots\; 0]\ ;\ G\in\mathbb{R}^{1\times 6n} \quad (44)$$

with

$$G_m=[L_1(z_i)\;\ldots\;L_n(z_i)]+\left[0\;\ldots\;0\;\;\frac{L_0(z_i)R}{1+R}\right]$$

Note that the non-zero elements of the matrix G, denoted G_m, depend on the kind of measurement (substrate, inorganic carbon, etc.)

In this paper, we explore the sensor location question with only the gas measurement: this corresponds to the most largely encountered situation in anaerobic digestion processes. More precisely, we shall concentrate on the availability of CH_4 gas measurements. Eq. (14) specializes that two sensor locations are needed:

$$y=h_M(\xi)=Q_M(z_C)-Q_M(z_i) \quad (45)$$

or

$$y=\frac{k_6A(z_C-z_i)\xi_2(z_C)\xi_6(z_C)}{K_{C2}\xi_6(z_C)+\xi_2(z_C)+\xi_2{}^2(z_C)/K_{I2}} \quad (46)$$

where z_c is the nearest collocation point above z_i, $0\le z_i<z_c\le$ H. In this case, we can write:

$$h_M=\left[0\;\ldots\;\left.\frac{\partial h_M}{\partial\xi_{2C}}\right|_{\xi=\bar{\xi}}\;\ldots\;\left.\frac{\partial h_M}{\partial\xi_{6C}}\right|_{\xi=\bar{\xi}}\;\ldots\right]\xi \quad (47)$$

with

$$\left.\frac{\partial h_M}{\partial\xi_{2C}}\right|_{\xi=\bar{\xi}}=\frac{k_6\cdot A\cdot(z_C-z_i)\cdot\mu_{2S}\cdot\bar{\xi}_{6n}\left(K_{C2}\bar{\xi}_{6C}-\frac{\bar{\xi}_{2C}{}^2}{K_{I2}}\right)}{\left(K_{C2}\bar{\xi}_{6C}+\bar{\xi}_{2C}+\frac{\bar{\xi}_{2C}{}^2}{K_{I2}}\right)^2} \quad (48)$$

$$\left.\frac{\partial h_M}{\partial\xi_{6C}}\right|_{\xi=\bar{\xi}}=\frac{k_6\cdot A\cdot(z_C-z_i)\cdot\mu_{2S}\cdot\bar{\xi}_{2C}{}^2\left(1+\frac{\bar{\xi}_{2C}}{K_{I2}}\right)}{\left(K_{C2}\bar{\xi}_{6C}+\bar{\xi}_{2C}+\frac{\bar{\xi}_{2C}{}^2}{K_{I2}}\right)^2} \quad (49)$$

Note that the choice of a collocation point as one of the sensor locations is arbitrary. Nevertheless, it allows to have z_c closed to z_i.

5. IMPLEMENTATION OF THE OBSERVABILITY MEASURES VIA NUMERICAL SIMULATIONS

For simulations, we consider a tubular reactor with a Peclet number, u_lH/ε_lE_z, equal to 20 (Schoefs, *et al.*, 2003), corresponding to a highly dispersed tubular reactor. Consider that the reactor height H is equal to 3.5 m and its diameter is equal to 0.6 m. The effective volume of the medium is 0.948 m^3 (i.e. ε_l = 0.96). The inlet flow rate is maintained constant, equal to 12.87 L/h. The reactor is operating with a recycle flow rate, i.e. 50 L/h. The atmospheric pressure P_{atm}= 1.04 atm, the pressure coefficient related to the gravity ρg = 0.096841 atm/m, and the temperature is maintained constant, i.e. 35°C.
The other parameters are as follows:

Parameter	Value
$\mu_{1\max}$ (day^{-1})	1.2
K_{C1} (g S_1/g VSS)	50.5
μ_{2s} (day^{-1})	0.74
K_{C2} (g S_2/g VSS)	16.6
K_{I2} (mmol/L)	256
k_La (day^{-1})	19.8
K_H (mol/(L.atm))	16
k_1 (g/g)	42.14
K_2 (mmol/g)	250
K_3 (mmol/g)	134
K_4 (mmol/g)	50.6
K_5 (mmol/g)	228.4
K_6 (mmol/g)	244.86
α	0.5
E_z	1

Once the model parameters are defined, we have then computed the three observability measures on the orthogonal collocation model by positioning the sensor at a point, denoted z_s, that moves from 0 to H.

Let us then consider 7 collocation points (including the boundaries, i.e. at 0, 0.16, 0.81, 1.75, 2.69, 3.34, 3.5 m), chosen as the zeros of a Jacobi polynomial with the parameters (α,β) = (0,0) (i.e. Legendre polynomial). It can be shown that this choice is sufficient to obtain a good approximation, especially for the system spectrum, see (Waldraff *et al.*, 1998). From the first observability matrix (see section 3.1) we can deduce the observability measure as the minimal/maximal singular values ratio for a varying sensor position, $0\le z_s\le$ H, as shown by the curves on Figure 1. Figure 2 shows the Gramian observability measure.

Note that both methods result in very low observability indices. These values are therefore not very reliable when related to the numerical accuracy. Nevertheless, the minimal downward "peaks" allow to detect the position where there are observability losses.

Let us now consider the PHB method. Figure 3 shows the results from the first and the second PHB methods. As we can see, the observability index values are "better" than the previous ones.

Using the three methods, we can note that by considering 7 collocation points there are 5 locations between 0 and H, where there are observability losses. They are 0.1, 0.8, 1.7, 2.7 and 3.3 m. If we consider 11 collocations points (i.e. 0, 0.06, 0.29, 0.68, 1.18, 1.75, 2.32, 2.82, 3.21, 3.44, 3.5 m), the PHB observability measure gives 9 positions of observability loss, i.e. 0.04, 0.28, 0.67, 1.16, 1.72, 2.31, 2.80, 3.20 and 3.43. Figure 4 illustrates the results of the PHB method. We have also applied the PHB method on a 16 point-model, and this gives 14 locations of observability loss.

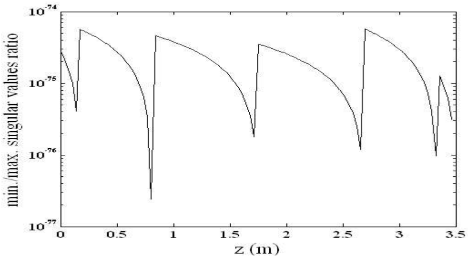

Figure 1. Observability measure based on the usual observability matrix using 7 points.

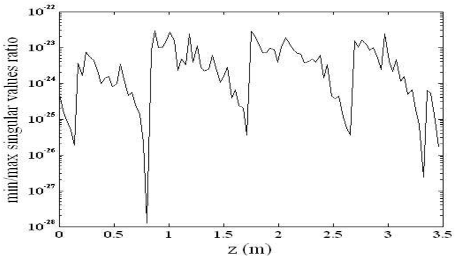

Figure 2. Gramian observability measure using 7 pts.

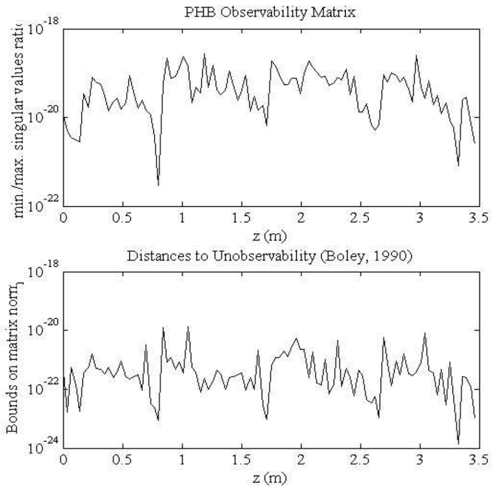

Figure 3. PHB observability measure using 7 points.

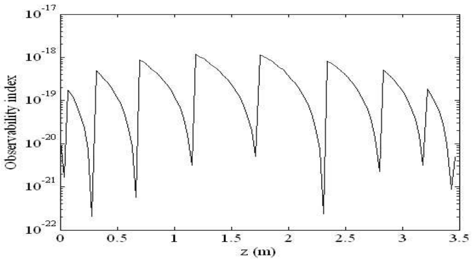

Figure 4. PHB observability measure using 11 points.

These locations approximately correspond to the zeros of the PDE model eigenfunctions (Waldraff, *et al.*, 1998). The larger the model dimension is, the more observability losses may exist.

6. LUENBERGER OBSERVER SYNTHESIS

For illustrating the observability results at different sensor positions, let us consider a state observer. Here, we propose to use the extended Luenberger observer, which is well adapted to a system evolving around an equilibrium point. The synthesis of the observer is as follows. If the nonlinear model of the process is given by the following state system:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y = g(x(t)) \end{cases} \tag{50}$$

the observer equation will satisfy the equation (51):

$$\frac{d\hat{x}}{dt} = f(\hat{x}(t), u(t)) + K(y(t) - g(\hat{x}(t))) \tag{51}$$

where K is a static gain that is calculated by placing the poles of:

$$\left.\frac{\partial f}{\partial x}\right|_{x_{eq}} - K\left.\frac{\partial g}{\partial x}\right|_{x_{eq}} \tag{52}$$

so as to let the estimation error $(\hat{x}(t) - x(t))$ converges to zero.

Assume that the process is initially in steady-state. The initial profiles of the substrates, biomasses and gas flow rates of the WWT process are illustrated on Figure 5. At a given time (here 10 days), steps on the influent concentrations are applied to the process as shown on Figure 6. In this case, the influent pH is maintained constant, equal to about 7, and so is the influent inorganic carbon, i.e. 0.525 mmol/L.

Using these data we have then simulated the model via the finite difference method with 50 nodes, so as to obtain a reference process, which will be considered as the measured variables for the state observer synthesis. Indeed a finite difference scheme with 50 nodes gives a relatively good accuracy.

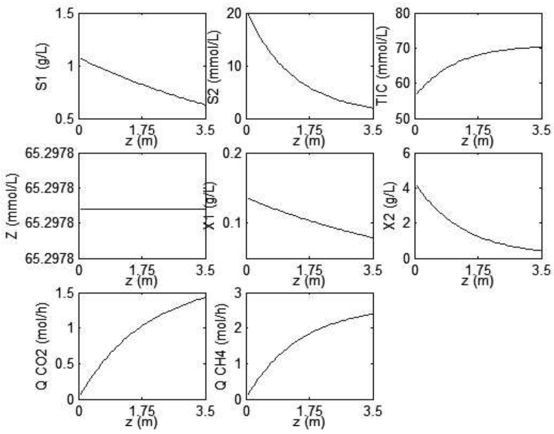

Figure 5. Initial profiles of all the state variables (substrates and biomasses) of a WWT process.

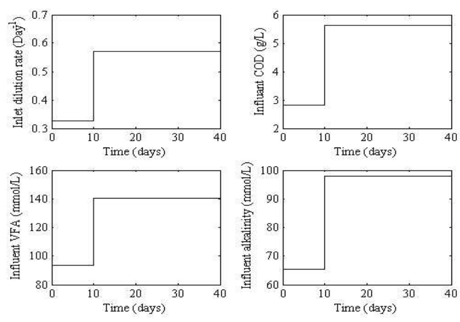

Figure 6. Profiles of the influent parameters.

First simulations are dedicated to CH_4 measurements when the sensor is located at the positions where observability losses are detected. Here, the Luenberger observer based on the 7-point-orthogonal collocation model does not perfectly track the simulated model variable. Figure 7, with a sensor placed on 1.715 m, gives an example of a bad observation. In this case, there is a risk of divergence when there is change of equilibrium, especially if the process is noisy.

On the other hand, Figure 8 is one example of a good sensor location. We can see in this example that the convergence time after the equilibrium change is good.

Note that small static errors can be noted in the curves between the reference (i.e. finite difference scheme) and estimated (i.e. collocation method) models in the steady-state conditions. Using the finite difference method, the mass conservation law is not totally satisfied because of Taylor approximation. On the other hand, the orthogonal collocation satisfies the mass conservation law.

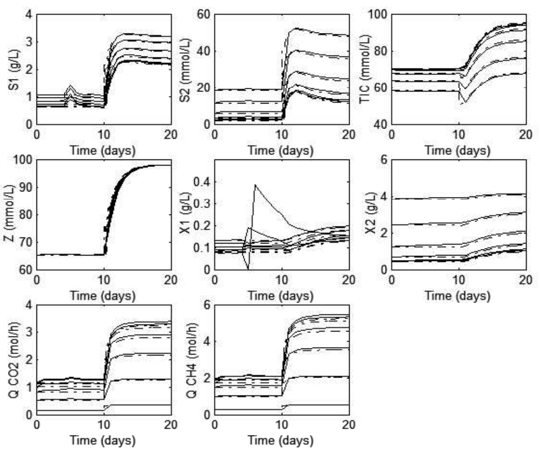

Figure 7. Comparison between the process (dash-dotted lines) and the observer (continuous lines) by placing the sensors at 1.715 m and 1.75 m.

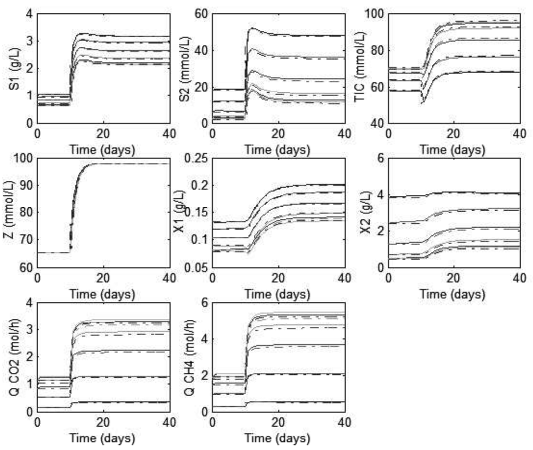

Figure 8. Comparison between the process (dash-dotted lines) and the observer (continuous lines) by placing the sensors at 3.4 m and 3.50 m.

7. CONCLUSION

In this paper, we have analyzed the sensor location for an anaerobic WWT where only the produced gas is on-line measurable (Bernard, *et al*., 2001; Schoefs, *et al*., 2003). The analysis is based on system observability tools for the PDE model of the process (Waldraff, *et al*., 1998)

In order to illustrate the sensor location results, the Luenberger state observer has been implemented in simulations. The state observer is designed by using the orthogonal collocation method, while the reference model is using a finite difference scheme.

The observability analysis has shown that the system is observable if we measure the CH_4 (methane) gas product at 2 positions, one is chosen at a collocation point and the other must be chosen so as to avoid losses of observability.

As the collocation method is a pseudo-spectral method, it transfers approximately the spectrum of the PDE equation. The observability losses are related to the zeros of the PDE model eigenfunctions. Therefore, their number and positions depend on the model dimension. A 7-point-collocation model is the best compromise between accuracy and least of observability losses.

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