INMA2380 Matrix theory

[30h+22.5h exercises] 5 credits

This course is taught in the 2nd semester

Teacher(s): Paul Van Dooren
Language: French
Level: Second cycle

Aims

In-depth study of some specific topics of matrix theory, with emphasis on applications and on underlying numerical aspects.

Main themes

- Matrices defined over a field: equivalence classes, Gaussian elimination, Hermitian forms. similarity and related questions (Courant-Fischer theorem, Schur lemma, QR algorithm, matrix functions, etc.), determinants (Binet-Cauchy theorem), generalized inverses and singular value decomposition with applications
- Matrices defined over a ring: Euclid's algorithm and applications in polynomial matrices, relation to the canonical forms of Hermite and Smith
- Norms and convexity: theory and applications of non-negative matrices, localization of eigenvalues
- Structured matrices: complexity of fast algorithms.

Content and teaching methods

After an introduction recalling some basic notions, we discuss the following topics:
1. Complements on determinants: theorems of Binet-Cauchy and Laplace
2. The singular value decomposition and its applications : polar decomposition, angles between subspaces, generalized inverses, projectors, least-squares problems, regularization
3. Eigenvalue decomposition: Schur and Weyr forms, Jordan form, QR algorithm
4. Approximations and variational characterization: Courant-Fischer and Wielandt-Hoffmann theorem, field of values and Gershgorin theorem
5. Congruence and stability: inertia, Sylvester theorem, Stein and Lyapunov equations, link to stability analysis of dynamical systems
6. Polynomial matrices: Euclid algorithm and the Smith and Hermite forms, link to the Jordan form
7. Non-negative matrices: Perron-Frobenius theorem, stochastic matrices.
8. Structured matrices: notion of displacement rank and fast algorithms for Toeplitz and Hankel matrices.

Other information (prerequisite, evaluation (assessment methods), course materials recommended readings, ...)

Basic knowledge (1st cycle) in linear algebra and numerical analysis

Other credits in programs

MAP22	Deuxième année du programme conduisant au grade d'ingénieur civil en mathématiques appliquées	(5 credits)	Mandatory