

# Faculté des sciences appliquées

## **FSA**

**MECA2660** 

### Méthodes numériques en mécanique des fluides

[30h+22.5h exercises] 5 credits

This course is taught in the 2nd semester

**Teacher(s):** Grégoire Winckelmans

Language: french

Level: 2nd cycle course

#### Aims

Enlarge the knowledge and skills of the students in numerical methods and initiate them to the numerical simulation in fluid mechanics (Computational Fluid Dynamics, CFD), the path followed focusing on the understanding of the physical problems and on their mathematical and numerical modelisation in an adequate formalism.

Develop the aptitude of the student to realize numerical programs (codes) that "put to work" some of the numerical schemes presented in the course, in order to produce a complete numerical simulation of a physical problem.

#### Main themes

- Reminder of the conservation equations in fluid mechanics; Reminder of the differents types of PDEs and of their classification.
- Finite differences et numerical schemes for ODEs and discretized PDEs : consistency, stability, convergence, explicit and implicit schemes.
- Case of 2-D and of 3-D flows, steady and unsteady.
- Incompressible flows: formulation in velocity-pressure and formulation in vorticity-velocity (streamfunction).
- Compressible flows, including capture of discontinuities.
- Structured grids, also with mapping from physical to computational space. Introduction to finite volumes approaches, and to unstructured grids.
- Lagrangian vortex element method (VEM) eventually combined with the boundary element method (BEM)

#### Content and teaching methods

- Reminder of the conservation equations in fluid mechanics.
- Reminder of the different types of partial differential equations (PDEs) and of their classification : hyperbolic, parabolic, elliptic; systems of equations; method of characteristics for hyperbolic cases.
- Finite differences and operators. Precision (order), dispersion (modified wavenumber), compact schemes.
- Reminder of numerical integration schemes for ordinary differential equations (ODEs). Numerical discretisation of PDEs in systems of ODEs. Consistance, stability, convergence. Explicit and implicit schemes.
- Model diffusion equation : explicit and implicit schemes, Alternate Direction Implicit (ADI) schemes for multidimensional problems.
- Model convection equation : explicit and implicit schemes, centered and upwind differences Model non-linear convection equation (Burgers), numerical capture of discontinuities.
- Model convection-diffusion equation, linear and non-linear (Burgers with diffusion).
- Hyperbolic systems in conservative form: Euler equations for inviscid compressible flows; discontinuities and their numerical capture; explicit schemes (Lax, Lax-Wendroff, Richtmeyer, Mac Cormak); implicit schemes (with linearisation of the convective term). Multidimensional problems et generalized ADI schemes.
- Finite differences on structured grids, also with mapping from physical to computational space.
- Introduction to finite volumes approaches, and to unstructured grids.
- Numerical methods for incompressible flows: formulation in velocity-pressure: discretisation using the staggered aproach (MAC), imposition of boundary conditions, method of artificial compressibility for steady flows, explicit and implicit (ADI) versions, methods for unsteady flows, energy conserving shemes; formulation in vorticity-velocity (through a streamfunction): vorticity boundary condition, artificial evolution method for steady flows, methods for unsteady flows, including the lagrangian vortex element method (VEM, using vortex "particles": vortex "blobs"), eventually combined with the boundary element method (BEM, using vortex "panels"), for flows in open domain (e.g., external flow aerodynamics).

## Other information (prerequisite, evaluation (assessment methods), course materials recommended readings, ...)

#### Pre-requisite:

- Knowledge and practice of numerical programming in a scientific language (e.g., C, Fortran).
- Basic knowledge of numerical methods and of fluid mechanics (e.g., course MECA 2321 "fluid mechanics and heat transfer I" or equivalent).

Modalities of organisation:

Practical exercice (homework) and final project. Those can be done using the equipment of the faculty.

Exam: written,; documentation allowed: personal notes, course notes, books.

Remark: the practical exercice and the project count for 40 % of the final course note, and the exam for the remaining 60 %, with the following condition: a minimal note of 30/60 must be obtained at the exam for the note of the practical exercice and project to be used in the final note; otherwise, only the exam note is reported as the final note (the note of the exercice and project still being "acquired", for use in an eventual secondary exam session).

#### Other credits in programs

MAP22 Deuxième année du programme conduisant au grade (5 credits)

d'ingénieur civil en mathématiques appliquées

MECA22 Deuxième année du programme conduisant au grade (5 credits)

d'ingénieur civil mécanicien