Strong Asymptotics of Planar Orthogonal Polynomials: Gaussian Weight Perturbed by Finite Number of Point Charges

Meng Yang

Abstract

We consider the planar orthogonal polynomial $p_n(z)$ with respect to the measure supported on the whole complex plane

$$\mathrm{e}^{-N|z|^2} \prod_{j=1}^{\nu} |z - a_j|^{2c_j} \, \mathrm{d}A(z) \tag{1}$$

where $\mathrm{d}A$ is the Lebesgue measure of the plane, N is a positive constant, $\{c_1, \cdots, c_\nu\}$ are nonzero real numbers greater than -1 and $\{a_1, \cdots, a_\nu\} \subset \mathbb{D} \setminus \{0\}$ are distinct points inside the unit disk. In the scaling limit when $n/N = 1$ and $n \to \infty$ we obtain the strong asymptotics of the polynomial $p_n(z)$. We show that the support of the roots converges to what we call the "multiple Szegő curve," a certain connected curve having $\nu + 1$ components in its complement. We apply the nonlinear steepest descent method on the matrix Riemann-Hilbert problem of size $(\nu + 1) \times (\nu + 1)$. This is the joint work with Seung-Yeop Lee(University of South Florida).