# Is Selling Immigration Rights Politically Sustainable ?

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## 1 Introduction

Rich nations face huge pressures from workers living in the rest of the world for opening their borders and increasing the number of immigrants.

Why do rich nations do not open borders ? One of the main reasons is that it is impossible to prevent immigrants from receiving social welfare benefits and accordingly, it is feared that illimited immigration would put public finance in jeopardy (implicitly, a lot of immigrants would come in order to get social benefits, and would not work at all). Moreover, free immigration could result in undesirable changes in factor prices (for instance, either a decrease in wage income or in capital income).

From a pure economic viewpoint, these concerns has led several nations to set immigration quotas (e.g., The United-States, Canada, Germany etc...) to regulate the flows of foreign workers, allowing an easy entry for those whose skills are must needed.

These quotas have been criticized by Gary Becker (1997)-(2005) because there are better tools to regulate inflows of foreign workers. These tools should rely more on market forces and less on bureaucracy. Becker proposes that countries should sell the right to immigrate. Under his proposal, a country would set a price for the right to immigrate and would allow entry to all applicants willing to pay the price.

There are several other possible ways to implement this idea. For instance, a country could auction the right to immigrate (see Ochel (2001)); it could also auction employment permits (by allocating work permits to the cheapest immigrants who apply for a certain period of time, see Felbermayr (2003))[1].

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1 De la Croix and Gosseries (2007) discuss tradable emigration and immigration quotas (respectively of skilled and unskilled workers) in relation to the procreation quotas advocated by Boulding (1964). On the use of these quotas in an international setting, see also De la Croix and Gosseries (2009).

What would be the effect of using a permits system ? First of all, applicants would probably be younger (since a relatively long period of time would be needed to earn enough to finance the immigration fee) and then healthier (as a consequence they would not rely on social welfare programs to live). Naturally, it would be easier to immigrate for skilled persons or ambitious individuals (because these agents earn the highest incomes and are in a better position to finance the immigration fee). Moreover, only those individuals who want to stay in a country for a rather long period of time would be interested in paying for the right to immigrate.

Ochel (2001) discusses several disadvantages of what he calls the auction model. There is no reason that immigration permits would always select the "right immigrants" (those whose skills are the most needed). It is also possible that potential migrants would lack information about the receiving country. Finally, one could raise ethical objections to the sale of immigration permits.

On the other hand, Becker (2005) suggests that "Economics analysis proves that there certainly exists a positive price (and I believe a significant one) that would have a larger number of immigrants than under the present quota system". But this implies that immigration permits would be politically sustainable, i.e., compatible with voters' preferences.

The present paper takes up this issue by focusing on the political process underlying the choice of the permits' price. It analyzes the conditions under which voters would prefer immigration permits over immigration quotas and which immigration price would be chosen.

This paper can be considered as a contribution to the literature that analyzes immigration from a political economy view point. The seminal article of this literature seems to be that of Benhabib (1996) where quotas are determined by majority voting (and where agents are differentiated according to their wealth-human capital). Magris and Russo (2004) have extended the Benhabib model by endogenizing migration decisions as well as by introducing border enforcement costs and imperfect screening of immigrants. Voting on immigration policy is also studied by Grether et al. (2001) and Bilal et al. (2003) in versions of the Ricardo-Viner model of international trade. Atsu Amegashie (2004) examines a model in which the number of immigrants allowed into a country is the outcome of a costly political lobbying contest between a firm and a union (the lobbying contest is an all-pay auction). Epstein and Nitzan (2006) analyze the endogenous determination of a migration quota by viewing it as the outcome of a two-stage political struggle between two interest groups: those in favor and those against the proposed migration quota. Bellettini and Berti Ceroni (2008) study the determinants of immigration policy in an economy with entrepreneurs and workers where a trade union has monopoly power over wages. Facchini, Razin and Willmann (2004) study the determination of immigration policy as the outcome of a lobbying game between domestic interest groups and the government when there is welfare leakage. Facchini and Willmann (2005) study the dermination of policy towards international factor mobility in a common agency setting (they compare tariffs and quotas and stress the importance of the degree of complementarity among inputs). Ortega (2005) studies the determination of immigration policy when there are heterogeneously skilled agents who anticipate that immigrants will have the right to vote and, hence, may affect future policies. Finally, Dulla, Kahana and Lecker (2006) have studied the political economy of the interactions between sources and receiving countries (they suggest that under certain conditions, the receiving country should direct some of the resources earmarked for coping with the problem of the illegal flow of workers to financially supporting the source countries, allowing them to compete among themselves for such aid).

In this paper, we rely on the model proposed by Benhabib (1996). This is a model of an open economy, with a unique production sector, two inputs, which are inelastically supplied by national agents. This is a simple but convenient tool for analyzing the political economy aspects of immigration when policies are set according to the preferences of the majority (formally, policies are determined by applying the median voter theorem). There are at least two limitations to using this model. First, since there is a single produced good, whose price is the numéraire, one cannot analyze the impact of immigration flows on the output price. Second, for the same reason, one cannot analyze the sectoral effects of immigration (and, in particular, its role in alleviating sectoral shortages of labor). In fact, the whole analysis concentrates on the effects of immigration flows on input prices (we disregard the effects of immigration on production levels, since as all national inputs are inelastically supplied, only the effects of immigration on factor prices matter, from a political economy view point). In the Benhabib model, analyzing the factor prices boils down to studying the capital-labor ratio obtained with a system of permits and quotas.

The idea that immigration has an effect on input prices, and in particular on wages, is a controversial one: for the USA, Borjas (2003) finds a substantial effect, while Card (2005) or Ottaviano and Peri (2008) find a negligible effect. However, Borjas (2009) criticizes the previous studies from a theoretical view point and asserts that immigration is likely to have a negative impact on wages. Thus, while our analysis will be mainly limited to analyzing the effects of immigration on factor prices, it will also be addressing an important issue.

The main results of this paper as well as its organization are as follows. After presenting the model in section 2, we will compare in section 3 the maximal and minimal values of the capital-labor ratios obtained with immigration permits and quotas. In section 4, we analyze the choice between immigration permits and quotas (agents first choose an immigration scheme, and then the way it is implemented (i.e., if immigrations permits (resp. quotas) are favored, the value of the permit (resp. of the quotas) is chosen in a second vote)). We first show that redistributing immigration fees to native agents can make them favor immigration permits over quotas (Here, we do not use the median voter theorem, since it is not easily applicable). This happens when the difference in the wage rates corresponding to both systems is low compared to the per capita value of immigration fees. When these fees are wasted, one may easily apply the median voter theorem. We then find that if the wealth of the median voter is low enough, immigration quotas will be chosen over immigration permits because they realize the highest capital-labor ratio (and then the highest wage rate). If the median voter's wealth is high enough, then the society will choose the system which delivers the lowest capital-labor ratio. We rely on an example to show that such a system could be the permits system. This suggests that political sustainability is an important issue when implementing immigration permits. Section 5 contains some concluding remarks.

# 2 The Model

## 2.1 The Receiving Economy

Following Benhabib (1996) we consider an economy where each agent is described by his wealth $s$ (which is equal to a quantity of capital). The density function describing the wealth distribution is denoted $N(z)$ and is defined in $[0, +\infty]$. That is, the number of agents whose wealth is no higher that $x$ is $\int_0^x N(s)\,ds$. The initial capital stock $K_0$ in the economy can be written as:

$$K_0 = \int_0^{+\infty} sN(s)ds \tag{1}$$

The size of the population in the economy (before immigration) is:

$$N_0 = \int_0^{+\infty} N(s)ds \tag{2}$$

The probability density function describing the wealth distribution of potential immigrants is denoted by $I(z)$. It is defined in $[\underline{z}, +\infty]$, where $\underline{z} > 0$, and it is differentiable[2]. Thus, the number of immigrants whose wealth is no higher that $x$ is $P(x) \equiv \int_{\underline{z}}^x I(z)\,dz$. The maximum number of potential immigrants is $I = P(\infty) < +\infty$.

[2] The assumption that $\underline{z}$ is positive is not important. It nevertheless enables us to work with Pareto distributions as in the example used in the next section .

Production is realized in a single firm whose technology displays constant returns to scale. There are two inputs, capital ($K$) and labor ($L$). Let $F : \mathbb{R}^2_+ \to \mathbb{R}_+$, $(K, L) \mapsto F(K, L)$ denote the production function. $F(.,.)$ is assumed to be increasing, concave, strictly concave with respect to each input, and twice continuously differentiable.

Denoting $R$ the capital price and $w$ the labor wage rate, one has $R = F_K^{'}(K/L, 1)$ and $w = F_L^{'}(K/L, 1)$. Let the capital labor ratio be $k = K/L$, then if $f(k)$ denotes $F(k, 1)$, one has: $R = f'(k)$ and $w = f(k) - kf'(k)$.

We shall assume that agents' payoffs are an increasing function of their incomes. Thus, for simplicity, we disregard the possibility that these preferences depend on the unemployment rate, ethnicity etc[3]... An agent with wealth $s$ has an income equal to $O_s(k) = w(k) + R(k)s$. Note that $O_s^{'}(k) = (s - k)f''(k)$. The function $O_s(k)$ reaches a minimum at $k = s$. An increase in $k$ brings about an increase in the wage rate as well as a decrease in the return to capital. When agents have a small capital, an increase in $k$ yields an increase in their income (since the increase in the wage rate more than compensates the decrease in the capital income). The reverse result is reached when agents are wealthy. These effects compensate exactly when $k = s$.

## 2.2 Immigration Policies

Immigration policies affect agents incomes because they affect the capital-labor ratio. The first policy that will be considered consists in selling immigration permits.

### 2.2.1 Permits

Permits are sold to immigrants (and not to firms in the host country). For the time being, we shall assume that the price of an immigration permit is fixed at a level $p$. This price must be paid before coming into the host economy. We also assume that every immigrant with wealth at least equal to $p$ chooses to immigrate. The number of migrants is then equal to:

$$\overline{P}(p) \equiv 1 - P(p) = \int_p^{+\infty} I(z)dz, \tag{3}$$

The determination of the post-immigration capital-labor ratio depends on the way permit sales are redistributed to native agents. To see this, first recall that the model used in this paper is a static one. There are no explicit savings decisions. Agents just supply labor and capital, then pro-

[3] For a study of the issues on which the preferences of the local population depend, see Boeri, Hanson and McCormick (2002), Mayda (2006) and Dustmann and Preston (2007). I thank a referee for giving me the last two references.

duction is realized. As there is a single good, the capital stock remaining at the end of the production period can be consumed or re-invested. However, the sharing of production (plus the non-depreciated part of thecapital) between consumption and investment is not detailed. Second, given the previous remarks, there seem to be two ways of conceiving the redistribution of permit sales. On the one hand, one may redistribute these sales before the production period. This means that these transfers must be invested by the native agents: they are like the capital stock $K_0$ (which cannot be consumed, at least not before having been used in the production process). On the other hand, one may redistribute the permit sales after the production period. In that case, agents simply receive transfers which can either be consumed or saved [4].

From a formal viewpoint, we shall assume that a share $1-\mu$, $\mu \in [0, 1]$ of permit sales are redistributed to native agents and re-invested. The remainder will either be redistributed to agents after the production period or wasted. To simplify the presentation, we will assume that when the transfers are not wasted they are consumed.

Building on these assumptions, we define the post-immigration capital-labor ratio in the economy a function $\tilde{k} : [\underline{z}, +\infty) \times [0,1] \to \mathbb{R}_{++}$:

$$\tilde{k}(p,\mu) = \frac{K_0 + \int_p^{+\infty} zI(z)dz - \mu p \int_p^{+\infty} I(z)dz}{N_0 + \overline{P}(p)} \tag{4}$$

This expression can be understood as follows. The net capital supplied by an immigrant with wealth $z$ is equal to $z-p$. When $\mu = 1$, the redistribution of the permit sales does not influence the capital stock: the native agents totally consume their transfers. The same expression of the capital stock is obtained when the permit sales are wasted. When $\mu = 0$, permit sales are completely re-invested. Intermediary cases are possible (in these cases, a fraction $1-\mu$ of the transfers is invested).

Equation (4) is obtained if the two following assumptions are satisfied (all of which will be standing throughout the paper). First of all, immigrants cannot borrow from agents in the host country. This assumption is made to simplify the analysis (its main implication is that savings in the host country are not diverted from capital accumulation). Second, it is assumed that the gross wealth $z$ that an immigrant brings into the host country does not depend on $p$ [5].

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4 In an overlapping generations model where life lasts two periods, our discussion would be boiled down to knowing how permit sales are redistributed between young and old agents.

### 2.2.2 Quotas

The second policy analyzed consists in choosing immigration quotas. A specification of quotas is a pair $(s, q)$, where $s$ is a positive real number, $q \in [s, \infty]$, which determines the types of agents that are allowed to enter the country. That is, the number of immigrants is equal to $\int_s^q I(z)\,dz = P(q) - P(s)$. The post immigration capital-labor ratio writes:

$$k(s,q) \equiv \frac{K_0 + \int_s^q zI(z)dz}{N_0 + P(q) - P(s)} \tag{5}$$

We assume that: $\underline{z} = K_0/N_0$. This is a reasonable assumption since $\underline{z}$ could be thought of as being close to zero.

## 2.3 Discussion of the modeling of immigration permits and quotas

What are the main differences between permits and quotas ? Concerning the structure of the immigrant population, specifying a quota amounts to fixing the number as well as the type of individuals in more detail than for permits. In particular, one can choose *both* the minimal and maximal wealth levels of immigrants. This is impossible with a system of permits. In the latter, one can only choose a minimal value for the wealth of the immigrants (since every immigrant who can afford the immigration fee is supposed to immigrate).

A second major difference between permits and quotas lies in the existence of permit fees. These fees may be redistributed and can be considered as a way of sharing the benefits of immigration between immigrants and native agents. Permit fees also have an impact on capital accumulation (since they influence the net inflows of capital).

It is not easy to take into account all these characteristics by studying the effects of immigration from a political economy viewpoint. We will study a special case ($\mu = 1$) by assuming that all immigration fees are wasted. This assumption *approximates* what seems to us the likely importance of immigration fees, at least for developed nations. We do not believe that redistributing immigration fees would yield a significant per capita

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[5] The gross wealth may depend on $p$ when immigrants borrow a part of their capital as well as the immigration fee. In such a situation, the gross wealth carried by an immigrant could be a decreasing function of $p$. To see this, assume that the amount $b$ borrowed by an immigrant solves the next problem: $max_b V(h+b-p) - C(b)$, where $V(.)$ is an increasing smooth concave function of the net wealth $w = h+b-p$ carried in the host country ($h$ is the wealth of an agent who does not borrow), while $C(.)$ is an increasing convex smooth function. The optimal choice of the immigrant with regard to $b$ satisfies $V'(h+b-p) = C'(b)$. It is easy to see that $db/dp = V''(h+p-b)/(V''(h+p-b) - C''(b)) > 0$. However, $dw(p)/dp = db/dp - 1 < 0$. Our assumption can be understood as supposing that $db/dp$ is small.

transfer for native agents (even if the number of immigrants is high and the immigration price is “significant” (Becker)). For instance, this seems unlikely in a developed country with a large population, as in the USA, or the European Union (it would matter, perhaps for a small country in the European Union, or Switzerland). We believe that the increase in the number of immigrants (which is expected by the proponents of the permit system, like Becker), may have important consequences on factor prices (and this will occur whether or not the destination country has a large population). We do not however deny that the redistribution of immigration fees is an important issue, especially from a political economy viewpoint (in particular if the redistribution of the permit sales is means-tested).

# 3 Comparing capital-labor ratios with immigration quotas and immigration permits

In this section we compare the maximal and the minimal values of the capital-labor ratios obtained with a system of permits and quotas. We first analyze the minimal and maximal values of the capital-labor ratio obtained with a system of quotas. Then, we compare these values with the capital-labor ratios obtained with a system of permits.These comparisons will have consequences on the political stability of permits since the capital-labor ratio is a key factor explaining the income of native agents.

## 3.1 Study of the capital-labor ratio obtained with a system of quotas

This topic was already taken up by Benhabib (1996) for the case $\underset{\sim}{z} = 0$. Here, we restate his result in the case $\underset{\sim}{z} > 0$ and we study the quotas maximizing or minimizing the capital-labor ratio (in particular with respect to $K_0/N_0$).

**Proposition 1.**

*a) There is a unique real number* $v$, $z < v < K_0/N_0$, *such that with immigration quotas* $(s, q) = (\underset{\sim}{z}, v)$, $k(\underset{\sim}{z}, v)$ *is the minimal value of the capital-labor ratio.*

*b) There is a unique real number* $\underline{s}$, $\underline{s} > K_0/N_0$, *such that with immigration quotas* $(s, q) = (\underline{s}, +\infty)$, $k(\underline{s}, +\infty)$ *is the maximal value of the capital-labor ratio.*

*Proof.* See Appendix A.

The intuition for the Proposition is as follows. A marginal increase in $q$ generates two opposite effects on the capital-labor ratio. On the one hand, an increase in $q$ allows wealthier immigrants to enter the country and this is conducive to an inflow of capital. On the other hand, the entry of new immigrants increases the labor supply. The marginal effects of an increase in $s$ can be analyzed in a similar way (but they work in opposite directions).

From point a) one sees that to achieve a minimal capital-labor ratio, one should not allow wealthy immigrants to enter. Only the poorest immigrants should enter the country. The reason for this is that poor migrants bring with them small amounts of capital. Hence, the effect of their entry on the supply of capital is positive but small whereas the effect on the labor supply is relatively high.

Symmetrically, as shown in point b), to realize a maximal capital-labor ratio, one should not allow the poorest migrants to enter.

## 3.2 Comparison of the capital-labor ratios obtained with a system of quotas and permits

We now compare the capital-labor ratios that can be obtained with a system of permits or quotas.

**Proposition 2.**

*a) If permit sales are completely re-invested ($\mu = 0$), the minimal value of the capital-labor ratio obtained with a system of quotas is always lower than the minimal value of the capital-labor ratio obtained with a system of permits. In the other case ($\mu > 0$), the comparison is inconclusive.*

*b) When $p = \underline{s}$ and permit sales are completely re-invested ($\mu = 0$), the maximal value of the capital-labor ratio obtained with a system of quotas is equal to the maximal value of the capital-labor ratio obtained with a system of permits. Otherwise, it is always higher.*

*Proof.* a) When $\mu = 0$, immigration fees - $p\overline{P}(p)$ - are rebated to native agents and completely re-invested. Therefore, the capital ratio is equal to:

$$\tilde{k}(p,0) = \frac{K_0 + p\overline{P}(p) + \displaystyle\int_p (z-p)I(z)\,dz}{N_0 + \overline{P}(p)} \tag{6}$$
$$= \frac{K_0 + \overline{P}(p)}{N_0 + \overline{P}(p)}$$
$$= k(p,\infty)$$

Hence, a capital-labor ratio with a system of permits can always be realized with a system of quotas. Now, from Proposition 1 a), we know that the minimal value of the capital-labor ratio with a system of quotas is obtained at $k(\underline{z}, v)$. Thus we must have : $k(\underline{z}, v) < \inf_p k(p,\infty) \leq \inf_{p \geq \underline{z}} k(p,\infty)$.

When $\mu$ is positive, the comparison of $\tilde{k}(p, \mu)$ and $k(\underset{\sim}{z}, v)$ is indeterminate. Below, we provide an example showing that when $\mu = 1$, one may have $\tilde{k}(p, 1) < k(\underset{\sim}{z}, v)$ (by continuity, this would also hold for $\mu$ close to 1).

b) Finally, when $\mu = 0$, unless $p = \underset{\sim}{s}$, Proposition 1 b) asserts that: $k(p, \infty) < k(\underset{\sim}{s}, \infty)$. Now since $\tilde{k}(p, \mu)$ is a decreasing function of $\mu$, we have : for all positive $\mu$, for all $p$, $\tilde{k}(p, \mu) < k(\underset{\sim}{s}, \infty)$. The conclusion follows.

Let us first consider the intuition of the first part of the Proposition.

An increase in the permit's price has two effects on the capital-labor ratio. First, there are less immigrants who are able to afford the price of entry. Thus there is a negative effect on the denominator of the capital-labor ratio. Second, each immigrant comes with a lower net capital (since they must pay the immigration fee). There is then a negative effect on the numerator of the capital-labor ratio. The net effect on the capital-labor ratio is therefore a priori ambiguous.

A system of quotas can always be designed to let in the immigrants who enter in the country with a system of permits. But it cannot always replicate their net inflows of capital, especially when the immigration fees are wasted. Allowing everyone to entry does not yield the lowest capital-labor ratio with a system of quotas. To achieve the latter, one must prevent rich immigrants from entry. But even if only relatively poor people do come in, this may not be sufficient to realize a capital-labor ratio lower than the lowest level achieved with permits. Indeed, with a system of permits, there are more people who enter and more capital, but this increase in capital is diluted by the increase in the number of immigrants; moreover, only the net inflow of capital matters if the permit sales are not completely re-invested.

Let us consider now the second part of the Proposition. Imagine that the permit's price $p$ maximizing the capital-labor ratio is equal to $\underset{\sim}{s}$. Then, necessarily, one would have $\tilde{k}(p, \mu) \leq k(\underset{\sim}{s}, \infty)$ since with a system of permits, immigrants must pay an immigration fee and, as a result, come with a lower net capital than if there were a system of quotas. Of course, there is no reason why $p$ would be equal to $\underset{\sim}{s}$. But whatever the value of $p$ may be, it is impossible that $\tilde{k}(p, \mu) > k(\underset{\sim}{s}, \infty)$. If this were the case, a system of quotas with $s = p$ would be feasible and, due to the immigration fee, would generate more capital inflows than the system of permits, contradicting the inequality.

When the number of immigrants follows a Pareto distribution and the permit sales are wasted, the capital-labor ratio is always an increasing function of the permit price. Thus, when the permit's price increases, there is relatively less capital than labor flowing in. Therefore, to achieve a minimal capital-labor ratio, one should allow in everybody who can afford the minimum price $\underset{\sim}{z}$. The next example illustrates the theoretical possibility that $\tilde{k}(\underset{\sim}{z}, 1) < k(\underset{\sim}{z}, v)$.

EXAMPLE. Assume that $P$ is a Pareto distribution, i.e., $P(z) = I(1 - \underline{z}/z^2)$. Then, one has:

$$\tilde{k}(p,1) = \frac{K_0 + I\frac{\underline{z}^2}{p}}{N_0 + I\frac{\underline{z}^2}{p^2}} \tag{7}$$

and:

$$k(\underline{z},q) = \frac{K_0 + 2I\underline{z}^2(\frac{1}{\underline{z}} - \frac{1}{q})}{N_0 + I(1 - \frac{\underline{z}^2}{q^2})} \tag{8}$$

Let us assume furthermore that $K_0 = 2$, $\underline{z} = 1/2$, $N_0 = 1$, $I = 1$. The graphs of $\tilde{k}(.,1)$ and $k(\underline{z},.)$ obtained with these values are depicted in figure 1 (see the end of this paper). This picture shows that $\tilde{k}(\underline{z},1) < k(\underline{z},q)$ for all $q$, $\underline{z} \le q$.

In appendix B, we show that the same conclusion is reached whenever $\underline{z} < K_0/N_0$.

Even if the example is robust, one might question the assumption that the number of potential immigrants follows a Pareto distribution. While many social phenomena follow power laws distribution (see, e.g., Gabaix (2009)), a slightly more general class than the Pareto distribution, only further empirical works can provide a sound answer[6].

## 4 Political sustainability of immigration permits

In this section, we discuss the choice between a system of quotas and permits. This choice is made by majority voting, which amounts formally to applying the median voter theorem. In the general case where immigration fees are redistributed, it is difficult to apply this theorem and we shall only make a few remarks. In contrast, when the permit sales are wasted, applying the median voter theorem is easy, and we can propose a more detailed analysis.

[6] One could for instance calibrate the distribution of immigrants' wealth by using the original dataset collected by Docquier and Marfouk (2006). Nevertheless, the data collected by the authors concern people who *have* immigrated: the data result from various quota policies and do not inform us exactly on the number of *potential* immigrants, who decided not to or could not immigrate. Also, only education attainments are taken into account (financial or physical capital are ignored). That said, education attainment could be considered as a proxy of wealth (when age is taken into account). As a consequence, using this data base would be an instructive exercise and a good starting point.

## 4.1 Remarks on the case where immigration fees are redistributed

Let us assume that immigration fees are redistributed to agents and that a fraction $1-\mu$ is re-invested. Then the income of a native agent with wealth $s$ would be equal to:

$$w\big(\tilde{k}(p,\mu)\big) + \big(s + (1-\mu)\frac{p\int_p^\infty I(z)dz}{N_0}\big)R\big(\tilde{k}(p,\mu)\big) + \mu\frac{p\int_p^\infty I(z)dz}{N_0} \tag{9}$$

This income has three components: 1) the wage rage, 2) the capital income (which derives in part from the re-investment of the permit sales), 3) the redistribution of the permit sales which is not re-invested. Since the expression above is not generally unimodal in $p$, it is difficult to apply the median voter theorem and then to analyze what permit price would be chosen by a majority of agents with redistribution of permit sales.

Let us assume now that the median voter prefers a high capital-labor ratio (and then a high wage rate). When immigration permit sales are rebated to agents, permits could be preferred over quotas even though it would be difficult to predict the value of $p$ chosen by the majority. Indeed, the transfers may compensate a lower capital-labor ratio (and a lower wage rate), compared to a system of quotas. The following Proposition formalizes this remark.

**Proposition 3.** *Let $p$ be any immigration fee such that:*

$$F'_L\big(k(\underline{s},\infty),1\big) - F'_L\big(\tilde{k}(p,\mu),1\big) < \frac{p\overline{P}(p)}{N_0}. \tag{10}$$

*Then every agent prefers immigration permits (sold at price $p$) over the quotas $(\underline{s},\infty)$.*

*Proof.* The preceding equation may be rewritten as:

$$\frac{1}{N_0}p\overline{P}(p) > w\big(k(\underline{s},\infty)\big) - w\big(\tilde{k}(p,\mu)\big) \tag{11}$$

Hence,

$$w\big(\tilde{k}(p,\mu)\big) + \frac{1}{N_0}p\overline{P}(p) > w\big(k(\underline{s},\infty)\big) \tag{12}$$

Consider an agent with wealth $s$. We have:

$$\begin{aligned} w(\tilde{k}(p,\mu)) + \frac{1}{N_0}p\overline{P}(p) + \big(s + \frac{p\int_p^\infty I(z)dz)}{N_0}\big)R(\tilde{k}(p,\mu)) &> w(k(\underline{s},\infty)) \\ &+ \big(s + \frac{p\int_p^\infty I(z)dz)}{N_0}\big)R(\tilde{k}(p,\mu)) \end{aligned} \tag{13}$$

Since $k(\underline{s}, \infty) \geq \tilde{k}(p, \mu)$ and the return to capital is a decreasing function, it follows that:

$$w\big(\tilde{k}(p,\mu)\big) + \frac{1}{N_0} p\overline{P}(p) + \big(s + \frac{p\int_p^{\infty} I(z)dz)}{N_0}\big) R\big(\tilde{k}(p,\mu)\big) > w\big(k(\underline{s},\infty)\big) + \big(s + \frac{p\int_p^{\infty} I(z)dz)}{N_0}\big) R\big(k(\underline{s},\infty)\big) \tag{14}$$

This proves that every agent prefers permits over quotas.

The Proposition above shows that if the per-capita value of immigration permit sales is greater than the difference between wages, then permits are preferred by all agents over quotas [7].

The condition in the above Proposition is likely to be satisfied when immigrants represent a substantial share of the population (this is the case, for instance for some Persian Gulf countries or Switzerland) [8]. In that case, the per capita values of the transfers to native agents may be important (and compensate the differential in the wage rates).

## 4.2 The case where immigration fees are wasted

We now concentrate on the case where immigration fees are not redistributed and are wasted ($\mu = 1$). What matters now is how the immigration policies directly affect the capital labor-ratio and thus the income of every native agent (let us recall that the income of agent $s$ can be written $O_s(k) = w(k) + R(k)s$).

To apply the median voter theorem, we need the next notion. We shall say that an agent with wealth $s$ is *indifferent* between two capital-labor ratios $k$ and $z$ if:

$$w(k) + R(k)s = w(z) + R(z)s \tag{15}$$

This agent always exists, is unique and such that:

$$s = \frac{w(k) - w(z)}{R(z) - R(k)} \tag{16}$$

The following Lemma, which is due to Benhabib (1996), is instrumental in studying the indifferent agent between two capital-labor ratios.

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7 In a model of international trade with endogenous growth *à la Grossman-Helpman*, Lundborg and Segerstrom (2002) show that mass immigrations can be welfare decreasing (both for laborers as well as capital owners). They also show that an immigration tax can compensate native workers. Such an immigration tax would be very similar to having to buy an immigration permit.

8 I thank a referee for drawing my attention to this fact.

**Lemma 1.** *(Benhabib) Let two positive capital-labor ratios* $z$ *and* $k$ *be given, with* $z < k$. *Let* $\Delta(z, k)$ *denote the capital stock of the agent which is indifferent between* $z$ *and* $k$. *Then,* $z < \Delta(z, k) < k$.

We also have:

**Lemma 2.** *Let two positive capital-labor ratios* $z$ *and* $k$ *be given, with* $z < k$. *Then:*

*a) Every agent with wealth* $s > \Delta(z, k)$ *(resp.* $s < \Delta(z, k)$ *) prefers* $z$ *(resp.* $k$ *) to* $k$ *(resp.* $z$ *):* $O_s(z) > (<) O_s(k) \Leftrightarrow s > (<) \Delta(z, k)$.

*b) For all capital stocks* $u$ *be in* $]z, k[$, *for all* $s$, $max\{O_s(z), O_s(k)\} > O_s(u)$.

*Proof.* a) The function $\phi(s) = w(k) - w(z) + (R(k) - R(z))$ is decreasing with $s$. Since it vanishes at $s = \Delta(z, k)$, one has $\phi(s) < 0 \Leftrightarrow s > \Delta(z, k)$. It follows that $O_s(z) > (<) O_s(k) \Leftrightarrow s > (<) \Delta(z, k)$.

b) Let $u$ be in $]z, k[$. Since $O_s(.)$ realizes its minimum at $s$, one has: if $s < u$, then $O_s(u) < O_s(k) \leq max\{O_s(k), O_s(z)\}$ and if $s > u$, then $O_s(u) < O_s(z) \leq max\{O_s(k), O_s(z)\}$.

Depending on their levels of wealth, agents will prefer either the smallest possible capital-labor ratio, or the highest. In the first case, they will choose to maximize the return to capital; in the second, they will look for the highest possible wage.

Let us now consider a vote whose issue is decided according to the majority principle. In our setting, the issue of the vote is decided by the median voter, i.e., the agent whose wealth $k_m$ is the solution to:

$$\frac{\int_0^{k_m} N(s)ds}{N_0} = 0.5 \tag{17}$$

We consider that the choice between immigration quotas and immigration permits is made in two steps. In the first step, voters have to choose between the two alternative systems; in the second one, an alternative having been chosen, they choose either the value of the quotas, or an immigration fee. Since immigration permits are an alternative to quotas (which are in use in several countries), they will only be implemented if they strictly increase the income of the median voter (when the outcomes of the two systems are equivalent, there is no reason to prefer permits over quotas).

The analysis presented in the preceding sections leads us to consider two cases. Either[9]:

9 Notice that since $sup_{p \geq z}\tilde{k}(p, 1) \geq K_0/N_0$, we always have: $k(z, v) < sup_{p \geq z}\tilde{k}(p, 1)$.

$$k(\underline{z}, v) \leq \inf_{p \geq \underline{z}} \tilde{k}(p, 1) < \sup_{p \geq \underline{z}} \tilde{k}(p, 1) < k(\underline{s}, \infty) \tag{18}$$

or[10]

$$\min_{p \geq \underline{z}} \tilde{k}(p, 1) < k(\underline{z}, v) < \sup_{p \geq \underline{z}} \tilde{k}(p, 1) < k(\underline{s}, \infty) \tag{19}$$

where $v$ is such that $(v, +\infty)$ minimizes $k(s, q)$ (see Proposition 1).

In the first case, we can see that in a vote with immigration quotas and immigration permits as alternatives, no agent will choose permits over quotas. Indeed, let us consider the indifferent agent $\Delta(k(\underline{z}, v), k(\underline{s}, \infty))$.

If an agent's wealth $i$ is such that $i > \Delta(k(\underline{z}, v), k(\underline{s}, \infty))$ this agent will favor immigration quotas $(\underline{z}, v)$ (Lemma 2 a)). These quotas generate the lowest possible capital-labor ratio and the agent prefers this ratio over $k(\underline{s}, \infty)$. From Lemma 2 b), we also know that this agent will favor quotas over immigration permits.

If an agent's wealth $i$ satisfies $\Delta(k(\underline{z}, q), k(s, \infty))$, this agent will prefer immigration quotas $(s, \infty)$ since immigration permits are not a strictly better alternative.

Depending on whether the median agent has a wealth higher or lo–wer than $\Delta(k(\underline{z}, v), k(\underline{z}, \infty))$, the vote will favor immigration quotas $(\underline{z}, v)$ or $(\underline{s}, \infty)$. But in the first place, immigration quotas will be chosen over immigration permits.

The conclusion turns out to be different in the second case. Indeed, in this case, the lowest capital-labor ratio is realized with immigration permits. The issue of the vote can be analyzed using the indifferent agent $\Delta(min_{p \geq \underline{z}}\tilde{k}(p, 1), k(\underline{s}, \infty))$. If the median agent has a wealth lower than $\Delta(min_{p \geq \underline{z}}\tilde{k}(p, 1), k(s, \infty))$, then immigration quotas will be preferred to immigration permits (Lemma 2 a)). Conversely, immigration permits will bechosen over immigration quotas whenever the median agent's wealth is higher than $\Delta(min_{p \geq \underline{z}}\tilde{k}(p, 1), k(\underline{s}, \infty))$.

We may now summarize the preceding discussion:

**Proposition 4.**

*a) If, $k(\underline{z}, v) \leq inf_{p \geq \underline{z}}\tilde{k}(p, 1) < sup_{p \geq \underline{z}}\tilde{k}(p, 1) < k(\underline{s}, \infty)$, immigration quotas will always be chosen over immigration permits by a majority of voters. If the median agent's wealth is higher (resp. lower) than $\Delta(\tilde{k}(\underline{z}, v), k(\underline{s}, \infty))$, the vote will favor immigration quotas $(\underline{z}, v)$ (resp. $\underline{s}, \infty$ ).*

*b) If, on the other hand, $min_{p \geq \underline{z}}\tilde{k}(p, 1) < k(\underline{z}, v) < sup_{p \geq \underline{z}}\tilde{k}(p, 1) < k(\underline{s}, \infty)$, immigration permits (resp. immigration quotas $(\underline{s}, \infty)$ ) will be preferred to immigration quotas (resp. immigration permits) whenthe median agent's wealth is higher (resp. lower) than $\Delta(min_{p \geq \underline{z}}\tilde{k}(p, 1), k(s, \infty))$.*

[10] Due to Lemma 3 in the appendix and the fact that $k(\underline{z}, v) < K_0/N_0$, the infimum of $\tilde{k}(.,1)$ is realized.

We have seen that when immigration permits are chosen by a majority of agents, it is because they generate the lowest capital-labor ratio. The capital-labor ratio is reduced further by charging a suitable fee and letting people come in who can pay the fee rather than by targeting immigration quotas toward those who have the lowest levels of capital.

This has striking implications when the price minimizing the capital-labor ratio is $\underline{z}$[11]. This is the lowest possible price for immigration permits. In this case, when permits are chosen over quotas, every potential immigrant is free to enter (whenever he pays the immigration fee).

But if $\underline{z} = 0$ (as could be the case with an exponential distribution), the lowest capital-labor ratio would be realized with a price equal to zero. This would amount to allowing free entry of immigrants. However, in such a situation, it is evident that immigration permits are useless since it suffices to open the borders.

# 5 Conclusion

In this paper, we have analyzed and compared immigration permits and immigration quotas by focusing on their effects on the capital-labor ratio. Two main conclusions can be drawn from the analysis.

First, the highest possible capital-labor ratio is almost always achieved with immigration quotas. This is because with immigration permits, immigrants' wealth is reduced by the amount of the permit price. We have also seen that it is not always the case that immigration quotas generate the lowest capital-labor ratio.

Second, the political process may be of considerable importance when designing a market for immigration permits. Immigration permits could not be politically sustainable in the sense that a majority of agents would prefer immigration quotas. This may happen when permit sales are wasted, because in this case agents prefer extreme values of the capital-labor ratio and immigration permits do not always yield these extreme values.

These conclusions rely on several assumptions which we present below.

We have assumed that borrowing in the destination country is infeasible. If this last assumption were relaxed, immigrants could enter the country with their gross wealth and paying the immigration fee would not reduce the inflow of capital. However, the effects of this operation on the capital-labor ratio would remain the same since immigration fees must be paid and this would reduce the amount of the capital stock in the destination country (provided that the fees are not re-invested). The same conclu-

[11] This is the case in the example presented above, see also figure 1.

sion would be reached if firms in the destination country were allowed to buy immigration permits.

We have not taken into account the fact that a system of permits could select younger and more skilled agents (with more human capital). To address this issue, using a dynamic model would be necessary.

We have also mainly assumed that immigration fees are wasted. While this is a strong assumption, it is nevertheless formally equivalent to the case where immigration fees would be redistributed to the origin countries. This could be a way to respond politically to the objection that "citizenship should not be for trade" (see Becker (1997).

Finally, potential migrants pay the same price to enter irrespective of their wealth levels. We could have also relied on more complex pricing schemes. We could imagine discriminating and charging different fees to people with different wealth levels. Similarly, we could have introduced a more complex system of quotas.

Relaxing the above assumptions is a natural topic for further research. But several other issues could be considered. For instance, it would be interesting to extend the study of immigration permits to models differing from that of Benhabib. This would allow us to take into account some agents like lobbies or trade-unions.

## References


Atsu Amegashie, J. (2004),. "A Political Economy Model of Immigration Quotas", *Economics of Governance*, Vol. 5, Nov, 255-267.

Becker, G. and N. Becker (1997). "Why Not Let Immigrants Pay for Speedy Entry", pages 58-59, in *The economics of Life*.

Becker, Gary, (2005). "Sell the Right to Immigrate", The Becker-Posner Blog, February 21.

Bellettini, G. and C. Berti Ceroni (2008). "Can Unions Hurt Workers: A positive Analysis of Immigration Policy", *Economics and Politics*, Vol. 20 (1), 106-124.

Benhabib, J. (1996). "On the Political Economy of Immigration", *European Economic Review*, 40, 1737-1743.

Bilal, S., Grether, J.M. and J. de Melo (2003), "Attitudes Towards Immigration: A Trade Theoretic Approach", *Review of International Economics*, 11, 1, 253-267.

Boeri, T., Hanson G. and B. McCormick (2002). *Immigration Policy and the Welfare System.* Oxford University Press.

Borjas (2003). "The Labor Demand Curve *Is* Downward Sloping: Reexamining the Impact of Immigration on the Labor Market", *Quarterly Journal of Economics*, 118, (4), 1335-1374.

Borjas (2009). "The Analytics of the Wage Effect of Immigration", Working paper, Harvard University.

Boulding K. (1964). *The Meaning of the Twentieth Century*, Harper and Row, New-York.

Card, D. (2005). "Is the New Immigration Really So Bad ?", *Economic Journal*, 115, (4), F300-F-323.

De la Croix, D. and A. Gosseries (2007). "Procreation, Migration and Tradable Quotas", in *Population Aging, Intergenerational Transfers and the Macroeconomy*, 227-249, R. Clark, A. Mason and N. Ogawa eds, Edward Elgar Publishing.

De la Croix, D. and A. Gosseries (2009). "Population Policy Through Tradable Procreation Entitlements", *International Economic Review*, Vol. 50, 2, 507-542.

Docquier F. and A. Marfouk (2006). "International migration by education attainment", in *International migration, remittances and the brain drain*, Caglar Ozden and Maurice Schiff Editors, Palgrave Macmillan: New York (2006) (available at: http://perso.uclouvain.be/frederic.docquier/oxlight.htm).

Dula G., Kahana N. and T. Lecker (2006). "How to partly bounce back the struggle against illegal immigration to the source countries", *Journal of Population Economics*, 933-1433.

Dustmann C. and I. Breton (2007). "Racial and Economic Factors in Attitudes to Immigration", *The B.E. Journal of Economic Analysis & Policy.* Vol 7, 1 (Advances), article 62.

Epstein, G. S. and S. Nitzan (2006). "The Struggle Over Migration Policy", *Journal of Population Economics*, forthcoming.

Facchini, G., Razin A. and G. Willmann (2004). "Welfare Leakage and Immigration Policy", *Cesifo Economic Studies*, 50 (4), 627-645.

Facchini, G. and G. Willmann (2005). "The Political Economy of International Factor mobility", *Journal of International Economics*, 67, 201-219.

Felbermayr, G.J. (2003). "On the possibility of Sustainable Immigration Policy in a Democratic Society", Working Paper.

Gabaix, X. (2009). "Power Laws in Economics and Finance", *Annual Review of Economics*, 1, p. 255-93. (Available at: http://pages.stern.nyu.edu/%7Exgabaix/).

Grether, J.M., de Melo, J. and T. Muller (2001). "The Political Economy of Migration in a Ricardo-Viner Model", in S. Djajic, ed., *International Migration: Trends, Policy, and Impact*, Routledge, London.

Lundborg, P. and P. S. Segerstrom (2002). "The Growth and Welfare Effects of International Mass Immigrations", *Journal of International Economics*, 56, 177-204.

Magris, F. and G. Russo (2004). "Voting on Mass Immigration Restriction", Working Paper 2004-27, Delta.

Mayda, A. M. (2006). "Who is Against Immigration ? A Cross-Country Investigation of Individual Attitudes Toward Immigrants", *Review of Economics and Statistics*, 88, 510-530.

Ochel, W. (2001). "Selective Immigration Policies: Point System versus Auction Model". CESifo Forum 2 (2), 48 - 52.

Ortega F. (2005). "Immigration Quotas and Skill Upgrading", *Journal of Public Economics*, 89, 1841-1863.

Ottaviano, G. and G. Peri (2005). "Rethinking the Gains from Immigration: Theory and Evidence from the U.S", N.B.E.R Working Paper 11672, October.

# APPENDIX A

**Lemma 3.** Assume that $\sup_{p\geq \underline{z}} \tilde{k}(p,1) > K_0/N_0$ *(resp.* $\inf_{p\geq \underline{z}} \tilde{k}(p,1) < K_0/N_0$*).* Then, there exists $\hat{p}$ such that: $\sup_{p\geq \underline{z}} \tilde{k}(p,1) = \sup_{p\in[\underline{z},\hat{p}]} \tilde{k}(p,1)$ (resp. $\inf_{p\geq \underline{z}} \tilde{k}(p,1) = \inf_{p\in[\underline{z},\hat{p}]} \tilde{k}(p,1)$ ) and the maximum (resp. minimum) of $\tilde{k}(p,1)$ is realized in $[\underline{z}, \hat{p}]$.

*Proof.* We shall only consider the existence of a maximum of $\tilde{k}(p,1)$ (the argument is similar for the realization of the minimum). Suppose that the first part of the Lemma is false. Then for all $\hat{p}$, there exists $p'(\hat{p}) > \hat{p}$ such that $sup_p \tilde{k}(p,1) > \tilde{k}(p'(\hat{p}),1) > sup_{p\in[\underline{z},\hat{p}]} \tilde{k}(p,1)$. Letting $\hat{p}$ go to infinity, one gets:

$$\sup_p \tilde{k}(p,1) \geq \lim_{\hat{p}\to+\infty} \tilde{k}(p'(\hat{p}),1) \geq \sup_p \tilde{k}(p,1) \tag{20}$$

Hence, $\sup_p \tilde{k}(p,1) = K_0/N_0 = \sup_p \tilde{k}(p,1)$ which is a contradiction. As for the last part of the Lemma, since $k(.,1)$ is a continuous function on $[\underline{z}, \hat{p}]$, Weierstrass Theorem ensures that it realizes its maximum.

**Proposition 1**

*a) There exists a unique real number $v$, $\underline{z} < v < K_0/N_0$, such that with immigration quotas $(s, q) = (\underline{z}, v)$, $k(\underline{z}, v)$ is the minimal value of the capital-labor ratio.*

*b) There exists a unique real number $\underline{s}$, $\underline{s} > K_0/N_0$, such that with immigration quotas $(s, q) = (\underline{s}, +\infty)$, $k(\underline{s}, +\infty)$ is the maximal value of the capital-labor ratio.*

*Proof.*

a) A simple computation yields:

$$\frac{\partial k(s,q)}{\partial s} = \frac{I(s)(k(s,q)-s)}{N_0 + \displaystyle\int_s^q I(z)dz} \tag{21}$$

Hence,

$$\frac{\partial k(s,q)}{\partial s} > 0 \iff k(s,q) > s \iff \frac{K_0}{N_0} + \frac{1}{N_0}\int_s^q (z-s)I(z)dz - s > 0 \tag{22}$$

Inspecting (22) reveals that:

- If $q \in [\underline{z}, K_0/N_0]$, then $k(s, q) > s$ for all $s$ in $[\underline{z}, q]$. Therefore, $k(s, q)$ realizes its minimum at $\underline{z}$, i.e., $k(\underline{z}, q)$.
- If $q \geq K_0/N_0$, then $k(s, q)$ realizes its maximum at a point $s(q) \geq K_0/N_0$. The minimizing value of $s$ is realized either at $z$ or $q$ (in which case, $k(s, q)$ is equal to $k(q, q) = K_0/N_0$).

To determine the minimum value of the capital-labor ratio, all we need to see is how $k(\underline{z}, q)$ changes with $q$. Notice that:

$$\frac{\partial k(\underline{z},q)}{\partial q}=I(q)\frac{q-k(\underline{z},q)}{N_0+\int_{\underline{z}}^{q}I(z)dz} \tag{23}$$

One has:

$$\frac{\partial k(\underline{z},q)}{\partial q}>0 \iff q>k(\underline{z},q) \iff \frac{K_0}{N_0}+\frac{1}{N_0}\int_{\underline{z}}^{q}(z-q)I(z)dz-q<0 \tag{24}$$

Inspecting (24), one may see that there exists $v$, $\underline{z}<v<K_0/N_0$ (because $\int_{\underline{z}}^{v}(z-v)I(z)dz\leq 0$), such that for all $q$ such that $q\leq v$, $\frac{\partial k(\underline{z},q)}{\partial q}\leq 0$, and for all $q$ such that $q\geq v$, $\frac{\partial k(\underline{z},q)}{\partial q}\geq 0$. Hence, $k(\underline{z},q)$ realizes its minimum at $q=v$.

We are now in a position to determine the minimal capital-labor ratio. This minimal value is realized at $(s,q)=(\underline{z},v)$ since $k(\underline{z},v)<k(\underline{z},q)$ for all $q\neq v$, so that, in particular, $k(\underline{z},v)<k(\underline{z},\underline{z})=K_0/N_0$.

b) The proof proceeds along similar lines to that of point a). Let $s\geq\underline{z}$ be fixed. We have:

$$\frac{\partial k(s,q)}{\partial q}=I(q)\frac{q-k(s,q)}{N_0+\int_{s}^{q}I(z)dz} \tag{25}$$

So,

$$\frac{\partial k(s,q)}{\partial q}>0 \iff q>k(s,q) \iff \frac{K_0}{N_0}+\frac{1}{N_0}\int_{s}^{q}(z-q)I(z)dz-q<0 \tag{26}$$

Notice that when $s>K_0/N_0$, one always has $q>k(s,q)$ for all $q\geq s$. Then, the ratio $k(s,q)$ is maximized by choosing $q=\infty$.

If not, as was seen in point $a$), the capital labor ratio reaches a minimum at a value $q(s)$. So, there are two potential maximizing choices for $q$, namely $q=s$ (and the ratio $k(s,s)=K_0/N_0$) or $q=+\infty$.

To determine a maximizing choice for $q$, we have to study $k(s,\infty)$. One has:

$$\frac{\partial k(s,+\infty)}{\partial s}=\frac{I(s)(k(s,\infty)-s)}{N_0+\int_{s}^{\infty}I(z)dz} \tag{27}$$

We have:

$$\frac{\partial k(s,+\infty)}{\partial s}>0 \iff k(s,\infty)>s \iff \frac{K_0}{N_0}+\frac{1}{N_0}\int_{s}^{\infty}(z-s)I(z)dz-s>0 \tag{28}$$

One can see that there exists $\underline{s}>K_0/N_0$ such that $k(s,\infty)$ reaches a maximum at $s=\underline{s}$.

# APPENDIX B

In this appendix, we show that if $K_0/N_0 > \underline{z}$, and $I(z)$ follows a Pareto distribution, then the minimal capital ratio is always realized with immigration permits.

Let us first briefly study the extrema of $\tilde{k}(p,1)$. At each interior extremum, one has:

$$\frac{d\tilde{k}(p,1)}{dp} = \frac{I(p)\tilde{k}(p,1) - \overline{P}(p)}{N_0 + \overline{P}(p)} = 0 \tag{29}$$

The second-order derivative is written:

$$\frac{d^2\tilde{k}(p,1)}{dp^2} = I'(p)\frac{\frac{d\tilde{k}(p,1)}{dp}}{I(p)} + I(p)\{\frac{\left(\frac{d\tilde{k}(p,1)}{dp} - \frac{d}{dp}\left(\frac{\overline{P}(p)}{I(p)}\right)\right)(N_0 + \overline{P}(p))}{(N_0 + \overline{P}(p))^2} + \frac{I(p)\left(\tilde{k}(p,1) - \frac{\overline{P}(p)}{I(p)}\right)}{(N_0 + \overline{P}(p))^2}\} \tag{30}$$

At an extremum, the preceding expression reduces to:

$$\frac{d^2\tilde{k}(p,1)}{dp^2} = -I(p)\frac{\frac{d}{dp}\left(\frac{\overline{P}(p)}{I(p)}\right)}{N_0 + \overline{P}(p)} \tag{31}$$

Hence, the existence of maxima or minima of the capital-labor ratio as a function of $p$ hinges on the sign of $\frac{d}{dp}\left(\frac{\overline{P}(p)}{I(p)}\right)$ $\left(\frac{\overline{P}(p)}{I(p)}\right.$ which is the inverse of the hazard rate.

For some distribution functions, the sign of this expression turns out to be constant. For instance, it is positive with a Pareto distribution, negative with a uniform distribution, either positive or negative with an exponential distribution. For some other distributions, like the log-normal law, the sign is not constant.

In our example $P$ is a Pareto distribution:

$$I(z) = 2\bar{I}\frac{\underline{z}^2}{z^3} \tag{32}$$

$$P(z) = \bar{I}\left(1 - \frac{\underline{z}^2}{z^2}\right) \tag{33}$$

$$\text{where } \bar{I} \equiv P(\infty) \tag{34}$$

After a little algebra, one gets:

$$\tilde{k}(p,1) = \frac{K_0 + I\frac{\underline{z}^2}{p}}{N_0 + I\frac{\underline{z}^2}{p^2}} \tag{35}$$

It is easy to see that $\tilde{k}(\underline{z}, 1) < K_0/N_0$ if and only if $\underline{z} < K_0/N_0$ which holds true by assumption. As a result of the discussion above, if $\tilde{k}(.,1)$ realizes a local minimum at $\underline{z}$, this will be in fact a global minimum.

There is a local minimum at $\underline{z}$ if:

$$\tilde{k}(\underline{z},1) > \frac{\int_{\underline{z}}^{\infty} I(z)dz}{I(\underline{z})} \tag{36}$$

or equivalently, if:

$$\frac{K_0 + I\underline{z}}{N_0 + I} > \frac{\underline{z}}{2} \iff \frac{K_0}{N_0} > \frac{\underline{z}}{2}(1 - \frac{I}{N_0}) \tag{37}$$

Under our assumptions, this condition is always satisfied.

We now consider the expression $k(\underline{z}, q)$ obtained with a system of quotas. After a few computations, one gets:

$$k(\underline{z},q) = \frac{K_0 + 2I\underline{z}^2(\frac{1}{\underline{z}} - \frac{1}{q})}{N_0 + I(1 - \frac{\underline{z}^2}{q^2})} \tag{38}$$

The value of $q$ that minimizes the capital-labor ratio satisfies $q = k(\underline{z}, q)$ which reduces to:

$$L(q) = (N_0 + I)q^2 - q(K_0 + 2I\underline{z}) + I\underline{z}^2 = 0 \tag{39}$$

This equation always has two real roots [12].

One can see that the highest root $q$ is such that: $\underline{z} < q < K_0/N_0$ [13]. Hence, the value of $q$ that we are looking for is the greatest root of the above equation, i.e.

$$q = v = \frac{K_0 + 2I\underline{z} + \sqrt{\triangle}}{2(N_0 + I)} \tag{40}$$

where:

$$\triangle = (K_0 + 2I\underline{z})^2 - 4(N_0 + I)\underline{z}^2 I \tag{41}$$

We can now compare $v = k(\underline{z}, v)$ and $\tilde{k}(\underline{z}, 1)$:

$$v - \tilde{k}(\underline{z},1) = \frac{1}{2(N_0 + I)}(\sqrt{\triangle} - K_0) \tag{42}$$

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12 To see this, notice that its discrimnant $\Delta$ is:

$$\begin{aligned} \triangle &= (K_0 + 2I\underline{z})^2 - 4(N_0 + I)I\underline{z}^2 \\ &> (\underline{z}N_0 + 2I\underline{z})^2 - 4(N_0 + I)I\underline{z}^2 = \underline{z}^2 N_0^2 > 0. \end{aligned}$$

13 Indeed, $L(\underline{z}) = N_0\underline{z}(\underline{z} - (K_0/N_0)) < 0$. Moreover, $L(K_0/N_0) = I((K_0/N_0)^2 - 2\underline{z}K_0/N_0 + \underline{z}^2)$. But considering the function $\psi(u) = u^2 - 2u\underline{z} + \underline{z}^2$, one sees that $\psi(\underline{z}) = 0$. Since, $\psi'(u) = 2(u - \underline{z})$, this proves that $L(K_0/N_0) > 0$.

The condition $v > \tilde{k}(\underset{\sim}{z}, 1)$ reduces to:

$$\frac{K_0}{N_0} > \underset{\sim}{z} \tag{43}$$

a condition which is always satisfied by assumption.

**Figure 1.** *A case where* $\tilde{k}(p, 1) < k(\underset{\sim}{z}, v)$ .