Postponing retirement age and labor force participation: the role of family transfers *

Pascal Belan **
Pierre-Jean Messe ***
François-Charles Wolff ****

1 Introduction

Many European countries have been characterized by a trend towards early retirement over the last three decades. This seems problematic in a setting where fertility rates tend to fall and life expectancy is steadily increasing, thereby putting a strong pressure on the financing of pension schemes. ¹

Given the expected growth of the demographic dependency ratio, solutions have to be found to make pay-as-you-go social security sustainable. While raising taxes or increasing the social security debt are clearly implausible solutions, several authors have suggested to increase the activity rates of older workers, ideally along with a reduction in social security benefits.

It has been shown that postponing retirement may lead to a so-called “double dividend” (Cremer and Pestieau, 2004). First, delaying retirement is expected to restore at least partially the financial balance of the pension

* We would like to thank two anonymous referees for their very helpful comments and suggestions. We are also indebted to Pierre Pestieau, Michael Kaganovich, André Masson and seminar participants at the Journées sur les Modèles à Générations Imbriquées (Paris), the LEN seminar (Nantes), the Séminaire scientifique de la Caisse des Dépôts (Bordeaux) and the Annual Conference of the European Society for Population Economics (Chicago). Any remaining errors are ours.

** Corresponding author. THEMA, Université de Cergy-Pontoise, 33 boulevard du Port, 95011 Cergy-Pontoise Cedex, France. Email: pascal.belan@u-cergy.fr

*** GAINS-TEPP, Université du Maine. Email: pierre-jean.messe@univ-lemans.fr

**** LEMNA, Université de Nantes, CNAV et INED, Paris, France. Email: francois.wolff@univ-nantes.fr

¹ Delaying retirement is subject to an implicit tax: prolonging activity implies paying additional payroll taxes and it also leads to a reduction in pension rights. Casamatta et alii (2006) show that this implicit tax on continued activity may result from some political process.
system. Second, it may lead to more income equality among the retirees, at least if the system operates redistribution within generations. A good understanding of the consequences of postponing retirement is then needed if the challenge of aging in developed countries may only be solved through a reform aimed at increasing age of retirement. Such a policy would itself have an impact on economic growth (Echevarria, 2004).

Assuming that delaying retirement will be helpful to avoid a financial crisis of social security means that this reform is indeed effective in terms of worker’s employment. Economists certainly agree that a reform of labor market rules is needed to prolong activity. However, even if we put this argument related to the functioning of the labor market aside, we argue that postponing retirement may strongly influence the pattern of employment not only of older workers, but also of younger workers. The idea is simply to account for intergenerational relationships. In many families, at a given date, two generations take part in the labor market. The labor force participation of these two generations cannot be disconnected as long as one observes substantial flows of family transfers.

If a change in the labor supply of old workers affects their own transfer decisions, this may in turn lead to a change in the employment rate of younger workers. That family matters in the context of retirement issues is not a new idea. For instance, it has been shown that there is a strong tendency of husbands and wives to retire together and that each spouse values retirement more once their spouse has retired (Gustman and Steinmeier, 2000). Instead of focusing on spousal relationships, we account here for intergenerational links and family transfers to investigate how a public policy aimed at postponing retirement influence employment rates.

We investigate in this paper the consequences of postponing retirement on the labor participation of both young and old workers when grandchild care matters. For that purpose, we consider an overlapping generations model with domestic production and intergenerational transfers. Both the old and young generations take part in the labor market. Young workers have children and have to care for them. They personally devote time to raise their children. During their period of activity, they may either pay for formal child care or benefit from grandchild care from their parents. In this setting, we study the relationship between the provision of parental transfers and retirement age.

Our results are twofold. First, we show that the children are more likely to work when older workers provide more grandchild care. Second, we prove that an increase in the length of the working period for older workers has an ambiguous effect on the labor participation of the younger workers. If the latter benefit from less time-related resources, then they will devote more time to their children and thus will reduce their labor participation. However, in some cases, we evidence the reverse result. Older people
will both work longer and care more for the grandchildren, which will in turn improve the employment rate of the children. Here, the main mechanism rests on the trade off of the old between time devoted to grandchild care and time devoted to their own domestic production. If later retirement brings higher income to grandparent and allows them to dramatically reduce time devoted to domestic production, time transfer to the young parent may increase.

A close look at this family externality matters in terms of public policy. Increasing the labor rates of older workers is most often viewed as a way for a government to spend less resources on pensions through two different channels. First, by postponing age of retirement, this will delay the receipt of the pension for all the workers who have to work longer. Second, in the case of a pay-as-you-go pension system, a government will receive additional taxes from the workers still involved in the labor market. Once private transfers are taken into account, a different picture emerges since expected changes in the labor participation of younger workers with children also affect payroll taxes.

The rest of the paper is organized as follows. We briefly review the existing literature in Section 2. In Section 3, we present a theoretical model with two generations, domestic production and grandchild care. We investigate in Section 4 the consequences of postponing age of retirement on the pattern of parental transfers and labor participation of the young workers. In Section 5, we perform a numerical analysis to illustrate the fact that delaying retirement may increase the labor force participation of both the young and old workers. Finally, Section 6 concludes.

2 Previous literature

Several microeconomic studies have focused on the interplay between private transfers and labor supply decisions. In the upward direction, individuals who have to care for their elderly parents may be induced to increase their number of worked hours if they have for instance to pay for formal services and housing retirement. Conversely, their labor participation may be reduced if they provide time transfers in the form of caregiving activities (Ettner, 1996, Wolf and Soldo, 1994). Family transfers are also likely to affect the behavior of recipients. Joulfaian and Wilhelm (1994) find that inheritances lead to a small decrease in the labor supply of women. Using samples of teenagers, Dustmann and Micklewright (2001) and Wolff (2006) examine whether the

---

2 For an overview of the consequences of endogenous labor supply in models of family transfers, see the survey of Laferrère and Wolff (2006).
receipt of financial transfers reduces the labor participation of children still enrolled in school.

Parents may also rely on non-financial transfers to help their children. Ermisch and Ogawa (1996) and Sasaki (2002) account for intergenerational co-residence and find that the labor supply of young women is much higher when they live with their parents. Dimova and Wolff (2010) focus on the role of time transfers in the form of grandchild care. Using the SHARE European data, they show that the receipt of grandchild care has a positive impact on the labor force participation of young mothers. Conversely, the receipt of monetary transfers does not affect the decision of young mothers to have a paid job. Using a unique sample on ageing first generation immigrants in France, Dimova and Wolff (2008) show that grandchild care is spread unequally across siblings and has a strong positive impact on the labor supply of care-receiving mothers. 3

In what follows, we argue that such transfers in the form of grandchild care matter when studying the consequences of postponing retirement age. It is thus important to shed light on the magnitude of these time transfers. In the U.S., Soldo and Hill (1995) and Cardia and Ng (2003) find that more than 40% of married women spend more than 100 hours per year caring for grand-children. Cardia and Ng (2003) provide additional evidence that intergenerational transfers of time are substantial both in the U.S. and Canada, grandparents playing an important role in child care. The SHARE data confirm that transfers in the form of grandchild care are frequent in all European countries. Regular grandchild care, defined as care provided on a daily or weekly basis, is received by exactly one third of mothers in working age with at least one child less than 10. South European countries are marked by the highest incidence of regular grandchild care. 4

Our attempt to account for private transfers in the context of retirement decisions using an overlapping generations model is innovative with respect to the previous literature. Despite of their importance, transfers in the form of grandchild care has been widely neglected by economists. As it stands, our contribution is more closely related to the work of Cardia and Ng (2003), who also consider an overlapping generations model with domestic production and both time and monetary transfers. However, contrary to these authors who calibrate this model to the US economy, we do not focus on capital accumulation and macroeconomic effects of child care policies. Our primary interest lies in the impact of postponing retirement and the labor participation consequences.

3 The unequal distribution of grandchild care is driven to a higher extent by better labor market potential than weaker financial status of the recipient.

4 In Europe, grandchild care is also much more frequent for women than the receipt of financial gift.
3 The theoretical model

We consider an overlapping generations model with two adult generations, the young (children) and the old (their parents). We assume that the young adults have themselves kids, so that there are implicitly three cohorts of agents in the model. However, as grandchildren are young kids, they do not make any economic decision, so that we focus in what follows on transfer and labor participation choices within a two-generations framework.\(^5\)

Following Cardia and Ng (2003), we introduce domestic production and time family transfers in the OLG model. The utility of each agent depends on a composite good, which is made of a consumed market good and a home produced good. Market goods and time are the two inputs necessary to home production. We assume that young adults will essentially devote their time to raise their children. The home-produced goods may thus be seen as child care for the young generation, educational expenditures and child care services being examples of market goods. Conversely, the domestic time of their parents is related to personal activities like cooking, shopping, housework and odd jobs among others. A crucial distinction between the two generations is related to the definition of the time inputs, since we allow for time transfers from the old to the young generation.

The setting is as follows. During the first period, denoted by 1 as superscript, the young adult allocates her time between a paid activity and home production, i.e. child care. Let \( h_1 \) be the labor supply and \( k_1 \) the number of hours devoted to the kids. Endowment of time for the young is normalized to one, so that

\[
h_t + k_t^1 = 1
\]

(1)

The second period, denoted by 2 and also normalized to one, is made up of both working time, retirement and domestic production. We denote by \( \theta_{t+1} \) the fraction of time devoted to labor activities, so that \( (1 - \theta_{t+1}) \) is the length of the retirement period. In our model, the retirement age is exogenous. Let \( e_{t+1}^2 \) be time spent in pure domestic production by the parent and let \( T_{t+1} \) be a transfer in the form of grandchild care. The time constraint for the parent may be expressed as

\[
\theta_{t+1} + e_{t+1}^2 + T_{t+1} = 1
\]

(2)

Two remarks are in order. First, we only consider transfers in the form of grandchild care provided by parents. In so doing, we neglect the possibility of home-sharing arrangements and financial transfers. This

\(^5\) The presence of grandchildren is in fact only necessary for grandchild care to take place. Another interpretation of our model is to assume that parents provide time to their children, even if the latter do not have themselves children. A very different framework would be to consider a three-generational framework, with specific investment decisions in grandchildren (related to human capital). On this issue, see for instance Duflo (2003).
restriction is in fact mainly empirically driven, as grandchild care transfers are much more frequent than cash gifts for young adults and represent a substantial number of hours per week (Cardia and Ng, 2003, Dimova and Wolff, 2008, 2010). Secondly, we clearly need a motivation for the provision of grandchild care: why would parents give time to their children since these transfers are presumably costly owing to transportation costs for instance? Several motives for private transfers have been suggested in the literature on intergenerational relationships (see Laferrière and Wolff, 2006).

A first possibility is that the parent is influenced by the utility level of the child, which is the spirit of the altruistic model (Becker, 1991). While the basic altruistic model usually only includes financial transfers, it can be easily extended to the case of services from parents to children (Sloan et alii, 2002). A second model relies on exchange considerations (Cox, 1987). This would lead to a framework where parents help their children through grandchild care, but expect a transfer of money in exchange of their services (for instance to pay for formal old-age support). A third possibility is the demonstration effect theory (Cox and Stark, 2005), according to which the child’s propensity to care for parents is conditioned by parental example.

In this paper, we choose an alternative strategy and rely on an impure form of the altruistic motive, i.e. the so-called warm-glow motive described in Andreoni (1990). The underlying idea is that parents obtain satisfaction not from the well-being of their children, but instead from the act of giving time per se. In so doing, we thus suppose that grandparents enjoy spending time with young kids. The donor’s utility is furthermore rising with the amount given. It should be noted that we depart here from the specification of Cardia and Ng (2003). Following the Beckerian altruistic model, these authors consider instead that grandparents take the well-being of their children when maximizing their own utility.

3.1 Consumer problem

We now describe the budget constraint for each generation. In the first period, the child’s resources are devoted to market good purchases \( c_t \) and savings \( s_t \). Denoting by \( w \) the wage rate and considering a pay-as-you-go pension scheme with payroll tax rate of \( \tau_t \), the budget constraint for the young is given by:

\[
c_t + s_t = (1 - \tau_t)h_t w
\]  

We consider a small open economy, so that the real interest rate \( r \) is exogenous. Let us denote by \( R = 1 + r \), the gross interest rate. During the second period of life, resources consist of wage income \( \theta_{t+1}(1 - \tau_{t+1})w \) during the working period and pension benefits \((1 - \theta_{t+1})b w \) once being retired,

\[6\] Furthermore, the labor supply decisions of young adults seem to be quite insensitive to the provision of financial transfers from parents. See for instance Wolff (2006).
where \( b \) represents the replacement rate. Accounting for the returns on first-period savings \( R_s \), the market good purchased by the household is \( d_{t+1} \) such that

\[
d_{t+1} = \omega_{t+1}(\theta_{t+1}, \tau_{t+1}) + R_s
\]

where \( \omega(\theta_{t+1}, \tau_{t+1}) = [(1 - \theta_{t+1})b + \theta_{t+1}(1 - \tau_{t+1})]w \) is the second period income.

For the young, we define \( \tau_i \) as the composite of market good \( c_i - z_i \) and domestically produced good \( q_i \). The latter comprises child care. Let \( g^1 \) be the family production function, whose arguments are purchased inputs \( z_i \) (like child care services), personal time to children \( k_i \) and grandchild care transfers \( T_i \). We consider a production function of the form \( q_i = g^1(z_i, k_i + T_i) \). We thus assume that time values \( k_i \) and \( T_i \) are perfect substitutes. What matters for instance for the young kid is to be with an adult, which is either a parent or a grandparent. Conversely, we make no assumption a priori concerning the complementarity or substitutability between \( z_i \) and \( k_i \). The composite good \( \tau_i \) is itself the result of a function \( f^1 \) such that

\[
\tau_i = f^1(c_i, z_i, q^1(z_i, k_i + T_i))
\]

In a similar way, the second-period composite good \( \tilde{d}_{t+1} \) is obtained by combining the market good \( d_{t+1} - z_{t+1} \) and the produced good \( q^2_{t+1} \). Family production is achieved through the production function \( q^2_{t+1} = g^2(z_{t+1}, e_{t+1}) \) where \( z_{t+1} \) and \( e_{t+1} \) denote respectively purchased inputs and time inputs. Then, \( \tilde{d}_{t+1} \) is given by

\[
\tilde{d}_{t+1} = f^2(d_{t+1} - z_{t+1}, q^2(z_{t+1}, e_{t+1}))
\]

Functions \( f^1, g^1, f^2 \) and \( g^2 \) are linear homogeneous. Marginal productivities are positive and decreasing, so that \( f^1_i > 0, f^1_{ii} < 0, f^1_2 > 0, f^2_2 < 0, f^2_{22} > 0 \) and \( g^1_i > 0, g^1_{ii} < 0, g^2_i > 0, g^2_{ii} < 0, g^1_{12} > 0, g^2_{12} > 0 \) \( (i = 1, 2) \).

The main difference between the parent and the child specifications for the extended consumption is related to the receipt of grandchild care, which is an additional input in the production function of the young. Given the paternalistic altruism, each individual seeks to maximize the intertemporal utility function \( U_i \) which is defined in the following way:

\[
U_i = u(\tau_i) + \delta[u(\tilde{d}_{t+1}) + \gamma \Phi(T_{t+1})]
\]

where \( \delta \) represents subjective discount factor. Utility functions \( u(.) \) and \( v(.) \) are twice differentiable, increasing and strictly concave. The term \( \gamma \Phi(T_{t+1}) \) picks up the warm-glow motive for providing grandchild care, the parameter \( \gamma \) being strictly positive (Andreoni, 1990).\(^7\)

\(^7\) The satisfaction function \( \Phi \) is also supposed to be twice differentiable, strictly concave and increasing.
The problem for an agent is thus to maximize the intertemporal utility function (7) with respect to $r_i$, $\bar{v}_t$, $c_t$, $d_{i+1}$, $s_t$, $h_t$, $z_t$, $k_t$, $z_{i+1}$, $c_{i+1}$ and $T_{i+1}$ subject to time constraints (1) and (2), financial constraints (3) and (4), and definitions of $\tau_i$ and $\bar{a}_{i+1}$, (5) and (6). All variables must be positive. The maximization program can be expressed as

$$\max_{k_t, z_t, z_{i+1}, T_{i+1}, s_t} \quad u\left(\frac{1}{1-k_t}(1-\tau_t)w - s_t - z_t, g^1\left(z_t, k_t + T_t\right)\right)$$

$$+ \delta u\left(\frac{1}{1-k_t}(1-\tau_t)w - s_t - z_t, g^2\left(z_{i+1}, 1 - \theta_{i+1} - T_{i+1}\right)\right) + \delta v(\tau_{i+1})$$

The five first-order conditions for an interior solution, i.e. $\partial U_i/\partial k_t = 0$, $\partial U_i/\partial z_t = 0$, $\partial U_i/\partial z_{i+1} = 0$, $\partial U_i/\partial T_{i+1} = 0$ and $\partial U_i/\partial s_t = 0$, lead to

$$-(1-\tau_t)wT_i = 0$$
$$-f_{i+1}(\cdot) + g_1^1(f_{i+1}(\cdot)) = 0$$
$$-f_2^1(\cdot) + g_1^2(f_2^1(\cdot)) = 0$$
$$-f_2^2(\cdot) + g_2^2(f_2^2(\cdot)) = 0$$
$$-g_2^1(f_2^1(\cdot))v_1(\cdot) + \Phi_1(\cdot) = 0$$
$$-f_1^i(\cdot)u_1(\cdot) + \delta R_1^i(\cdot)v_1(\cdot) = 0$$

The interpretation of the first-order conditions is as follows. From (8), the marginal benefit of a rise in leisure $g_2^1/f_2^1$ is equal to its marginal cost resulting from the loss of income $(1-\tau_t)wT_i$. From (9), the marginal disutility $f_1^i$ of the young adult involved by an increase in purchased inputs $z_t$ used to produce the non-market good is equal to its marginal benefit $g_1^1/f_2^1$, while (10) has the same interpretation for the old parent. According to (11), the marginal cost of giving time to the young $g_2^1/f_2^1v_1$ is equal to its marginal benefit $\Phi_1$ resulting from the joy-of-giving motivation. Finally, (12) represents the trade-off between the two periods of life.

### 3.2 Dynamics

Consumer’s choice may be expressed as a function of four variables: the net wage when young $(1-\tau_t)w$, the time-transfer received from his parent $T_t$, the second-period income $\omega(\theta_{i+1}, \tau_{i+1})$, and the retirement age $\theta_{i+1}$. In particular, the time-transfer of the consumer born in $t$ to his child writes

$$T_{i+1} = \bar{T}((1-\tau_t)w, T_t, \omega(\theta_{i+1}, \tau_{i+1}), \theta_{i+1})$$

---

8 For any function $F$ in the following, $F_i$ stands for the first-order derivative with respect to the $i^{th}$ argument.
Similarly, labor supply when young is
\[ h_t = 1 - h^1_t = \hat{h}((1 - \tau_t) w, T, \omega(\theta_{t+1}, \tau_{t+1}), \theta_{t+1}) \]

In order to disentangle the multiple effects of postponing retirement and isolate the relevant ones for our purpose, we shall focus on an increase in the current period retirement age under two assumptions. First, we keep the retirement age in future periods at its initial value. Second, the contribution rate of the pay-as-you-go pension system is constant and set at its initial value.

The second assumption means that we do not take into account the government budget constraint. Normalizing the size of each generation to unity, the gap between contributions receipts and pension benefits
\[ D_t = (1 - \theta_t) b(\theta_t) w - \tau(h((1 - \tau) w, T, \omega(\theta_{t+1}, \tau), \theta_{t+1}) + \theta_t) w \]
is added to the public deficit, that is, we consider the case of a country with a large public deficit (as in almost all European countries) and assume that part of the deficit is due to the pension system.\(^9\)

With a constant retirement age \( \theta_t \), dynamic evolution of time transfers is backward and writes
\[ T_{t+1} = \Gamma((1 - \tau_t) w, T, \omega(\theta, \tau), \theta) \]

Starting the economy at period 0, time-transfer \( T_0 \) of the old born in period \(-1\) results from the maximization program
\[
\max_{s_{-1}} v(f(\omega(\theta, \tau) + R_{s_{-1}} - z_{0_s} g(z_{0_s}, 1 - \theta - T_0)) + y(\Phi(T_0))
\]
where saving \( s_{-1} \) is given.

We then study the consequences of postponing the current retirement age, i.e., an increase in \( \theta_0 \), as an economic policy aimed at reducing the public budget deficit, keeping both the contribution rate \( \tau \) and the future retirement age \( \theta_t \) set at their initial value before the reform. A particular attention is put on labor supply within the family \( h_0 + \theta_0 \).

A convenient interpretation of our framework is thus the case of a government that postpones retirement in order to get additional resources for the public pension system. Nevertheless, it could be argued that a temporary rise in retirement age is not totally convincing in a context where policy aging is expected to be steady. If the government raises the retirement age for the financial aspect of the pay-as-you-go program facing population aging, then the retirement age is expected to raise in all periods. We shall give some insights on this matter in Section 5.

---

\(^9\) Note that there is no difficulty to finance the deficit since we consider a small open economy.
4 The optimal pattern of labor supply and transfers

Let us now investigate the consequences of postponing retirement on the pattern of family transfers and labor participation. We proceed in the following way. First, we study how a change in parental transfers affects the labor supply of the young. Then, we focus on the relationship between an increase in the retirement age and the provision of time transfers by the grandparents. Finally, we analyze the overall effect of prolonged activity on the labor participation of both generations.

4.1 Grandchild care and child’s labor supply

When the young makes her decisions, she takes the grandchild care transfer \( T_t \) from her parents as given. For convenience, we omit time indices in this section and denote by \( T \), time transfer received by the consumer from his parent. We focus on the relationship between the parental transfer and the child’s labor participation. We deduce from (8) and (9) that:

\[
g_1^1(z^1, k^1 + T^{-1}) \quad g_1^1(z^1, k^1 + T^{-1}) = (1 - \tau)w
\]

which means that the marginal rate of substitution between child care \( k^1 \) and purchased input \( z^1 \) is equal to the net wage \((1 - \tau)w\), i.e. the marginal cost of reallocating time from labor to child care. Let \( \eta = \frac{z^1}{k^1} \) be the ratio between the purchased and time inputs devoted to domestic production. Since \( g^1 \) is linear homogeneous, \( \eta^1 \) is the solution of \( g_1^1(\eta^1, 1)/g_1^1(\eta^1, 1) = (1 - \tau)w \).

Now, let \( \zeta^1 = \frac{z^1}{k^1} \) be the consumption per unit of time devoted to domestic production. Using the first-order condition (9) and linear homogeneity of \( f^1 \) and \( g^1 \), \( \zeta^1 \) is the solution of

\[
\frac{f_1^1(z^1 - \eta^1, g_1^1(\eta^1, 1))}{f_1^1(z^1 - \eta^1, g_1^1(\eta^1, 1))} = g_1^1(\eta^1, 1)
\]

Moreover, since \( \zeta_1^1(k^1 + T) = c = (1 - k^1)(1 - \tau)w - s \) and \( h + k^1 = 1 \), one obtains that labor supply of the young satisfies

\[
h = 1 - k^1 = \frac{s + \zeta_1^1(1 + T)}{(1 - \tau)w + \zeta^1}
\]

(13)

From the linear homogeneity of \( f^1 \) and \( g^1 \), we get

\[
\bar{c} = f^1(\zeta^1 - \eta^1, g^1(\eta^1, 1))(k^1 + T).
\]
The augmented consumption of the young adult may then be expressed as follows

\[ \tilde{c} = \tilde{c}[(1 - \tau) w(1 + T') - s] \]  
(14)

with \( \tilde{c} \equiv \frac{f(\tilde{c} - \tilde{c}(\tilde{c}, \tilde{c}))}{1 - \tilde{c} + \tilde{c}} \).

**Proposition 1.** Savings increase with the intensity of grandchild care, i.e. \( \frac{\partial s}{\partial \tau} > 0 \).

*Proof.* From equation (12) and using the expression (14) of \( \tilde{c} \), we get

\[ 0 = -f'(\tilde{c} - \tilde{c}(1 - \tau) w(1 + T') - s) \]

By concavity of the consumer problem, the right-hand side is decreasing with respect to \( s \). Thus

\[ \text{sgn} \frac{ds}{dT} = \text{sgn}[-f'(\tilde{c}) \tilde{c}(1 - \tau) w u_{11}(. \cdot)] \]

so that the derivative \( \frac{ds}{dT} \) is positive.

**Proposition 2.** The child’s labor supply is an increasing function of grandchild care, i.e. \( \frac{\partial h}{\partial \tau} > 0 \).

*Proof:* Using (13), we get the following derivative

\[ \frac{\partial h}{\partial T} = \frac{1}{(1 - \tau) w + \tilde{c}(1 - \tau) w u_{11}(\tau, . \cdot)} \]

Since \( \frac{\partial h}{\partial \tau} + \tilde{c} > 0 \) from Proposition 1, this implies \( \frac{\partial h}{\partial T} > 0 \). A rise in the provision of grandchild care leads to a decrease in parental child care \( k^1 \) and then to a rise in the child’s labor participation \( h \).

Not surprisingly, with more grandparental services devoted to the care of young babies, young adults are able to spend more time on paid employment. Much simpler models of transfers with endogenous labor supply lead to similar conclusions. Interestingly, this prediction receives empirical support in the literature. In Europe, estimates from simultaneous equation models indicate that the coefficient of the endogenously treated grandchild care variable has a positive impact on the labor force participation of young mothers (Dimova and Wolff, 2010).
4.2 The parental decision of transfer

We now study the parental decision of giving time to the young generation. As mentioned before, we consider an increase in the retirement age in period 0 that was not expected by generation -1 in the preceding period. Savings $s_{-1}$ accumulated during period -1 is given. Omitting time indices, let $\Omega = \omega + Rs$ be the second-period income of the old of period 0, and $L = 1 - \theta$ the length of their retirement period. The problem for the parent may be expressed as

$$\max_{z^2, T} \Psi \left( z^2, T, \Omega, L \right) = v \left( f^2 \left( \Omega - z^2, g^2 \left( z^2, L - T \right) \right) \right) + \gamma \Phi \left( T \right)$$

We seek to understand how an increase in retirement age influences the provision of grandchild care. It is straightforward to see that there are two effects:

- a time constraint effect: with an increase in labor participation, the parent has less time to devote to grandchild care, which will reduce $T$;
- an income effect: when the government increases the value of $\theta$, the second-period parental income also increases since $\frac{d\Omega}{d\theta} > 0$. As a consequence, the old can devote more financial resources to purchase input $z^2$ related to domestic production. The consequence on time $e^2$ is ambiguous and depends on the substitutability of time and market good in domestic production. Then, as we shall see, the income effect leads grandparent to spend more or less time with grandchildren.

Accounting for time constraint effect and income effect implies that the impact of $\theta$ on $T$ cannot be signed in the general case. Considering the parental maximization program, the first-order conditions for an interior solution are equations (10) and (11) and may be expressed as

$$\Psi_z \left( z^2, T, \Omega, L \right) = 0$$
$$\Psi_T \left( z^2, T, \Omega, L \right) = 0$$

with $\Psi_z \left( z^2, T, \Omega, L \right) = (-f_1^2 + f_2^2 \theta^2) v_1$ and $\Psi_T \left( z^2, T, \Omega, L \right) = -v_1 f_2 g^2_{\Omega} + \gamma \Phi_1$. By concavity of the consumer problem, $\Psi_{zz} < 0$, $\Psi_{TT} < 0$, and the determinant $\Delta = (\Psi_{zz} \Psi_{TT} - (\Psi_{zT})^2)$ of the Hessian matrix of $\Psi$ is positive. Differentiating the first-order conditions with respect to $z^2$, $T$, $\Omega$ and $L$, we deduce that

$$\frac{dT}{d\Omega} = \frac{\Psi_{zT} \Psi_{\Omega} - \Psi_{z\Omega} \Psi_{T}}{\Delta}$$
$$\frac{dT}{dL} = \frac{\Psi_{zT} \Psi_{L} - \Psi_{zL} \Psi_{T}}{\Delta}$$
Proposition 3. A decrease in the length of the retirement period reduces time devoted to grandchild care, i.e. \( \frac{dT}{dL} > 0 \). A rise in parental resources during the second period has an ambiguous impact on time transfers, i.e. \( \frac{dT}{d\Omega} \geq 0 \) or \( \frac{dT}{d\Omega} \leq 0 \).

Proof: We denote by \( \sigma_y^2 \) and \( \sigma_f^2 \) the elasticity of substitution respectively for the production function \( q^2 \) and \( f^2 \). In the Appendix, we calculate the second-order derivatives \( \Psi_{zz}, \Psi_{z\bar{\bar{\Omega}}}, \Psi_{\bar{\bar{\Omega}}z}, \Psi_{TT}, \Psi_{T\bar{\bar{\Omega}}} \) and \( \Psi_{Tz} \). It is then straightforward to obtain \( \frac{dT}{dL} \) and \( \frac{dT}{d\Omega} \). After some manipulations, we get

\[
\frac{dT}{dL} = \frac{\Delta}{\sigma_y^2 \sigma_f^2 \left[ \Omega - z^2 + \frac{g^2}{g_1} \right]^2 \left( v_1 f_{12} g_{12} \right)}
\]

\[
= -\frac{v_1 f^2}{\Omega - z^2} \frac{1}{g_1} \left[ \frac{g^2}{g_1} \left( L-T \right) g_2 - \sigma_y^2 \left( L-T \right) g_2 \right]
\]

\[
+ \left[ 1 + \frac{1}{\Omega - z^2} \frac{g^2}{g_1} \left( L-T \right) g_2 \right] \frac{z^2 g_1^2}{\Omega - z^2} \frac{1}{g_1} \left[ 1 + \frac{1}{\Omega - z^2} \frac{g^2}{g_1} \left( L-T \right) g_2 \right]
\]

that implies \( \frac{dT}{dL} > 0 \).

In a similar way, calculation of \( \frac{dT}{d\Omega} \) leads to:

\[
\frac{dT}{d\Omega} = \frac{\Delta}{\sigma_y^2 \left( \sigma_f^2 \right)^2 \left( v_1 f_{12} \right)^2 \left( g_{12} \right)^2 \left( \Omega - z^2 + \frac{g^2}{g_1} \right)^2 \left( \Omega - z^2 + \frac{g^2}{g_1} \right)^2}
\]

\[
= -\frac{v_1 f^2}{\Omega - z^2} \frac{1}{g_1} \left[ \frac{g^2}{g_1} \left( \sigma_y^2 \sigma_f^2 \right) + \frac{g^2}{g_1} \left( \sigma_y^2 - \sigma_f^2 \right) g_1 \right] - \frac{g^2}{\Omega - z^2 + \frac{g^2}{z^2}}
\]

so that \( \frac{dT}{d\Omega} \) can be either positive or negative depending on the magnitude of the two terms on the right-hand side of the previous equality.

To study the consequences of Proposition 3, we first express the effect of an increase in retirement age on grandparental transfer as

\[
\frac{dT}{d\theta} = \frac{dT}{d\Omega} \frac{d\Omega}{d\theta} - \frac{dT}{dL}
\]
When retirement age is postponed, the old worker receives more labor income, meaning that \( \frac{\partial \Omega}{\partial \theta} > 0 \). Hence, there are three cases depending on the derivatives \( \frac{dT}{d\Omega} \) and \( \frac{dT}{dL} \).

1. When \( \frac{dT}{d\Omega} < 0 \), the grandparental transfer is an inferior good. In that case, an increased labor participation during old age reduces the provision of grandchild care. Both the time constraint and income effects are negative, which implies that \( \frac{dT}{d\theta} \) is unambiguously negative.

2. When \( \frac{dT}{d\Omega} \geq 0 \), i.e. the transfer is a superior good, then the derivative \( \frac{dT}{d\Omega} \) can be either positive or negative, depending on the magnitude of \( \frac{dT}{d\Omega} \) and \( \frac{dT}{dL} \).

2.1. when \( \frac{dT}{d\Omega} \) is smaller than \( \frac{dT}{dL} \), the time constraint effect is more important than the income effect and a rise in \( \theta \) has a negative impact on \( T \); 

2.2. conversely, when \( \frac{dT}{d\Omega} \) is larger than \( \frac{dT}{dL} \), the income effect dominates the time constraint effect and the provision of grandchild care will increase when retirement is postponed.

Both the time constraint and income effects may be more closely related to the possibility of substitution between inputs of the different production functions. As evidenced in the proof of Proposition 3, the sign of \( \frac{dT}{d\Omega} \) depends on the values of \( \sigma_{g}^{2} \) and \( \sigma_{f}^{2} \).

Specifically, \( \frac{dT}{d\Omega} \) will be positive only if \( \sigma_{g}^{2} \) is sufficiently high. When the purchased input \( z^{2} \) and time devoted to domestic activities \( 1 - \theta - T \) are strongly substitutable, postponing retirement will lead to a rise in private downward transfers. With the rise in \( \theta \), grandparents have now more financial resources (income effect). Since \( z^{2} \) is a normal good and owing to the substitutability, grandparents will essentially devote this supplement of revenue to purchase additional units of \( z^{2} \) and lessen time devoted to domestic tasks. They will therefore spend more time with their grandchildren. Conversely, in situations where \( z^{2} \) and leisure \( 1 - \theta \) are not very substitutable, then the reverse conclusion holds. Parents have less time to devote to their own consumption and not enough additional resources to buy purchased domestic inputs instead of providing domestic time. Children will then benefit from less time transfers related to grandchild care.
4.3 The labor participation of young and old workers

We now investigate the overall impact on the labor market of a rise in retirement age $\theta$. Since the child’s labor participation depends on the family transfer and since postponing retirement influences grandchild care, this means that delaying retirement will affect the labor supply decision of both the parent and the child. Labor supply is given by $h$ for the young and $\theta$ for the old. The impact of a change in $\theta$ is then given by:

$$\frac{\partial (h + \theta)}{\partial \theta} = 1 + \frac{\partial h}{\partial T} \left[ \frac{\partial T}{\partial \omega_b} - \frac{\partial T}{\partial L} \right]$$

Combining results from Propositions 1, 2 and 3 sheds light on the interplay between the receipt of family transfers and employment on the labor market. Our main result is that postponing retirement may either increase or decrease the labor participation of the young parents.

Since $\frac{\partial h}{\partial T} > 0$, if the time constraint effect is dominant, postponing retirement age decreases time transfer and therefore labor supply of the young, i.e. $\frac{\partial (h + \theta)}{\partial \theta} < 1$. But, when the income effect dominates the time constraint effect, grandparents are expected to devote more time to grandchild care when retirement age is postponed. Specifically, the derivative $\frac{\partial (h + \theta)}{\partial \theta}$ will be higher than 1 when the following condition holds:

$$\frac{\partial T}{\partial \omega_b} > \frac{\partial T}{\partial L}$$

that is, $z^2$ and $\epsilon^2$ are strongly substitutable and the additional income effect due to the postponed retirement has a large impact on grandchild care.

With respect to the existing literature (Cremer and Pestieau, 2004), our model exhibits the potential for a new “dividend” of a public policy aimed at postponing retirement. The provision of grandchild care transfers gives rise to an interdependency within the family. A change in the labor participation of the old generation will have a direct impact on the labor supply of the young generation. In many cases, grandparents will reduce time devoted to their adult children in response to a rise in $\theta$, as they have less time for non-working activities ($\frac{\partial \omega_b}{\partial \theta} < 0$). This will in turn reduce the participation of children on the labor market, since the latter have now to care for their own children instead of relying on the parental support.

However, postponing retirement may also increase the provision of time transfers ($\frac{\partial \omega_b}{\partial \theta} > 0$). By working longer, grandparents are in a position to buy more purchased inputs related to domestic production. If these
purchased inputs and their own domestic time are strongly substitutable, they will spend more time with the grandchildren. This is then the reverse story. Adults with young children have now more time to devote to non-domestic activities, and they will presumably spend part of this extra-time in paid activities. A modification in \( \theta \) has now an unintended, albeit beneficial, consequence. Delaying retirement increases employment of both the old and young generations.

Although we do not focus on the sustainability and the financial balance of the pension scheme in our model, it is clear that delaying retirement will be helpful for the pension system in terms of additional payroll taxes. A question worth is then to assess the magnitude of the ‘crowding-out’ or ‘crowding-in’ effects occurring through intergenerational linkages. If postponing retirement improves the labor participation of the old, but reduces at the same time employment for young adults, then the expected increase in payroll taxes will be much lower than initially expected. Knowing the overall effect of a change in \( \theta \) on \((h + \theta)\) deserves further attention and we perform in the next section numerical simulations to assess the possibility that postponing retirement may improve the labor participation of both the young and old generations.

5 Numerical illustration

We rely on the following functional forms for utility and production functions. First, we consider Cobb-Douglas functions for the extended levels of consumption of the young and the old, i.e. \( \tau_t \) and \( \tilde{\tau}_{t+1} \). This implies that in both periods, the elasticities of substitution \( \sigma_f \) and \( \sigma_d \) between the market good and the domestically-produced good are equal to one. Second, we assume CES forms for the domestic production functions \( g^1 \) and \( g^2 \). Hence, \( \tau_t \) can now be expressed as

\[
\tau_t = ((1 - \tau_t)\omega(1 - \kappa_t^1) - s_t - z_t^1)^{1 - \alpha_{11}}(a_{11}(z_t^1)^{\rho_1} + a_{12}(k_t^1 + T_t)^{\rho_1})^{1 - \alpha_{11}},
\]

where \( a_{11} \) and \( a_{12} \) measure the respective contributions of input and time devoted to child care. In a similar way, we get \( \tilde{\tau}_{t+1} \)

\[
\tilde{\tau}_{t+1} = (\omega^1(\theta_{t+1}, \tau_{t+1} + R s_{t+1} - z_{t+1}^2)^{\alpha_{21}}(a_{21}(z_{t+1}^2)^{\rho_2} + a_{22}(1 - \theta_{t+1} - T_{t+1})^{\rho_2})^{1 - \alpha_{21}},
\]

with \( a_{21} \) and \( a_{22} \) the respective weights of purchased input and time devoted to domestic tasks in the second-period production function. At last, we rely on a logarithmic utility function

\[
\ln \tau_t + \delta (\ln \tilde{\tau}_{t+1} + \gamma \ln T_{t+1})
\]
With these specifications and for an interior solution of the consumer problem in each period, the evolution of time transfers is characterized by the following recurrence equation

\[(1 - \alpha_2) T_{t+1} - \gamma \equiv 1 - \theta_{t+1} - T_{t+1}^{-1} \frac{\rho_2}{\theta_2} \left( R \left( 1 - \tau_t \right) w \left( 1 + T_t \right) + \omega \left( \theta_{t+1}, \tau_{t+1} \right) \right) \left( \frac{1}{\theta_2} + 1 + \gamma \right) T_{t+1}^{-1} - \gamma (1 - \theta_{t+1}) \]  \[= (15) \]

where the interval of admissible values for \( T_{t+1} \) is \( I = \left( \frac{1}{1 - \alpha_2 + \gamma}, 1 - \theta_{t+1} \right) \).

Since \( \rho_2 < 1 \), straightforward calculus shows that the left-hand side is an increasing function of \( T_{t+1} \) and grows from 0 to \( +\infty \) when \( T_{t+1} \) goes from the lower to the upper bound of \( I \). Thus, for any time transfer \( T_t \) received by an individual born in \( t \), there exists a unique time transfer \( T_{t+1} \) left to his offspring.

Our numerical illustration draws on the parameter values summarized in Table 1.\(^{10}\) We suppose that domestic production functions \( g^1 \) and \( g^2 \) are the same in both periods. Recalling that the elasticity of substitution between the purchased input and time devoted to domestic activities is a crucial parameter, we do not set its value for the moment.

### Table 1. Parameters

<table>
<thead>
<tr>
<th>First period of life</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>weight of the market good in the consumption function</td>
<td>( \alpha_1 )</td>
<td>0.5</td>
</tr>
<tr>
<td>weight of the purchased input in production function</td>
<td>( a_{11} )</td>
<td>1</td>
</tr>
<tr>
<td>weight of time in production function</td>
<td>( a_{12} )</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second period of life</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>weight of the market good in the consumption function</td>
<td>( \alpha_2 )</td>
<td>0.5</td>
</tr>
<tr>
<td>weight of purchased input in the domestic production function</td>
<td>( a_{21} )</td>
<td>1</td>
</tr>
<tr>
<td>weight of time in the domestic production function</td>
<td>( a_{22} )</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other parameters for preferences</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>discount factor</td>
<td>( \delta )</td>
<td>0.3</td>
</tr>
<tr>
<td>altruism</td>
<td>( \gamma )</td>
<td>0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Economic environment</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>net interest rate</td>
<td>( R )</td>
<td>( (1.04)^{30} = 3.24 )</td>
</tr>
<tr>
<td>wage</td>
<td>( w )</td>
<td>1</td>
</tr>
<tr>
<td>replacement rate</td>
<td>( b )</td>
<td>0.4</td>
</tr>
<tr>
<td>contribution rate</td>
<td>( \tau )</td>
<td>0.15</td>
</tr>
<tr>
<td>retirement age in the benchmark case</td>
<td>( \Theta )</td>
<td>( \frac{1}{3} )</td>
</tr>
</tbody>
</table>

\(^{10}\) Following de la Croix and Michel (2002), we choose \( \delta \) in the line of the RBC literature, with a quarterly psychological discount factor of 0.99. With period length of thirty years, the parameter \( \delta \) is thus set equal to \( 0.99^{30} = 0.3 \).
In the benchmark case, we set retirement age \( \theta \) to one-third. Assuming, for instance, economic life starts at twenty and period length is thirty years, this means that people enter the labor force at twenty and leave it at sixty. Then, we assess the consequences of an increase in retirement age at period 0 from one-third to one-half, i.e. from sixty to sixty-five. As in the preceding section, we consider a one-shot policy, i.e. an increase in \( \theta_0 \), holding \( \theta_t, \ t \geq 1 \), constant. Retirement age is postponed in order to reduce the deficit of the pay-as-you-go pension system for given contribution rate and replacement rate.

For permanent retirement age \( (\Theta = 1/3) \), replacement rate \( (b = 0.4) \) and contribution rate \( (\tau = 0.15) \), we compute the steady-state time transfer \( T \), solution to the following equation \(^{11}\)

\[
\left( \frac{(1 - \alpha_2) T - \theta}{1 - \theta - T - \gamma} \right)^{1-\rho_2} = \frac{a_{21} \gamma}{a_{22}} \left[ \frac{R (1 - \tau) w (1 + T) + \omega (\theta, \tau)}{(\frac{1}{\delta} + 1 + \gamma) T - \gamma (1 - \theta)} \right]^\rho_2
\]

Then, we compute \( T_0 \) and \( h_0 \) that result from a one-shot increase in the retirement age from 1/3 to 1/2 in period 0. Time transfer of the old in period 0 is determined by \(^{13}\)

\[
\left( \frac{(1 - \alpha_2) T_0 - \theta}{1 - \theta_0 - T_0 - \gamma} \right)^{1-\rho_2} = \frac{a_{21} \gamma}{a_{22}} \left[ \frac{RS_{-1} + \omega (\theta_0, \tau)}{(1 + \gamma) T_0 - \gamma (1 - \theta_0)} \right]^\rho_2
\]

As shown in Proposition 3, an increase in \( \theta_0 \) may have two offsetting effects on time transfer \( T_0 \). Indeed, the effect of \( \theta_0 \) associated with retirement duration \( (L) \) appears on the left-hand side, while the effect of \( \theta_0 \) through the second-period income \( (\Omega) \) appears on the right-hand side. From Proposition 3, we know that the former effect (time constraint effect) is negative on time transfer, while the latter one (income effect) is ambiguous. Nevertheless, from the expression of \( \frac{dT}{d\Omega} \) in the proof of Proposition 3, we deduce that the specifications of the utility function that we have chosen imply that the sign of the income effect is the same as the sign of \( \rho_2 \). This is confirmed by the equation that characterizes \( T_0 \) in this example.

From equation (13), the resulting labor supply of the young born in 0 is

\[
h_0 = \frac{s_0 + c_0 (1 + T_0)}{(1 - \tau_0) w + c_1}
\]
where

\[ \zeta_1 = \frac{\eta_1}{1 - \alpha_1} \left[ 1 + \frac{\alpha_1 a_{12}}{a_{11} (\eta_0)^2} \right] \quad \text{and} \quad \eta_0 = \left( \frac{a_{11} (1 - \tau_0) w}{a_{12}} \right)^{1 - \theta_1} \]

and savings write

\[ s_0 = \frac{\delta \left( 1 + \gamma - \gamma \frac{1 - \theta_1}{T_1} \right) (1 - \tau_0) w (1 + T_0) - R^{-1} \omega (\theta_1, \tau_1)}{1 + \delta \left( 1 + \gamma - \gamma \frac{1 - \theta_1}{T_1} \right)} \]  \hspace{1cm} (17)

In the last expression, since we consider a one-shot increase, we have \( \theta_1 = 1/3 \) and \( T_1 \) results from equation (15).

Figure 1 plots the impact of delaying retirement on the time transfer and labor supply, as a function of the elasticity of substitution between purchased input and time in the domestic production.

When considering low values of the elasticity of substitution \( \sigma_2 \approx 0.1 \), we find that grandparents reduce their time transfers of 0.12 in response to an increase in the retirement age of 0.16. This leads in turn to a significative fall in labor participation of young adults. Nevertheless, the overall labor participation is higher since the rise in \( \theta \) (+0.16) is higher than the fall in \( h_0 \) (-0.1). These effects are attenuated for higher values of the elasticity of substitution. In fact, the income effect becomes positive for \( \sigma_2 \) higher than 1, i.e. time transfer becomes a superior good. But, the substitution effect is still dominant and leads to a fall in time transfer and labor supply of the young.

A positive effect of the retirement age on time transfer is obtained when the three following conditions are simultaneously satisfied : (i) high values of the ratio \( a_{21}/a_{22} \), (ii) high elasticity \( \sigma_2 \) and (iii) low degree of altruism. For instance, as plotted on Figure 2, if \( a_{21} = 5 \), \( a_{22} = 1 \), \( \sigma_2 \geq 9 \), \( \gamma = 0.02 \), and keeping the values of Table 1 for the other parameters, the retirement age has a small positive effect on time transfer. Since the weight of the purchased input in domestic production is high, the income effect of the rise in the retirement age allows the grandparent to sharply reduce time input \( e^2 = 1 - \theta - T \). Grandparent chooses to reduce time input \( e^2 \) of an amount higher than the fall in time endowment \( 1 - \theta \). This results in a rise in time transfer to the young, and consequently in higher labor supply of the young in period 0.
Figure 1. Increasing retirement age in period 0 \( (a_{21} = 1, \gamma = 0.2) \)

![Graph showing the impact of increasing the retirement age in period 0.](image)

Figure 2. Increasing retirement age in period 0 \( (a_{21} = 5, \gamma = 0.02) \)

![Graph showing the impact of increasing the retirement age in period 0.](image)
Let us now discuss the effect of a permanent increase in retirement age on labor force participation. The difference with respect to the one-shot increase is that young workers, born in period 0, expect an increase in their second-period income. As equations (16) and (17) show, labor supply of the young in period 0 depend on $\theta_1$ through two channels. First, a direct effect appears in equation (17): a permanent increase in the retirement age raises the second-period income $\omega(\theta_1, \tau)$. Since both extended consumptions $\bar{c}_t$ and $\bar{d}_{t+1}$ are normal goods in our example, first-period consumption of the young increases and savings $s_0$ is reduced. Through this channel, the increase in $\theta_1$ implies a rise in $h_0$. Nevertheless, there also exists an indirect effect that passes through time transfer $T_1$ and that may be ambiguous.\footnote{Indeed, it is straightforward to see that $s_0$ is increasing with $T_1$, but $T_t$ is affected by $\theta_1$ and $\theta_0$.} It is then clear that permanent postponing of retirement has also an ambiguous effect on labor participation of young parents.

6 Conclusion

The purpose of this paper was to study the consequences of prolonging activity in a setting where family transfers matter. We show that owing to intergenerational linkages, a change in the labor participation of one generation is expected to affect the employment rate of the other generation. Interestingly, we find that this family externality may be either positive or negative. In some circumstances, postponing retirement may have a boosting effect on the labor market, in that it increases the labor force participation of both the young and the old workers. This finding, which is innovative with respect to the previous literature, is of importance with respect to the financing of pension scheme, as delaying retirement will impact the amount of expected additional payroll taxes.

A shortcoming of our analysis is that our approach, followed to avoid the government budget constraint, means that in the long run public debt will explode and goes towards infinity. This is due to the interest rate paid by the government, which is exogenous in our model. In fact, in our analysis, retirement postponing is expected to only reduce the deficit of the pay-as-you-go pension system. In order to restore balance in the budget, the government could additionally implement a lump-sum taxation, or raise the contribution rate, or reduce the replacement rate. But such policies would affect the optimal allocation of resources of the different generations in a complex way.

Moreover, as our primary aim was to show that intergenerational relationships and family support have to be taken into account when studying the functioning of the labor market, we have restricted our attention to
changes in the retirement age. An interesting extension would be to explore the consequences of alternatives changes in the pay-as-you-go pension system as increasing contribution rate or decreasing replacement rate and compare the results with those from changing the retirement age.

Several other extensions of this model may come to mind. First, it would be useful to account for financial gifts made by the parents to their children. Second, transfers may flow in the reverse direction and it could be that children pay for the services and time transfers provided by the parent. Third, older workers may themselves have alive parents, which would give rise to a trade-off between caring for elders and helping children. This suggests that the consequences and magnitude of the underlying family externality have to be examined within an extended framework, and we leave this issue for future research.

References

Appendix

We calculate the second-order derivatives of $\Psi$. From the following definition of $\sigma^2_y$:

$$\sigma^2_y = -\frac{d \left( \frac{x^2}{L-\tau} \right) / \left( \frac{x^2}{L-\tau} \right)}{d \left( \frac{g_1^2}{g_2^2} \right) / \left( \frac{g_1^2}{g_2^2} \right)}$$

and the properties of linear homogeneous functions, we deduce that

$$\sigma^2_y = \frac{g_2^2 g_1^2}{g_2^2 g_1^2}$$

Similarly, we get the following formula for $\sigma^2_f$:

$$\sigma^2_f = \frac{f_2^2 f_1^2}{f_1^2 f_2^2}$$

Using these definitions of $\sigma^2_y$ and $\sigma^2_f$ and recalling that $\Psi(z^2, T, \Omega, L) = (-f_1^2 + f_2^2 g_1^2) v_1$ and $\Psi_T(z^2, T, \Omega, L) = -v_1 f_2^2 g_2^2 + \gamma \Phi_1$, we get

$$\Psi_{zz} = v_1 \left[ f_1^2 - 2 f_1^2 g_1^2 + f_2^2 (g_1^2)^2 + f_2^2 g_2^2 \right] = -v_1 f_2^2 g_2^2 \left[ \frac{g_1^2}{\Omega - \tau} \left( 1 + \frac{\Omega - \tau}{g_2^2 g_1^2} \right)^2 + \frac{f_2^2}{f_1^2} \sigma^2_f \frac{L - T}{\tau^2} \right] < 0$$

$$\Psi_{zT} = v_1 \left[ (f_2^2 g_2^2) - f_2^2 g_2^2 \right] = v_1 \left[ (1 + \frac{\Omega - \tau}{g_2^2 g_1^2}) g_2^2 f_1^2 - f_2^2 g_1^2 \right]$$

$$\Psi_{z\Omega} = v_1 \left[ -f_1^2 + f_2^2 g_2^2 \right] = v_1 \left[ \frac{g_1^2}{\Omega - \tau} + g_2^2 \right] f_2^2 g_1^2 > 0$$

$$\Psi_{zL} = v_1 \left[ -f_1^2 g_2^2 + f_2^2 g_2^2 \right]$$

$$\Psi_{TT} = v_1 \left( f_2^2 g_2^2 \right) - v_1 \left( -f_2^2 (g_2^2)^2 - f_2^2 g_2^2 \right) + \gamma \Phi_1 < 0$$

$$\Psi_{T\Omega} = -v_1 f_2^2 g_2^2 - v_1 f_2^2 g_2^2 = -v_1 \left[ \frac{v_1}{f_1^2} + 1 \right] f_2^2 g_2^2$$

$$\Psi_{TL} = -v_1 \left( f_2^2 g_2^2 \right) - v_1 \left( f_2^2 (g_2^2)^2 - v_1 f_2^2 g_2^2 \right) > 0$$

$$= -v_1 \left( f_2^2 g_2^2 \right) + v_1 \left[ \frac{\Omega - \tau}{g_2^2} f_1^2 (g_2^2)^2 + f_2^2 \frac{z^2}{L - T} \right]$$

---

15 Since $f_i$ is linear homogeneous, first-order derivatives $f_i$ and $f_i^2$ satisfy $f_i'(\lambda x, \lambda y) = f_i(x, y)$ and $f_i^2'(\lambda x, \lambda y) = f_i^2(x, y)$. Differentiating with respect to $\lambda$, we get $f_i'(\lambda x, \lambda y) + y f_i(x, y) = 0$ and $x f_i'(\lambda x, \lambda y) + y f_i(x, y) = 0$ from which we deduce $v_1 f_1'(x, y) = f_1(x, y) = \frac{d f_1(x, y)}{d \lambda}$. In the same way, we get $v_1 f_1(x, y) = -g_2^2 f_1(x, y) = \frac{d g_2^2(x, y)}{d \lambda}$.