AK growth models: new evidence based on fractional integration and breaking trends

J. Cunado, L.A. Gil-Alana, F. Pérez de Gracia *
Universidad de Navarra
Faculty of Economics
Edificio Biblioteca, Entrada Este
E-31080 Pamplona, SPAIN
Phone: 00 34 948 425 625
Fax: 00 34 948 425 626
E-mail: fgracia@unav.es

1 Introduction

Although the study of the causes of economic growth has always been an important research area in economics, interest in this subject has increased with the literature on endogenous growth since the mid-80s (see, for example, Romer, 1986; Lucas, 1988). These models propose different explanations for some of the features of the long run path of the economies for which the neoclassical Solow (1956) model was unable to account. As it is well known, the diminishing returns to capital assumed in the neoclassical model implies that in the absence of continuing improvements in technology, per capita growth eventually ceases. However, according to the models of the initial wave of the new research, AK growth models, (Romer, 1986; Lucas, 1988; Rebelo, 1991), economic growth could go on indefinitely because the returns on investment in a broad class of capital goods do not necessarily diminish

* We would like to thank the Editor and two anonymous referees for improving this paper through their comments and suggestions. Juncal Cunado and Luis A. Gil-Alana gratefully acknowledge financial support from the Spanish Ministry of Science and Technology (SEJ2005-07657/ECON). Fernando Perez de Gracia acknowledges research support from the Spanish Ministry of Science and Technology and FEDER through grant SEJ2005-06302/ECON and from the Plan Especial de Investigacion de la Universidad de Navarra.

DOI: 10.3917/rel.752.0131

as economies develop. In this framework, active government policies affecting investment rates are desirable, since they would have permanent effects on growth rates.

Another key distinct prediction between neoclassical and endogenous growth models is related to the convergence hypothesis: while the neoclassical model predicts that per capita output in an economy will converge to each country's steady-state or to a common steady-state regardless of its initial per capita output level, in endogenous growth models there is no tendency for income levels to converge.

Within the wide debate on the validity of the different growth models, empirical papers try to analyze these two testable predictions of endogenous growth models. On the one hand, recent years have witnessed an emerging body of empirical literature on convergence in per capita output across different economies. On the other hand, research on growth models also includes papers which analyze the so called "growth effects" predicted by the endogenous growth models (e.g. Barro, 1991; Mankiw, Romer and Weil, 1992; Jones, 1995; McGrattan, 1998; Binder and Pesaran, 1994), following both cross-country and time series techniques. In a time series framework, Jones (1995) tests the "growth effects" prediction by comparing the univariate properties of the rate of growth of per capita GDP and investment shares of GDP in fifteen OECD countries using data for the post-World War II period (1950-1988). Applying ADF test to these series, the results show that while investment rates are non-stationary I(1), per capita GDP growth rates are I(0) variables. Therefore, empirical evidence for the post-war period seems to support neoclassical growth models in contrast to AK-type models. However, McGrattan (1998), by means of graphical analysis, shows that the "growth effects" prediction of AK theory may be consistent with the data when using longer historical data sets. Kocherlakota and Yi (1997) test whether changes in policy variables have permanent effects on growth rates in a regression of growth rates on current and lagged policy variables (public capital and tax rates) using time series data for the US and the UK. They find empirical evidence against exogenous growth models since the policy variables they include in the regression have an economically and statistically significant effect on growth rates in both countries.

The goal in this paper is to re-examine whether permanent changes in investment rates have lead to permanent changes in growth rates, as predicted by the AK growth models. We examine the cases of Canada, the UK and the US for the time period 1870-2002, and Japan for 1885-2002. However, instead of using classic approaches, which are all based on the strict distinction between I(0) stationarity and nonstationarity I(1), we consider the possibility of I(d) processes, where d can be any real number. In doing so, we allow for a much richer flexibility in the dynamic behavior of the series, not achieved by these two classic representations. In the context of growth models, long memory arises naturally as the result of the inclusion of crosssectional heterogeneity in a growth model (i.e. Solow growth model) and thus, justifies the use of fractional techniques in analyzing the univariate properties of per capita GDP and investment series (see, for example, Michelacci and Zaffaroni, 2000) [1]. Moreover, it is a well known fact that traditional unit-root procedures (e.g., Dickey and Fuller, 1979; Phillips and Perron, 1988) have very low power in the context of fractional alternatives (e.g., Diebold and Rudebusch, 1991; Hassler and Wolters, 1994; Lee and Schmidt, 1996), and, in that respect, a proper study of this topic based on fractional integration seems overdue. The evidence in favor of long memory may also be due to the effect of aggregation. In fact, aggregation is one of the main sources of long memory. The key idea is that aggregation of independent weakly dependent series can produce a strong dependent series. Robinson (1978) and Granger (1980) show that fractional integration can arise as a result of aggregation when: a) data are aggregated across heterogeneous autoregressive (AR) processes, and b) data involving heterogeneous dynamic relationships at the individual level are then aggregated to form the time series.

The paper is organized as follows. Section 2 contains some economic foundations of one of the key AK growth model predictions as opposed to the neoclassical model. Section 3 presents the procedures employed in the paper. In Section 4, the methods are applied to per capita GDP growth rates and investment output ratios in Canada, Japan, the UK and the US. Section 5 incorporates the possibility of structural breaks in the series. Finally, Section 6 contains some concluding comments.

2 The AK growth models

In this section we show that one of the implications of the absence of the diminishing returns assumption in the AK growth models (in contrast to neoclassical models) is that a permanent change in the investment output ratio generates a permanent change in the growth rate of an economy [2]. For this purpose, we consider the basic Solow growth model, with an exogenous saving rate, s, and a Cobb-Douglas production function given by:

$$Y = AK^{\alpha}L^{1-\alpha} \tag{1}$$

In per capita terms,

$$y = Ak^{\alpha} \tag{2}$$

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1 Lau (1999), for example, derives time series properties of stochastic endogenous growth models by relating economic growth, unit roots and cointegration.

2 For a more detailed version of the neoclassical and AK growth models, see, for example, Barro and Sala-i-Martin (1995).

where y and k are output and capital per worker, A is a positive constant that reflects the level of technology and α is a number between 0 and 1, that is, there are diminishing returns on capital per worker. In this model, the long-run per capita growth rate is pegged at the exogenous rate of technological change, and any change in the saving or investment ratio shows up in only a higher level of capital and output per effective worker, but not in a change in the steady-state per capita growth rate.

However, if $\alpha = 1$, that is, in the absence of diminishing returns to capital, the key property of AK growth models, the per capita production function can be written as:

$$y = Ak \quad [3] \tag{3}$$

In this case, the average and marginal products of capital are constant at level $A > 0$, and per capita output grows at the rate given by:

$$\gamma_k^{*} = \gamma_y^{*} = s - n + \delta, \quad [4] \tag{4}$$

where n is the population growth rate and δ is the depreciation rate.

In contrast to the neoclassical growth models, an increase in the savings output ratio (or investment output ratio), s, will cause an increase not only in the level of capital per capita but in its steady state growth rate as well. Based on this prediction and in a time series context, we would say that according to AK growth models, a permanent change in investment rates will have a permanent effect on steady state growth rate, but only a temporary effect if growth is driven according to neoclassical models.

From an econometric viewpoint, the distinction between permanent and transitory changes can be determined by looking at the order of integration of the series. Thus, for example, if it is I(0) stationary, shocks affecting the series will be transitory and the degree of decay will depend on the structure describing the short run dynamics. On the other hand, if the series is I(1), shocks will be permanent and thus persisting forever. In this paper, however, we allow for a much richer degree of flexibility in the dynamic behaviour of the series adopting methodologies based on fractional integration. If a series is described in terms of an I(d) process, the parameter d plays a crucial role in the analysis of the persistence of shocks: if $d \in (0, 0.5)$, the series is covariance stationary and mean reverting, with the effect of shocks disappearing in the long-run; if $d \in [0.5,1)$, the series is no longer covariance stationary but is still mean reverting, while $d \geq 1$ means nonstationary and non mean reverting. In conclusion, the higher the d, the higher the degree

[3] There exist different growth models in the literature that provide a justification for the presence of constant returns in the aggregate production function, such as Romer (1986) and Barro (1990).

[4] We still assume that the saving rate is exogenous and equal to s. However, the results do not change when an endogenous saving rate is assumed. In this case, the steady-state growth rate of the economy is given by $(1/\varepsilon)(A-\rho-\delta)$, where ρ is the rate of time preference and ε measures the willingness to substitute consumption intertemporally when $u(c) = (c^{1-\varepsilon} - 1)/(1 - \varepsilon)$. Then, a decrease in the rate of time preference, ρ, which leads to an increase in investment will also have a permanent effect on the steady state growth rate.

of association will be between the observations and thus, the higher the persistence of shocks.

3 Testing for fractional integration

There exist many different methods of testing for unit-root models. The most common ones are the ADF test (Dickey and Fuller, 1979) and the PP (Phillips and Perron, 1988) [5]. Ng and Perron (2001) propose a modified version of the ADF and PP tests which tries to solve the main problems present in these more conventional tests for unit root. Conspicuous features of the above methods are the non-standard nature of the null asymptotic distributions which are involved, and the absence of Pitman efficiency [6]. However, these properties are not automatic, rather depending on what might be called a degree of "smoothness" in the model across the parameters of interest. That is, these properties hold if the limit distribution of the tests does not change in an abrupt way with small changes in the parameters. Thus, they do not hold in the case of unit root tests against AR alternatives. This is associated with the radically variable long run properties of AR processes around the unit root. Under the AR structure: $y_t = \rho y_{t-1} + \varepsilon_t$, for $|\rho| > 1$, y_t is explosive; for $|\rho| < 1$, y_t is covariance stationary; and for $\rho = 1$, it is nonstationary but non-explosive. In view of these abrupt changes, the fractional processes have become a rival class of alternatives to the AR model in the case of unit-root testing. Robinson (1994) proposes a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o : d = d_o, \tag{5}$$

in a model given by:

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, ..., \tag{6}$$

where d_o can be any real number and where u_t is I(0). The x_t in (6) can be the time series we observe, though it may also be the errors in a regression model of the form:

$$y_t = \beta' z_t + x_t, \tag{7}$$

where $\beta = (\beta_1, \ldots, \beta_k)'$ is a (kxl) vector of unknown parameters, and z_t is a (kxl) vector of deterministic regressors that may include, for example, an intercept, (e.g., $z_t \equiv 1$), an intercept and a linear time trend ($z_t = (1, t)'$) or even dummy variables. The functional form of the test statistic, (denoted

5 Another standard procedure is Kwiatkowski et al. (KPSS, 1992). They test for stationarity against the alternative of a unit root. See also Müller (2005) for a recent overview of stationary tests in the context of highly autocorrelated series.

6 This refers to the power of the tests against local departures from the null. Most of unit root tests are not efficient against local alternatives. Exceptions are the tests of Dufour and King (1991) and Elliot et al. (1996).

here by $\hat{R}$) can be found in any of the numerous applications of the tests (e.g., Gil-Alana and Robinson, 1997; Gil-Alana, 2000).

Robinson (1994) shows that under certain regularity conditions,

$$\hat{R} = \hat{r}^2 \rightarrow_d \chi_1^2, \qquad as \quad T \rightarrow \infty. \tag{8}$$

These conditions are very mild, implying that u_t must belong to the class of sequences $\{v_t, t = 0, \pm1, \ldots\}$ of stationary random variables satisfying $E(v_t|B_{t-1}) = 0$ and $E(v_t^2|B_{t-1}) = \sigma^2$ almost surely, where $0 < \sigma^2 < \infty$, and B_t is the σ-filed of events generated by v_s, $s \le t$. There are also other technical conditions which are satisfied by the model in (6) and (7). Thus, according to (8), we are in a classical large-sample testing situation and the conditions on u_t in (6) are far more general than Gaussianity, with a moment condition only of order 2 required. Because $\hat{R}$ involves a ratio of quadratic forms, its exact null distribution could have been calculated under Gaussianity via Imhof's algorithm. However, a simple test is approximately valid under much wider distributional assumptions. An approximate one-sided 100α%-level test of H_o (5) against the alternative: H_a: $d > d_o$ ($d < d_o$) will reject H_o (5) if $\hat{r} > z_\alpha$ ($\hat{r} < -z_\alpha$), where the probability that a standard normal variate exceeds z_α is α. Furthermore, Robinson (1994) shows that the above test is efficient in the Pitman sense, i.e., that against local alternatives of form: H_a: $d = d_o + \delta T^{-1/2}$, with $\delta \neq 0$, the limit distribution of $\hat{r}$ is normal with variance 1 and mean that cannot (when u_t is Gaussian) be exceeded in absolute value by that of any rival regular statistic.

In the final part of this article we also perform a procedure recently developed by Gil-Alana (2007) for estimating structural breaks and fractional integration. In particular, we consider models of form:

$$y_t = \alpha_1 + \beta_1 t + x_t; \quad (1 - L)^{d_1} x_t = u_t, \quad t = 1,\ldots,T_b \tag{9}$$

$$y_t = \alpha_2 + \beta_2 t + x_t; \quad (1 - L)^{d_2} x_t = u_t, \quad t = T_b+1,\ldots,T, \tag{10}$$

where the α's and the β's are the coefficients corresponding respectively to the intercepts and the linear trends; d_1 and d_2 may be real values, and they are the orders of integration for each subsample, u_t is again I(0), and T_b is the time of the break that is supposed to be unknown. Note that the model in equations (9) and (10) can also be written as:

$$(1 - L)^{d_1} y_t = \alpha_1 \tilde{1}_t(d_1) + \beta_1 \tilde{t}_t(d_1) + u_t, \quad t = 1, \ldots, T_b, \tag{11}$$

$$(1 - L)^{d_2} y_t = \alpha_2 \tilde{1}_t(d_2) + \beta_2 \tilde{t}_t(d_2) + u_t, \quad t = T_b+1, \ldots, T, \tag{12}$$

where $\tilde{1}_t(d_i) = (1 - L)^{d_i} 1$, and $\tilde{t}_t(d_i) = (1 - L)^{d_i} t$, i = 1, 2 [7].

[7] For the empirical work we assume that $\tilde{1}_t = \tilde{t}_t = 0$ for $t \le 0$, which is consistent with the Type II definition of fractional integration. For an alternative definition (Type I) see Marinucci and Robinson (1999) and Gil-Alana and Hualde (2008).

The method presented in Gil-Alana (2007) is based on the least squares principle applied to equations (11) and (12). First we choose a grid for the values of the fractionally differencing parameters d_1 and d_2, for example, $d_{io} = 0, 0.01, 0.02, \ldots, 1$, $i = 1, 2$. Then, for a given partition $\{T_b\}$ and given d_1, d_2-values, (d_{1o}, d_{2o}), we estimate the α's and the β's by minimizing the sum of squared residuals,

$$\min_{w.r.t.\{\alpha_1,\alpha_2,\beta_1,\beta_2\}} \sum_{t=1}^{T_b}\left[(1-L)^{d_{1o}} y_t - \alpha_1 \tilde{1}_t(d_{1o}) - \beta_1 \tilde{t}_t(d_{1o})\right]^2 + \sum_{t=T_b+1}^{T}\left[(1-L)^{d_{2o}} y_t - \alpha_2 \tilde{1}_t(d_{2o}) - \beta_2 \tilde{t}_t(d_{2o})\right]^2$$

Let $\hat{\alpha}(T_b; d_{1o}^{(1)}, d_{2o}^{(1)})$ and $\hat{\beta}(T_b; d_{1o}^{(1)}, d_{2o}^{(1)})$ denote the resulting estimates for partition $\{T_b\}$ and initial values $d_{1o}^{(1)}$ and $d_{2o}^{(1)}$. Substituting these estimated values on the objective function, we have $RSS(T_b; d_{1o}^{(1)}, d_{2o}^{(1)})$, and minimizing this expression across all values of d_{1o} and d_{2o} in the grid we obtain: $RSS(T_b) = \arg\min_{\{i,j\}} RSS(T_b; d_{1o}^{(i)}, d_{2o}^{(j)})$. Then, the estimated break date, $\hat{T}_k$, is such that: $\hat{T}_k = \arg\min_{i=1,\ldots,m} RSS(T_i)$, where the minimization is taken over all partitions $T_1, T_2, \ldots, T_m$, such that $T_i - T_{i-1} \geq |\varepsilon T|$. Then, the regression parameter estimates are the associated least-squares estimates of the estimated k-partition, i.e., $\hat{\alpha}_i = \hat{\alpha}_i(\{\hat{T}_k\})$, $\hat{\beta}_i = \hat{\beta}_i(\{\hat{T}_k\})$, and their corresponding differencing parameters, $\hat{d}_i = \hat{d}_i(\{\hat{T}_k\})$, for i = 1 and 2. Several Monte Carlo results based on the model in (9) and (10) are provided in Gil-Alana (2007). In that paper the author shows that the method performs relatively well even with small samples.

4 Data and test results

The data used in this section are annual log real GDP per capita in 1990 Geary-Khamis PPP-adjusted dollars and non-residential gross domestic investment as percentage of GDP. Purchasing Power Parity (PPP) are currency conversion rates that both convert to a common currency and equalise the purchasing power of different currencies. That is, they eliminate the differences in price levels between countries in the process of conversion and this explains why most of the analysis comparing different economics use data adjusted by (see, for example, Jones, 1995; Michelacci and Zaffaroni, 2000 among others) [8]. The series run from 1870 to 2002 for Canada, the UK and the US, and from 1885 to 2002 for Japan. The real per capita GDP series for the period 1870-1994 have been obtained from Maddison (2001)

8 Jones (1995) and Michelacci and Zaffaroni (2000) also use GDP per capita from Maddison.

and these series were updated using the GGDC (Groningen Growth and Development Center) Database 2002. The non-residential gross domestic investment as percentage of GDP series for the period 1870-1989 have been obtained from Maddison (1992) and were updated using the OECD Statistical Compendium 2003. We use non-residential investment instead of total investment since in a growth model framework, productive investment is the investment component which should be taken into account. Figure 1 shows the temporal evolution of the growth rate of per capita GDP and non residential investment output ratios for Canada, Japan, the UK and the US. The data show that per capita GDP growth rates have remained roughly constant (except during the war period), and they seem to exhibit a stationary behavior throughout the whole period. However, the analysis of the stationarity in the investment output ratios is not so clear from the plots. Moreover, these variables may have suffered one or more structural breaks along the time period examined in the paper. For example, and as shown in Figure 1, per capita GDP growth in Japan suffers a clear break in 1945 due to WWII, which have been taken into account in the empirical analysis carried out below.

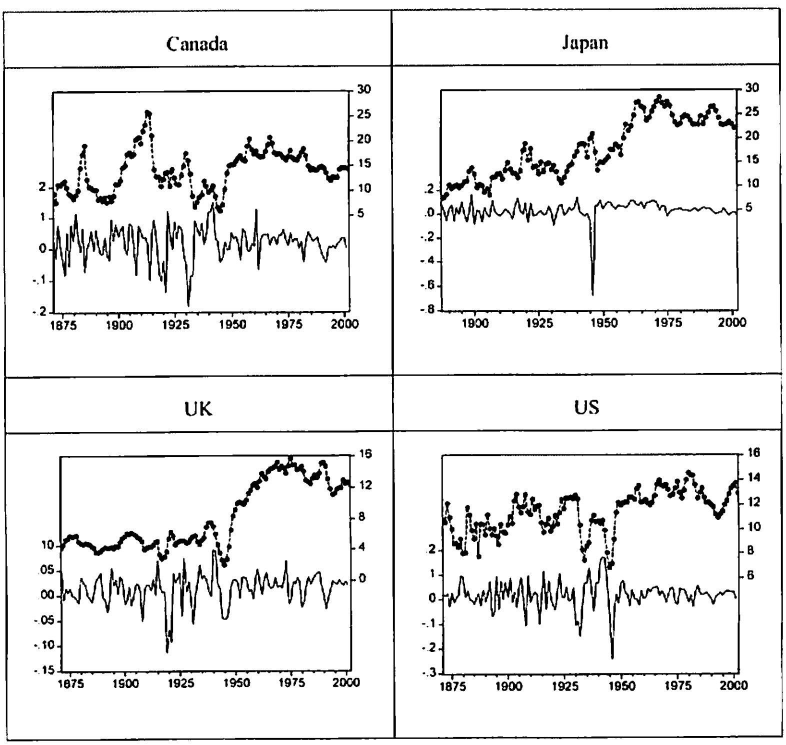

Figure 1 : Per capita GDP growth rates and investment output ratios, 1870-2002

Annual per capita GDP growth rates (solid) in left axis and investment ratios (dashed) in right axis.

The first thing we do is to perform the tests of Robinson (1994) described in Section 3 to the individual series. Robinson's (1994) parametric approach does not require preliminary differencing, and it allows us to test any real value d encompassing stationary and nonstationary hypotheses. Denoting each of the time series by y_t, we employ the model given by (6) and (7), with $z_t = (1,t)'$, $t \geq 1$, $z_t = (0,0)'$. Thus, under the null hypothesis H_o (5):

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, \ldots \tag{13}$$

$$(1 - L)^{d_o} x_t = u_t, \qquad t = 1, 2, \ldots, \tag{14}$$

and we treat separately the cases $\beta_0 = \beta_1 = 0$ a priori; β_0 unknown and $\beta_1 = 0$ a priori (i.e., with an intercept); and β_0 and b_1 unknown, (i.e., with a linear time trend). The results were fairly similar in the three cases. Thus, we only report the cases corresponding to the model with an intercept, since that coefficient was found to be statistically significant at conventional statistical levels, unlike what happened with the time trend coefficients that were insignificant in most cases. We model the I(0) process u_t to be both white noise and to have parametric autocorrelation. We test H_o (5) for d_o-values equal from 0 to 2 with 0.01 increments

Table 1 shows the test results; the numbers in parenthesis are the maximum likelihood estimates of d obtained with the Whittle function. Table 1 also shows the 95% confidence bands for the non-rejections of d_o using Robinson's (1994) parametric approach. Starting with the white noise case, we observe that for the GDP per capita growth rates, the null hypothesis of I(0) stationarity cannot be rejected in the cases of Canada, Japan and the US, while this hypothesis is rejected in favour of $d > 0$ (i.e., long memory) for the UK. Allowing autocorrelation throughout the model of Bloomfield (1973) [9] the I(0) hypothesis cannot be rejected for any series and evidence of anti-persistence (i.e., $d < 0$) is found in many cases.

If we look now at the investment output ratios (in the lower part of Table 1), we observe evidence of nonstationarity in practically all cases. If u_t is white noise, the I(1) hypothesis cannot be rejected for Canada, Japan and the US, while UK is the only country where d is found to be higher than 1. Using the model of Bloomfield (1973) the values are smaller than in the white noise case, and d, though positive, is smaller than 1 for Canada, Japan and the US. On the other hand, UK presents the highest estimate of d (0.82) and the unit root null hypothesis cannot be rejected for this country. We can now summarize the results in this table by saying that GDP per capita growth rates are stationary (and thus, mean reverting) for the four countries examined. For the investment output ratios the results substantially vary depending on how we model the I(0) disturbance term. Thus, if it is white noise, the unit root is rejected in favour of higher orders of integration for the UK, while this hypothesis cannot be rejected for the

9 The model of Bloomfield (1973) is a non-parametric approach of modelling the I(0) disturbances that produces autocorrelations decaying exponentially as in the AR(MA) case.

remaining three countries. However, if we permit weak dependence for the error term, mean reversion (d < 1) is found for these three countries and the unit root is non-rejected for the UK case. Therefore, and in contrast to the AK growth models prediction, a permanent increase in the UK investment output ratio will not have a permanent effect on per capita GDP growth rate.

i) Per capita GDP growth rates		
	White noise disturbances	Bloomfield (m = 1) disturbances
Canada	[-0.01 (0.14) 0.34]	[-0.39 (-0.14) 0.21]
Japan	[-0.04 (0.06) 0.22]	[-0154 (0.02) 0.29]
UK	[0.07 (0.23) 0.49]	[-0.28 (-0.13) 0.11]
US	[-0.06 (0.14) 0.39]	[-0.72 (-0.48) 0.02]
ii) Investment output ratios		
	White noise disturbances	Bloomfield (m = 1) disturbances
Canada	[0.96 (1.17) 1.46]	[0.40 (0.59) 0.89]
Japan	[0.72 (0.85) 1.04]	[0.55 (0.65) 0.88]
UK	[1.20 (1.40) 1.69]	[0.68 (0.82) 1.08]
US	[0.64 (0.79) 1.01]	[0.29 (0.42) 0.66]

Table 1: 95% confidence intervals along with the estimates of d using Robinson's (1994) parametric approach

We also perform traditional unit root tests to the series. We use a modified version of the DF and PP tests proposed by Ng and Perron (2001) which tries to solve the main problems present in these more conventional tests for unit root. The results are displayed in the upper part of Table 2 and suggest that per capita GDP growth rates are stationary I(0) variables, while the null hypothesis of I(1) cannot be rejected in any of the cases for the UK, that is, the results are in accordance with those obtained in Table 1 with the Bloomfield model. Therefore, and as before, we only find evidence against the AK growth models for the UK case, while the "growth effect" prediction of these types of models cannot be rejected for the remaining three countries.

We also apply the Ng and Perron (2001) unit root tests for the post-war period, that is, from 1950 to the end of the sample. The results displayed in Table 2 suggest that when analyzing the post-war period, the unit-root hypothesis is rejected for all per capita GDP growth rates, while we cannot reject this hypothesis for investment output ratios, except for the case of the US. That is, we find more evidence against the "growth effect" prediction of AK growth models for the post-war period, although not for the US economy.

	Ng-Perron			
	Intercept		Intercept and linear trend	
Country	MZ_α	MZ_t	MZ_α	MZ_t
Canada	-61.69**	-5.55**	-61.69**	-5.55**
Japan	-55.07**	-5.23**	-56.23**	-5.30**
UK	-42.32**	-4.59**	-50.99**	-5.04**
US	-60.61**	-5.50**	-60.58**	-5.50**
Testing the order of integration of non-residential investment output ratios, 1871 or 1885-2002				
	Ng-Perron			
	Intercept		Intercept and linear trend	
Country	MZ_α	MZ_t	MZ_α	MZ_t
Canada	-14.25**	-2.64**	-25.15**	-3.53**
Japan	-0.96	-0.52	-15.91*	-2.72*
UK	-2.45	-0.94	-13.8	-2.62
US	-17.41**	-2.90**	-25.61**	-3.58**
Testing the order of integration of per capita GDP growth rates, 1950-2002				
	Ng-Perron			
	Intercept		Intercept and linear trend	
Country	MZ_α	MZ_t	MZ_α	MZ_t
Canada	-25.09**	-3.51**	-25.38**	-3.56**
Japan	-8.06*	-1.80*	-19.97**	-3.15**
UK	-23.06**	-3.39**	-50.03**	-5.00**
US	-17.08**	-2.86**	-21.72**	-3.29**
Testing the order of integration of non-residential investment output ratios, 1950-2002				
	Ng-Perron			
	Intercept		Intercept and linear trend	
Country	MZ_α	MZ_t	MZ_α	MZ_t
Canada	-3.73	-1.34	-6.35	-1.78
Japan	-1.84	-0.93	-3.56	-1.22
UK	-2.83	-1.17	-5.49	-1.58
US	-20.54**	-3.19**	-23.85**	-3.45**

Table 2: Testing the order of integration of per capita GDP growth rates, 1871 or 1885-2002

Critical values are taken from Ng-Perron (2001) - Table 1.

Model with intercept: (1) MZ_α asymptotic critical values for 5% and 10% are -8.1 and 5.7; (2) MZ_t asymptotic critical values for 5% and 10% are -1.98 and -1.62.

Model with intercept and trend: (1) $MZ\alpha$ asymptotic critical values for 5% and 10% are -17.3 and -14.2; (2) MZ_t asymptotic critical values for 5% and 10% are -2.91 and -2.62.

** and * denote rejection of the null hypothesis of stationarity at the 5% and 10% significance levels.

In view of the different results obtained when analyzing separately the post-war period and the whole sample period, it is of interest to examine the potential presence of structural breaks in the data.

5. A potential presence of a structural break

In this section we are concerned with the effect that a potential break in the data may have had on the above results. The relation between structural change and fractional integration is a topic that has been investigated recently. Lobato and Savin (1998), Engle and Smith (1999), Bos, Frances and Ooms (1999, 2002), Diebold and Inoue (2001) and Granger and Hyung (2004) are some the papers relating these two concepts.

The possibility of a structural break is clear in Figure 1, in which investment ratios seem to have suffered a change in the trend in the post-war period, especially in the cases of Japan and the UK. We first examine the possibility of structural breaks by means of including deterministic dummies in the regression model (7). An advantage of this procedure is that the limit distribution of the test statistic is unaffected by the inclusion of such components. On the other hand, a drawback of this approach is that we have to specify a priori the time and the type of the break [10]. In this paper, based on the time evolution of investment output ratios showed in Figure 1 and on the different results obtained when the analysis is carried out only for the post-war period, we choose 1950 as the time of the break. Moreover, the choice of 1950 as the time of the break may help us to understand the differences between our results and those in Jones (1995) [11]. Thus, we consider the following regression model:

$$y_t = \beta_0 + \beta_1 t + \beta_2 D_{1t} + \beta_3 D_{2t} + x_t, \qquad t = 1, 2, \dots,$$

where $D_{1t} = 1I(t > T_b)$, (mean shift); $D_{2t} = t\, I(t > T_b)$ (slope shift); $I(x)$ is the indicator function, and $T_b = 1950$. We examine separately the cases of a shift dummy (i.e., $\beta_3 = 0$ a priori); a slope dummy (i.e., $\beta_2 = 0$ a priori), and a combination of both (β_2 and β_3 unknown), for the different cases of white noise and autocorrelated disturbances. In Table 3 we report the results only for the case of a shift dummy. Very similar results were obtained in the other two cases.

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10 Beran and Terrin (1996) and Bos et al. (1999) also proposed LM tests for fractional integration with breaks.

11 Other time periods for the break were also employed, and the results were completely in line with those reported here.

i) Per capita GDP growth rates			
Country	White noise	Bloomfield (m = 1)	Bloomfield (m = 2)
Canada	[-0.01, 0.34]	[-0.16, 0.19]	[-0.17, 0.21]
Japan	[-0.14, 0.15]	[-0.19, 0.18]	[-0.21, 0.21]
UK	[0.03, 0.47]	[-0.21, 0.02]	[-0.23, 0.11]
US	[-0.06, 0.45]	[-0.23, 0.01]	[-0.24, 0.06]
ii) Investment output ratios			
Country	White noise	Bloomfield (m = 1)	Bloomfield (m = 2)
Canada	[0.94, 1.43]	[0.37, 0.86]	[0.35, 0.86]
Japan	[0.73, 1.08]	[0.38, 0.94]	[0.33, 0.84]
UK	[1.11, 1.64]	[0.55, 1.08]	[0.51, 1.11]
US	[0.56, 0.93]	[0.11, 0.67]	[0.12, 0.63]

Table 3: 95%-confidence intervals of the non-rejection values of d, including dummy variables for a break at World War II

The results are completely in line with those reported in the previous section. Thus, for the GDP growth rates, the results strongly support the hypothesis of stationarity in all cases. With respect to investment, the unit root cannot be rejected if the disturbances are white noise. However, if we permit autocorrelations, the unit root is rejected in favour of $d < 1$ for all cases except the UK. Therefore, we can conclude that when we analyze the plausibility of the "growth effect" implication of AK growth models using a long span of data (1870-2002 for Canada, the UK and the US and 1885-2002 for Japan), we do not find strong evidence against these models. In fact, we can only reject this theory for the case of the UK, even when we allow for a structural break in 1950.

As mentioned above, a limitation of the above approach is that the break-date is fixed a priori, which may be unrealistic in some cases. Thus, in what follows we apply the methodology presented in Section 3, that is, we estimate the break-dates along with the fractional differencing parameters and their associated parameters for each subsample. By simplicity, we only consider here the case of a changing intercept, though practically the same break-dates were obtained in all cases if a changing trend is allowed. The results are displayed in Table 4.

i) Per capita GDP growth rates							
Country	Break date	First sub-samples			Second sub-samples		
		d_1	α_1		d_2	α_2	
Canada	1917	-0.17 [-0.38, 0.11]	**0.0215 (5.491)**	0.0026	0.32 [0.08, 0.54]	0.0055 (0.293)	0.0027
Japan	1946	-0.09 [-0.35, 0.14]	0.0104 (1.135)	0.0106	0.54 [0.32, 0.61]	**0.0621 (3.619)**	0.0005
UK	1921	0.45 [0.19, 0.58]	0.0027 (0.497)	0.0010	0.21 [-0.07, 0.56]	**0.0197 (2.966)**	0.0007
US	1929	-0.47 [-0.71,-0.20]	**0.0171 (17.224)**	0.0020	0.65 [0.19, 0.84]	-0.0582 (-1.310)	0.0036
ii) Investment output ratios							
Country	Break date	First sub-samples			Second sub-samples		
		d_1	α_1		d_2	α_2	
Canada	1914	1.08 [0.78, 1.65]	**7.362 (3.429)**	5.2650	1.11 [0.88, 1.39]	**10.843 (7.648)**	2.0364
Japan	1946	0.80 [0.53, 1.03]	**8.077 (5.090)**	2.9333	1.11 [0.89, 1.41]	**13.027 (10.407)**	1.5608
UK	1914	1.28 [0.78, 1.51]	**4.498 (15.276)**	0.1619	1.45 [1.20, 1.79]	**3.429 (5.509)**	0.4703
US	1931	0.51 [0.36, 0.85]	**10.615 (16.236)**	1.1243	1.26 [0.99, 1.79]	**8.331 (12.619)**	0.4542

Table 4: Estimates of the break-dates along with the fractional differencing parameters and intercepts

The values in brackets in columns 3 and 6 refers to the confidence bands for the d's at the 95% level. The values in parenthesis in columns 4 and 7 are t-values.

First we note that none of the breaks take place at 1950, implying that the results presented above may be biased as a consequence of the misspecification of the break date. The breaks occur at similar dates for the two variables in the four countries. It is around 1917 for Canada; at around 1920 for the UK; 1929 for the US, and at 1946 for Japan. Starting with the per capita GDP growth rates, most of the estimated orders of integration are in the stationary interval. The only two values which are above 0.5 correspond to Japan and the US in the second subamples, though the confidence bands include many stationary cases. For Canada, Japan and the US the values of d increase during the second subsample, being below 0 before the break in the three cases. Nevertheless, for Canada and Japan in the first subsamples, the I(0) hypothesis cannot be rejected at the 5% level.

If we look now at the orders of integration for the investment output ratios, we observe that the values of d are substantially higher. For Canada, and especially for the UK, the values are above 1. For Japan and the US, they are below unity during the first subsample and for the US, the unit root is rejected in favor of $d < 1$. After the break, the values of d are above 1 in all cases, and for the UK, the unit root is rejected in favor of $d > 1$. We also note that the intercepts are statistically significant in the majority of cases, especially for the investment output ratios [12]. These results are completely in line with those obtained using other parametric approaches like Sowell (1992) and Beran (1995) applied separately for each subsample. The results suggest the following. First, and as before, there is clear evidence against the "growth effect" prediction of the AK growth models for the UK case, since this result remains the same for all the models and unit root tests applied in the paper. Finally, there seems to be more evidence against the AK theory for the second subperiod of the sample, not only for the UK case but also for the three remaining countries.

6 Concluding comments

According to AK growth models, permanent changes in investment-output ratios have permanent effects on a country's per capita GDP growth rate. In a time series context, a series is said to be mean-reverting when a shock on the series has only a transitory effect on its behaviour, while it is not mean reverting when the shock has a permanent effect on it. Therefore, the prediction of AK models stated above can be rejected if investment-output ratios are found to be non-mean-reverting when per capita output growth rates are. Jones (1995) tests this hypothesis using traditional unit root tests, that is, testing the unit root hypothesis ($d = 1$) against the alternative of $d = 0$ in fifteen OECD countries for the period 1950-1988. He finds that economic growth rates are I(0), while investment rates follow I(1) processes, and thus he concludes that the evidence is inconsistent with AK models. In this paper, we test the same hypothesis as in Jones (1995) for a sample of four OECD countries using a longer span of data (1870-2002 for Canada, the UK and the US, and 1885-2002 for Japan). However, instead of using classic techniques, based on integer differentiation, we use a methodology based on fractional integration.

The results obtained in this paper suggest that the inconsistency found in the literature with the AK theory is not so evident. In fact, we only find clear evidence against the "growth effect" prediction of these models for the

[12] Note that these coefficients do not correspond to the mean of the processes since they refer to the d-differenced processes. (See, equations (11) and (12)). Ljung-Box statistics were also performed and evidence of no serial correlation was found in practically all cases.

UK case, while in the remaining three countries this evidence depends on the proposed model and the analyzed subperiod. First, we apply Robinson's (1994) parametric methodology to estimate the integration order of both per capita GDP growth rates and non-residential investment output ratios and find that the first variable is mean reverting in all cases while the unit root hypothesis for investment output ratios can be rejected for Canada, Japan and the US when we allow for autocorrelated disturbances. Therefore, we can only reject the AK prediction for the UK when analyzing data for the period 1871-2002 (1885-2002 for Japan). When we apply Ng-Perron unit root tests, the evidence suggests again that the AK growth model prediction does not hold for the UK case for the 1871-2002 period.

In order to allow for structural breaks in the data, we follow two approaches. First, and in order to compare our results with those in Jones (1995), we analyze the post-war period (1950-2002), and obtain more evidence against the AK growth models, since we cannot reject the unit root hypothesis for the US case when applying Ng and Perron tests. However, when we apply fractional techniques and allow for autocorrelated disturbances, we only find evidence against the AK theory for the UK case. In the second approach, we do not impose the date of the break exogenously but estimate it along with the fractional differencing parameters. In this case, we find evidence of structural breaks in per capita GDP growth rates in 1917, 1946, 1921 and 1929 for each of the countries (Canada, Japan, UK and US) and in 1914, 1946, 1914 and 1931 for investment output ratios. When analyzing the "growth effect" prediction of AK models, we find more evidence against this theory for the second subsample. Again, evidence against this theory is strong for the UK case.

The paper can be extended in several directions. First the procedure used for fractional integration with a single break could be easily extended to the case of multiple breaks (see again Gil-Alana, 2007). However, in the present study we focus instead on a single break to explain the stochastic nature of series. The reason is the following: for the validity of the type of long-memory (fractional integration) model we use for these series it is necessary that the data span a sufficiently long period of time to detect the dependence across time of the observations; given the sample size of the series employed here, the inclusion of two or more breaks would result in relatively short subsamples, thereby invalidating the analysis based on fractional integration. On the other hand, the analysis of the stationarity/nonstationarity of the per capita GDP growth rates and investment output ratios in terms of I(d) statistical models could also have been studied by means of semiparametric procedures, i.e., using a model exclusively specified by (6) and without imposing any functional form for the I(0) disturbances u_t. However, the use of semiparametric methods (e.g., Geweke and Porter-Hudak, 1983; Robinson 1995a,b, etc.) produced results which were very sensitive to the choice of the bandwidth parameter numbers. In that respect, the use of a fully-parametric approach like the one proposed in this paper leads us to clear and concise results, finding evidence against the AK models only for the UK case.

References

Barro, R. (1990), "Government spending in a simple model of endogenous growth", *Journal of Political Economy*, vol. 98, pp. 103-125.

Barro, R. (1991), "Economic growth in a cross section of countries", *Quarterly Journal of Economics*, vol. 106, pp. 407-443.

Barro, R. and X. Sala-i-Martin, (1995), *Economic Growth*, McGraw Hill, New York.

Beran, J., (1995), "Maximum likelihood estimation of the differencing parameter for invertible short and long memory ARIMA models", *Journal of the Royal Statistical Society*, Series B, vol. 57, pp. 659-672.

Beran, J. and N. Terrin, (1996), "Testing for a change of the long memory parameter", *Biometrika*, vol. 83, pp. 627-638.

Binder, M. and M.H. Pesaran, (1999), "Stochastic growth models and their econometric implications", *Journal of Economic Growth*, vol. 4, pp. 139-183.

Bloomfield, P., (1973), "An exponential model in the spectrum of a scalar time series", *Biometrika*, vol. 60, pp. 217-226.

Bos, C.S., P.H. Franses and M. Ooms, (1999), "Long memory and level shifts: reanalyzing inflation rates", *Empirical Economics*, vol. 24, pp. 427-449.

Bos, C.S., P.H. Franses and M. Ooms, (2002), "Inflation, forecast intervals and long memory regression models", *International Journal of Forecasting*, vol. 18, pp. 243-264.

Dickey, D.A. and W.A. Fuller, (1979), "Distribution of the estimators for autoregressive time series with a unit root", *Journal of the American Statistical Association*, vol. 74, pp. 427-431.

Diebold, F.S. and G.D. Rudebusch, (1991), "On the power of Dickey-Fuller tests against fractional alternatives", *Economic Letters*, vol. 35, pp. 155-160.

Diebold, F.S. and A. Inoue, (2001), "Long memory and regime switching", *Journal of Econometrics*, vol. 105, pp. 131-159.

Dufour, J.M. and M. King, (1991), "Optimal invariant tests for the autocorrelation coefficient in linear regressions with stationary or nonstationary errors", *Journal of Econometrics*, vol. 47, pp. 115-143.

Elliott, G., T. Rothemberg and J.H. Stock, (1996), "Efficient tests for an autoregressive unit root", *Econometrica*, vol. 64, pp. 813-839.

Engle, R.F. and A.D. Smith, (1999), "Stochastic permanent breaks", *Review of Economics and Statistics*, vol. 81, pp. 553-574.

Fuller, W.A., (1976), *Introduction to statistical time series*, Wiley, New York, NY.

Geweke, J. and S. Porter-Hudak, (1983), "The estimation and application of long memory time series models", *Journal of Time Series Analysis*, vol. 4, pp. 221-238.

Gil-Alana, L.A., (2000), "Mean reversion in the real exchange rates", *Economics Letters*, vol. 16, pp. 285-288.

Gil-Alana, L.A., (2007), "Fractional integration and structural breaks at unknown periods of time", forthcoming in Journal of Time Series Analysis.

Gil-Alana, L.A. and J. Hualde, (2008), "Fractional integration and cointegration. An overview and an empirical application", In *Palgrave Handbook of Econometrics*, Vol. 2, forthcoming.

Gil-Alana, L.A. and P.M. Robinson, (1997), "Testing of unit roots and other nonstationary hypotheses in macroeconomic time series", *Journal of Econometrics*, vol. 80, pp. 241-268.

Granger, C.W.J., (1980), "Long memory relationships and the aggregation of dynamic models", *Journal of Econometrics*, vol. 14, pp. 227-238.

Granger, C.W.J. and N. Hyung, (2004), "Occasional structural breaks and long memory with an application to the S&P 500 absolute stock returns", *Journal of Empirical Finance*, vol. 11, pp. 399-421.

Hassler, U. and J.Wolters, (1994), "On the power of unit root tests againts fractional alternatives", *Economic Letters*, vol. 45, pp. 1-5.

Jones, C.I., (1995), "Time series tests of endogenous growth models", *Quarterly Journal of Economics*, vol. 110, pp. 495-525.

Kocherlakota, N.R. and K.M. Yi, (1997), "Is there endogenous long-run growth? Evidence from the United States and the United Kingdom", *Journal of Money, Credit and Banking*, vol. 29, pp. 235-262.

Kwiatkowski, D., P.C.B. Phillips, P. Schmidt, and Y. Shin, (1992), "Testing the null hypothesis of stationarity against the alternative of a unit root", *Journal of Econometrics*, vol. 54, pp. 159-178.

Lau, S.-H. P., (1999), "I(0) In, integration and cointegration out: time series properties of endogenous growth models", *Journal of Econometrics*, vol. 93, pp. 1-24.

Lee, D. and P. Schmidt, (1996), On the power of the KPSS test of stationarity against fractionally-integrated alternatives, Journal of Econometrics 73, 285-302.

Lobato, I.M. and N.E. Savin, (1998), "Real and spurious long memory properties of stock market data", *Journal of Business and Economics Statistics*, vol. 16, pp. 261-268.

Lucas, R., 1988, "On the mechanics of economic development", *Journal of Monetary Economics*, vol. 22, pp. 3-41.

Maddison, A., (1992), "A long run perspective on saving", *Scandinavian Journal of Economics*, vol. 94, pp. 181-196.

Maddison, A., (2001), *The world economy: A millennial perspective*, Paris, OECD.

Mankiw, G., P. Romer, and D.N. Weil, "1992, A contribution to the empirics of economic growth", *Quarterly Journal of Economics*, vol. 107, pp. 407-437.

Marinucci, D. and P.M. Robinson, (1999), "Alternative forms of fractional Brownian motion", *Journal of Statistical Planning and Inference*, vol. 80, pp. 111-122.

Michelacci, C. and Zaffaroni, P, (2000), "(Fractional) beta convergence", *Journal of Monetary Economics*, vol. 45, pp. 129-153.

McGrattan, E.R., (1998), "A defense of AK growth models", *Federal Reserve Bank of Minneapolis Quarterly Review*, vol. 22, pp. 13-27.

Müller, U., (2005), "Size and power of tests for stationarity in highly autocorrelated time series", *Journal of Econometrics*, vol. 128, pp. 195-213.

Ng., S. and P. Perron, (2001), "Lag length selection and the construction of unit root tests with good size and power", *Econometrica*, pp. 69, pp. 1529-1554.

Phillips, P.C.B. and P. Perron, (1988), "Testing for a unit root in a time series regression", *Biometrika*, vol. 75, pp. 335-346.

Rebelo, S., (1991), "Long-run policy analysis and long-run growth", *Economic Journal*, vol. 38, pp. 543-559.

Robinson, P.M., (1978), "Statistical inference for a random coefficient autoregressive model", Scandinavian Journal of Statistics, vol. 5, pp. 163-168.

Robinson, P.M., (1994), "Efficient tests of nonstationary hypotheses", *Journal of the American Statistical Association*, vol. 89, pp. 1420-1437.

Robinson, P.M., (1995a), "Log-periodogram regression of time series with long range dependence", *Annals of Statistics*" vol. 23, pp. 1048-1072.

Robinson, P.M., (1995b), "Gaussian semiparametric estimation of long range dependence", *Annals of Statistics*, vol. 23, pp. 1630-1661.

Romer, P., (1986), "Increasing returns and long-run growth", *Journal of Political Economy*, vol. 94, pp. 1002-1037.

Solow, R.M., (1956), "A contribution to the theory of economic growth", *Quarterly Journal of Economics*, vol. 70, pp. 65-94.

Sowell, F. (1992), "Maximum likelihood estimation of stationary univariate fractionally integrated time series models", *Journal of Econometrics*, vol. 53, pp. 165-188.