Advertising and endogenous exit in a differentiated duopoly

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1 Introduction

The role of advertising in the competition among firms has always represented an interesting issue that has been studied following the different aspects of its nature. Advertising is *informative* when its function is to provide information about the availability of a certain product/brand and its characteristics. But advertising has also a *persuasive* content, when the investing firm aims at convincing customers that what they really want is its particular variety, thus increasing product differentiation. Butters (1977), Grossman and Shapiro (1984) and Schmalensee (1978) deal with advertising that carries basic product information, while Milgrom and Roberts (1986) and Nelson (1974) focus on information as a signal of quality. The issue of persuasive advertising is instead discussed in Dixit and Norman (1978) and von der Fehr and Stevik (1998).

Moreover, advertising could give rise to barriers to entry for newcomers that would need to spend a substantial amount of money to overcome the reputation of the incumbents. Many authors focused on the issue of strategic advertising as an instrument to deter entry (Bagwell and Ramey, 1988 and 1990). Schmalensee (1983) considered a duopoly two-stage Cour-

* We thank Rabah Amir, Claude d'Aspremont, Luca Lambertini, Jacques Thisse, Vincent Vannetelbosch, Xavier Wauthy as well as two ananymous referees for useful comments and suggestions. We are also grateful to seminar participants at the University of Bologna, the 2003 SMYE and the 2003 EARIE meeting. Both authors gratefully acknowledge financial support from CORE. The usual disclaimer applies.

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not model where an initial investment in advertising was able to deter the entry of new rivals. More recently, Ishigaki (2000) found that Schmalensee's results did not hold in a similar Bertrand setting.

In this paper we consider a two stage model of duopolistic competition with horizontally differentiated goods. Firms first decide whether to invest in advertising or not and then compete in the market by setting prices. We deal with advertising that shifts demand curves, and in particular we focus on two effects, namely the market enlargement of a "non well-known" product and the predatory interaction that arises in advertising games.

The first effect comes from the consideration that consumers might not be fully aware of the presence of certain types of products in the market. A firm that develops a "novelty" must invest resources to explain which kind of product has become available. The creation of a new market, or the enlargement of an existing one, could represent nonetheless an advantage for a potential rival, that would benefit from an information spill-over that *shifts the demand curve upward for all those kinds of goods.* The overall effect, that we call "market enlargement effect", provides an advantage to all firms as sellers of that type of product. This case is particularly suited to describe informative advertising for search goods like new hi-tech products. A good example can be traced in the DVD market expansion boosted by Sony's massive advertising campaign.

The second effect is related to the conventional view that advertising creates "brand loyalty" and "goodwill" that stick to a determined brand (Kaldor, 1950). In fact, by engaging in advertising, a firm increases its own demand while at the same time *it reduces the demand of the rival,* giving rise to what we call "stealing effect". An example is given by the use of comparative advertising, through which a firm compares the characteristics of its product with those of the competitors.[1]

We depart from the standard assumption that advertising is either *cooperative* or *predatory* (see Friedman, (1983); Martin, (2002)). In the literature, in fact, advertising is defined as cooperative when it shifts the demand curve for the product at large, thus creating a benefit for all firms in an industry. It is defined as predatory when a firm's advertising shifts its own demand curve at the expenses of the rivals. A very simple way to represent such a distinction is to adopt a unique parameter that, depending on the sign, determines which effect is at stake.

By contrast, we analyse situations where advertising has *both* cooperative and predatory features by modeling simultaneously the market enlargment and the stealing effect.[2] As we will see, the relative strength of these two components determine the outcome of the game in such a rich way that

[1] The use of comparative advertising progressively increased both in the United States and, more recently, also in the European Union. According to Muehling *et al.* (1990), in the United States around 40% of all advertising is comparative.

[2] For an alternative (dynamic) framework in which advertising is both cooperative and predatory, see Piga (1998).

cannot be reduced to a single parameter story. Depending on particular values, both symmetric and asymmetric equilibria could arise. Among them, two outcomes are of particular interest : *(i)* a *coordination game*, in which both firms deciding to invest and both deciding not to invest are simultaneously equilibria of the game; *(ii)* a *chicken game*, in which only one firm invests in equilibrium with the second one possibly driven (endogenously) out of the market. Furthermore, we provide some insight about the Pareto optimality (from firms' standpoint) of market outcomes that will enable us to identify *prisoner dilemma* situations.

In our setting, particular attention will be paid to the role of product differentiation in determining the equilibrium level of advertising, as well as to shed some light on the problem of coordination. Crucially, and that is why we decided to deal with both price competition and differentiation, the degree of product substitutability has a direct impact on advertising decisions, and in particular on the capability to steal market shares from rivals, that could not be easily modeled in the standard quantity competition framework. In some settings, the degree of product differentiation is affected by the advertising strategy. However, it is reasonable to think that each market has some "predeterminated" degree of product substitutability that influence firms decisions and in particular the advertising effort. Furthermore, price competition is a more natural assumption in a market where products are differentiated. Grossman and Shapiro (1984), for example, consider a differentiation duopoly model with price competition and show that advertising is positively related to the degree of product differentiation. Other advertising models dealing with these two features can be found in Butters (1977), Wolinsky (1984) and von der Fehr and Stevik (1998). In our framework we will see that (by taking into account asymmetric outcomes), the relation between the equilibrium level of advertising and the degree of product differentiation is not as simple.

The remainder of the paper is organized as follows. In the following section we will introduce the analytic features of the model. Section 3 analyzes the second stage price game while in Section 4 we solve (backward) the first stage advertising game. Section 5 provides a complete characterization of the equilibria of the game in terms of parameters and then turns to their economic interpretation. Section 6 finally provides conclusions and directions for further research.

2 The Model

Consider an industry composed of two a priori identical firms that produce a differentiated good. They are engaged in the following two-stage game. In the first stage, each firm decides the resources to be invested in advertising, while in the second stage they compete in prices. We further assume that firms can only decide to advertise or not, incurring a fixed cost that we normalize to one, while the strategy set of each firm for the second stage price game is the entire $\Re^+$. As for the solution of the game, we restrict our attention to subgame perfect Nash equilibria.

Ideally, it would be preferable to use a more sophisticated set of alternatives for advertising decisions. However, as some other types of investments, advertising has a discrete nature in the sense that it is sometimes more important to decide whether to invest or not rather than the exact amount to be spent on it. Indeed, as pointed out is Sutton (1991), advertising belongs to those "endogenous" sunk costs that determine the strategic features of a market. Furthermore, as we will see afterwards, our simple binary assumption will allow for a complete characterization of all possible equilibria of the game.

In order to keep things as simple as possible, marginal costs are supposed to be zero and there are no fixed costs in production. The demand structure turns out to be extremely important in our analysis. As we want to deal with both product differentiation and price competition, a natural starting point is the linear demand function: [3]

$$q_i = a - bp_i + c\,(p_{-i} - p_i) = a - (b + c)\,p_i + c\,p_{-i} \tag{1}$$

where a stands for market size and b represents the surplus of the own price over the cross price effect. The parameter c is an (inverse) measure of product differentiation; the higher c, the higher the substitutability between the products, given the stronger impact of a price difference.

As we mentioned in the Introduction, the kind of advertising we are interested in gives rise simultaneously to two separate effects. First, demand curves shift outward, thus enlarging the market for the product at large. The market enlargement effect is then modeled through a symmetric shift of the parameter a in the demand function of the two firms. Secondly, by doing advertising, each firm creates "brand loyalty" and "goodwill" for its own product, thus drawing consumers away from rivals. In our setting, firms cannot choose the nature of their advertising effort, i.e. they cannot determine how much market enlargment vs stealing effect they can induce. This is certainly an issue that deserves attention but which is beyond the scope of this paper.

Before proceeding further, we should mention that, in general, any demand enhancing investment activity gives rise to complex issues regarding the utility function from which the demand is derived. This is a well known microfoundated problem that many models share. Another problem, initially pointed out by Dixit and Norman (1978), arises when advertising shifts the individual utility function from which the demand curve is derived. The usual criticism regarding the welfare comparison also applies to the profit comparisons that we will carry out in our model.

[3] The proposed linear demand function is consistent with utility maximizing consumers with quadratic utility functions (see Shubik and Levitan (1980)).

The lower the degree of product differentiation (high values of c), the higher the impact of this second effect. In fact, as long as products are perceived as highly substitutes, firms have a strong incentive to attach an element of differentiation on their own good through advertising. In order to capture the stealing effect, we make the hypothesis that a firm doing advertising receives a demand gain $c\alpha$, while imposing at the same time an equivalent demand cut $-c\alpha$ to its rival.[4]

Although our two firms are *a priori* identical, the game could admit asymmetric equilibria. In particular, we have to account for the possibility that firm i sets a price lower or equal to a limit price p_i^l, pushing the other firm (endogenously) out of the market. This clearly raises the problem of defining the demand received by the remaining firm. Starting from equation (1), the solution that we adopt is to define demand in the limit pricing domain in such a way that continuity is preserved for all admissible price strategies. This leads to the following demand system:

$$q_i(p_i,p_{-i},I_i,I_{-i}) = \left\{ \begin{array}{ll} \max\{a(I_i,I_{-i}) - bp_i + c[p_{-i} - p_i + \alpha_i(I_i) - \alpha_{-i}(I_{-i})], 0\}, & \text{if } p_i > p_i^l \\ \max\{2a(I_i,I_{-i}) - bp_i - b\varphi(p_i), 0\}, & \text{if } p_i \leqslant p_i^l \end{array} \right\} \tag{2}$$

where:

$$I_i = \{0,1\} \quad \text{for } i = 1,2$$

$$a(I_i,I_{-i}) = a(I_i + I_{-i}) = \begin{cases} \underline{a} & \text{if } I_i + I_{-i} = 0 \\ \underline{a} + \gamma & \text{if } I_i + I_{-i} = 1 \\ \underline{a} + 3\gamma/2 & \text{if } I_i + I_{-i} = 2 \end{cases}$$

$$\alpha_i = \begin{cases} 0 & \text{if } I_i = 0 \\ \alpha & \text{if } I_i = 1 \end{cases} \quad \text{for } i = 1,2$$

$$\varphi(p_i) = \max\left\{ \frac{a(I_i + I_{-i}) + c[\alpha_{-i}(I_{-i}) - \alpha_i(I_i)]}{b+c} + \frac{c}{b+c}p_i, 0 \right\}$$

$$\underline{a}, b, \alpha, \gamma, c > 0$$

The demand system (2) makes clear the limit pricing issue by considering the two alternatives that depend on firm i' price decision. When $p_i > p_i^l$ both firms are active on the market, while for $p_i \leqslant p_i^l$ only firm i makes positive sales, being the rival driven out of the market. However, it is important to stress that the requirements of continuity and non-negativity of demands for all admissible prices leads to a 'unique' definition of demand in the limit price domain.[5]

As for the different components entering into demand system (2), the binary variable I_i represents advertising strategies: firm i could either

[4] A more realistic modelisation would require, for example, a loss for the rival equal to $-\theta c\alpha$, with $\theta \in [0,1]$. However, this would burden the calculations with another parameter that does not add particular insight in our analysis. The aim of this paper is in fact to stress the interaction between the two aformentioned effects more than their exact effectiveness.

[5] Further details on how demand function (2) is obtained from equation (1) as well as a complete discussion on the continuity requirements are given in Appendix A.1.

advertise ($I_i = 1$) or not ($I_i = 0$). The market enlargement effect is captured by $a(I_1, I_2)$ and depends upon total investment : I_1+I_2. If no firm advertises, then $a(I_1 + I_2)$ is stuck to a basic level $\underline{a}$. If only one firm advertises, then $a(I_1+I_2)$ increases to $\underline{a}+\gamma$, while if the other does the same the new marginal increase is just $(1/2)\gamma$. This series of (geometric) diminishing increments is a simple way to model the fact that there cannot be unlimited expansion of the market.[6] We also tried alternative specifications for the decreasing expansion of the market finding very similar results. The stealing effect is instead parameterized by α_i, that could be either zero or α. If only one firm advertises then, as long as the other one is actually on the market ($p_i > p_i^l$), its demand increases by $c\alpha$ while the demand of the rival decreases by the same amount. As argued in the previous section, the magnitude of the stealing effect is in fact positively related to product substitutability. Indeed, if two goods are perceived as very different from each other, the capability of advertising to shift demand from one firm to the other is very limited. On the other hand, if prices and advertising strategies are such that only firm i makes positive sales ($p_i \leqslant p_i^l$), its demand depends only on p_i in such a way that continuity in prices is guaranteed.

3 The second stage price game

In equilibrium, there can obviously be just two possibilities : either the two firms sell a positive amount of goods, or just one of them receives a positive demand while the other has a zero output. As we already pointed out, our demand system (2) is continuous for all $p_1, p_2 \in [0, \infty)$ and it is clearly monotone decreasing (increasing) in each firm own (cross) price whenever a firm's demand is positive. Anyway, first-order conditions alone do not suffice to characterize Nash Equilibria in the price game because demand functions have kinks. In fact, demands are just piece-wise linear in both own and cross price, and their slope changes in view of limit pricing. However, in Appendix A.1 we show that this change is "well-behaved" in the sense that the slope is lower (in absolute value) when only one firm sells on the market.

This change in price responsiveness comes from the fact that, when both firms are active on the market, a price reduction by one firm induces not only new customers to buy the product, but also usual clients of the rival to drift to the price-reducing firm. This has a very useful implication for firms' profit functions which, by linearity of demands and absence of variable costs, turn out to be strictly concave whenever quantities are positive. Therefore, we can state the following :

Claim 1 : *If a NE in the second stage price game with both firms making positive sales exists, then first-order conditions are necessary and sufficient to identify it.*

6 In particular, we adopt the geometric series $a(n) = \underline{a} + \sum_i^n \frac{1}{i}\gamma$.

3.1 Case A : None invests

We start by considering the symmetric case where no firm invests in advertising. If both firms receive positive demands, then the demand curves of each firm are (with $I_1 = I_2 = 0$) :

$$q_1 = \underline{a} - b\,p_1 + c\,(p_2 - p_1) \qquad (3)$$
$$q_2 = \underline{a} - b\,p_2 + c\,(p_1 - p_2).$$

Since we assume that marginal costs are zero, and there are no fixed costs in production, profits are :

$$\pi_1 = p_1 q_1 = p_1 \left[\underline{a} - b\,p_1 + c\,(p_2 - p_1)\right] \qquad (4)$$
$$\pi_2 = p_2 q_2 = p_2 \left[\underline{a} - b\,p_2 + c\,(p_1 - p_2)\right].$$

Profits are quadratic in each firm's own price, and by first-order conditions we get equilibrium prices :

$$p_1^A = p_2^A = \frac{\underline{a}}{2b + c} > 0. \qquad (5)$$

As one can see, the demands corresponding to these prices are always positive and the equilibrium profits (obtained using equilibrium prices p_1^A and p_2^A) are :

$$\pi_1^A = \pi_1(p_1^A, p_2^A) = \pi_2^A = \pi_2(p_1^A, p_2^A) = \frac{\underline{a}^2(b + c)}{(2b + c)^2} > 0. \qquad (6)$$

According to Claim 1, the pair $\{p_1^A, p_2^A\}$ thus represents the unique "SPE" characterized by both firms making positive sales. On the other hand, it is straightforward to check that each firm can always find here, whatever the other does, a strictly positive price such that it receives some demand and makes strictly positive profits. This clearly means that there is no room for equilibria with just one active firm, i.e. :

Lemma 1 : *In the subgame where no firm invests, there is a unique Nash equilibrium in pure strategies given by* $\{p_1^A, p_2^A\}$.

3.2 Case B : Only one firm invests

We now examine the case where only one firm invests in advertising. Without loss of generality, we assume that firm 1 invests while firm 2 does not : $I_1 = 1$ and $I_2 = 0$. In case of positive sales for both firms, the demand curves are given by :

$$q_1 = \underline{a} + \gamma - b\,p_1 + c\,(p_2 - p_1 + \alpha) \qquad (7)$$
$$q_2 = \underline{a} + \gamma - b\,p_2 + c\,(p_1 - p_2 - \alpha).$$

Compared to the previous case, where none of them invested in advertising, both firms enjoy here an increase in demand equal to γ due to the market enlargement effect. However, due to the stealing effect, firm 1 receives an additional gain $c\alpha$, while imposing a penalty $-c\alpha$ to the rival. Intuitively, this stealing component allows firm 1 to increase its price with respect to firm 2. Furthermore, if α is strong enough, the latter could (eventually) be unable to get any market share. Profits are given by:

$$\begin{aligned} \pi_1 &= p_1 q_1 - 1 = p_1 \left[\underline{a} + \gamma - b p_1 + c(p_2 - p_1 + \alpha)\right] - 1 \\ \pi_2 &= p_2 q_2 = p_2 \left[\underline{a} + \gamma - b p_2 + c\,(p_1 - p_2 - \alpha)\right] \end{aligned} \tag{8}$$

By first-order conditions we get prices:

$$p_{1Ac}^{B} = \frac{(\underline{a} + \gamma)\,(2b + 3c) + c\alpha(2b + c)}{(2b + c)\,(2b + 3c)} > 0 \tag{9}$$

$$p_{2Ac}^{B} = \begin{cases} \dfrac{(\underline{a} + \gamma)(2b + 3c) - c\alpha(2b + c)}{(2b + c)(2b + 3c)} & \text{if } \alpha < \alpha_a \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where the subscript Ac indicates that both firms are active on the market. One can easily check that $q_2^B(p_{1Ac}^B, p_{2Ac}^B)$ is positive iff $\alpha < \alpha_a = \dfrac{(2b + 3c)(\underline{a} + \gamma)}{(2b + c)c}$. Following Claim 1, when such a condition on α is satisfied, then the pair of strategies $\{p_{1Ac}^B, p_{2Ac}^B\}$ that we have identified by first-order conditions is the unique NE characterized by both firms making positive sales. Not surprisingly, the relative advantage of the investing firm should be small enough ($\alpha < \alpha_a$) in order to have both firms selling at equilibrium. The associated equilibrium profits are:

$$\begin{aligned} \pi_{1Ac}^{B} = {} & \frac{1}{(2b + c)^2(2b + 3c)^2} \\ & \left\{ \left(\underline{a}^2 + \gamma^2\right)(b + c)(2b + 3c)^2 + (2b + c)^2 \left[c^2\alpha^2(b + c) - (2b + 3c)^2\right] + \right. \\ & \left. +2(b + c)(2b + 3c)\left[c\alpha(2b + c)\,(\gamma + \underline{a}) + \underline{a}\gamma(2b + 3c)\right] \right\} \end{aligned} \tag{11}$$

$$\pi_{2Ac}^{B} = \frac{(b + c)\left[(\underline{a} + \gamma)\,(2b + 3c) - c\alpha(2b + c)\right]^2}{(2b + c)^2(2b + 3c)^2}. \tag{12}$$

We should now turn to the study of equilibria characterized by just one active firm. It is easy to check that only firm 1 can always find, whatever the other does, a strictly positive price such that it still receives some demand. Therefore, the only possibility is that firm 2, which does not advertise its product, finds itself out of business. Furthermore, we can prove that (without loss of generality) one could simply focus on equilibria in which $p_2 = 0$:

Lemma 2 : *Consider the subgame where just firm 1 invests. If* $\{p_1^*, p_2^*\}$ *is a NE with* $p_1^*, p_2^* > 0$ *and* $q_2(p_1^*, p_2^*) = 0$*, then also* $\{p_1^*, p_2\}$ *is, for any* $p_2 \in [0, p_2^*]$*, a NE with* $q_2(p_1^*, p_2) = 0$*. Furthermore,* $\pi_1(p_1^*, p_2^*) = \pi_1(p_1^*, p_2)$ *and* $\pi_2(p_1^*, p_2^*) = \pi_2(p_1^*, p_2) = 0$ *for any such* $p_2 \in [0, p_2^*]$.

Proof. Suppose that $\{p_1^*, p_2^*\}$ is a NE with $p_1^*, p_2^* > 0$, and $q_2(p_1^*, p_2^*) = 0$. For p_2^* to be a best reply to p_1^*, there should not exist any $p_2 > 0$ such that $q_2(p_1^*, p_2) > 0$. By continuity of our demand system, this implies that also $q_2(p_1^*, 0)$ cannot be positive, and so all $p_2 \in [0, p_2^*]$ are certainly best replies to p_1^*. On the other hand, for any $p_2 \in [0, p_2^*]$, we have that $q_2(p_1^*, p_2) = 0$ and so firm 1's demand does not certainly depend on such p_2 for prices lower or equal than p_1^*, i.e. $q_1(p_1, p_2) = q_1(p_1, p_2^*) = q_1(p_1)$ $\forall p_1 \in [0, p_1^*]$, while for prices $p_1 \in (p_1^*, \infty)$ it satisfies the inequality $q_1(p_1, p_2) \leqslant q_1(p_1, p_2^*)$ that comes from the fact that demand is non-decreasing in the cross price. Being p_1^* a best reply to p_2^* we have $q_1(p_1^*, p_2^*) \geqslant q_1(p_1, p_2^*)$ $\forall p_1$, and using the previous relations we obtain that, for any $p_2 \in [0, p_2^*]$, $q_1(p_1^*, p_2) \geqslant q_1(p_1, p_2)$ $\forall p_1$ so that p_1^* is also a best reply to any such p_2 and in particular to $p_2 = 0$.

Lemma 2 means that, whenever firm 1 pushes the rival out of the market, we are in the same situation (in terms of equilibrium price p_1 and payoffs) as if firm 2 charged a price equal to zero.

In order to study such equilibria we first figure out how firm 1's profit function looks like. Indicating with $p_{1Dt}^B = \alpha - \dfrac{\underline{a} + \gamma}{c}$ the limit price p_1^l corresponding to $p_2 = 0$ we have that, depending on its price p_1, firm 1 may find itself in the domain in which both firms sell something ($p_1 > p_{1Dt}^B$), or in the domain in which it is the only active firm ($p_1 \leqslant p_{1Dt}^B$). As we already know, the two domains correspond to different analytical expressions of the demand function.

If firm 1 prices above the limit price p_{1Dt}^B, its demand (for $p_2 = 0$) will be given by :

$$q_1 = \underline{a} + \gamma - bp_1 + c(-p_1 + \alpha) \tag{13}$$

with profits :

$$\pi_1 = p_1 q_1 - 1 = p_1 \left[\underline{a} + \gamma - bp_1 + c(-p_1 + \alpha)\right] - 1. \tag{14}$$

By first-order conditions we get the unique maximum :

$$\hat{p}_1^B = \frac{\underline{a} + \gamma + c\alpha}{2(b + c)}. \tag{15}$$

Nonetheless, this solution rests on the hypothesis that $q_2 > 0$, which has to be checked. If firm 1 instead prices below the limit price p_{1Dt}^B, it gets a demand (for $p_2 = 0$) and a profit respectively given by :

$$q_1 = 2(\underline{a} + \gamma) - bp_1 \tag{16}$$

$$\pi_1 = p_1 q_1 - 1 = p_1 \left[2(\underline{a} + \gamma) - b p_1\right] - 1. \tag{17}$$

By first-order conditions we obtain the unique maximum :

$$p_{1Mp}^B = \frac{\underline{a} + \gamma}{b} > 0. \tag{18}$$

It is easy to check that, for $\alpha = \alpha_a$, $\hat{p}_1^B = p_{1Dt}^B > 0$ while, for $\alpha > \alpha_a$ ($\alpha < \alpha_a$), $\hat{p}_1^B$ is strictly lower (higher) than p_{1Dt}^B and they are still both positive.

In case of $\alpha > \alpha_a$, this means that for prices bigger than p_{1Dt}^B ($\geqslant \hat{p}_1^B$), the "true" demand firm 1 faces is given by (13) and then profits, given by the concave parabola (14), are decreasing in this range of prices precisely because we are to the right of $\hat{p}_1^B$. We can thus exclude all prices $p_1 > p_{1Dt}^B$ from equilibrium. If firm 1 instead charges a price lower or equal than p_{1Dt}^B, its "true" demand is given by (16), with relative profits given by (17) which is again a concave parabola in p_1 with a unique maximum p_{1Mp}^B. There are consequently 2 possible scenarios, represented respectively in Figures 1a and Figure 1b, referring to firm 2 being out of the market :

- When $\alpha_a \leqslant \alpha < \alpha_b = \dfrac{(b+c)(\underline{a}+\gamma)}{bc}$, we have $p_{1Mp}^B > p_{1Dt}^B$. Consequently, for prices higher than p_{1Dt}^B, firm 1's profit corresponds to the decreasing branch of the parabola (14), while in the other case it corresponds to the increasing branch of the parabola (17). The two parabolas touch each other at $p_1 = p_{1Dt}^B$, that is the unique maximum.
- When $\alpha \geqslant \alpha_b$, we have $p_{1Mp}^B \leqslant p_{1Dt}^B$. Consequently, for prices higher than p_{1Dt}^B firm 1's profit corresponds again to the decreasing branch of the parabola (14), while in the other case it corresponds to the decreasing branch of the parabola (17). The two parabolas touch each other at $p_1 = p_{1Dt}^B$, and the unique maximum is reached for $p_1 = p_{1Mp}^B$.

Obviously, $p_2 = 0$ is a best reply to limit prices p_{1Dt}^B and p_{1Mp}^B and so all the conditions needed in order to have a Nash Equilibrium are satisfied.[7] We can finally state the following :

Lemma 3 : *in the subgame when only firm one invests, there is a "unique" equilibrium (in terms of payoffs and price p_1) in pure strategies given by*

1. $\{p_{1Ac}^B, p_{2Ac}^B\}$ *when* $\alpha < \alpha_a$;
2. $\{p_{1Dt}^B, 0\}$ *when* $\alpha_a \leqslant \alpha < \alpha_b$;
3. $\{p_{1Mp}^B, 0\}$ *when* $\alpha \geqslant \alpha_b$.

[7] For the sake of completeness, in case $\alpha < \alpha_a$ we have that for prices bigger than p_{1Dt}^B ($< \hat{p}_1^B$), the 'true' demand firm 1 faces is still given by (13) but profits, represented by the concave parabola (14), are increasing in this range of prices because we are now to the left of $\hat{p}_1^B$, which represents the unique maximum. Consequently, as limit pricing is never a best strategy for firm 1, we can exclude equilibria with only one active firm whenever $\alpha < \alpha_a$.

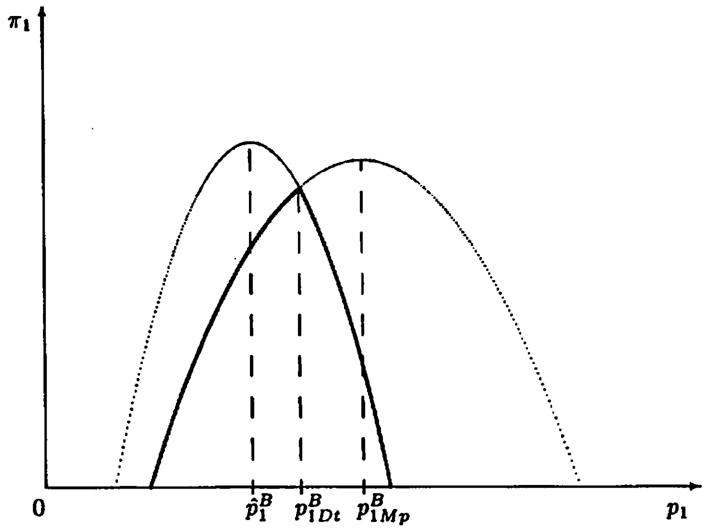

Figure 1a : *Firm 1's profit function for $\alpha_a < \alpha \leqslant \alpha_b$ (bold line).*

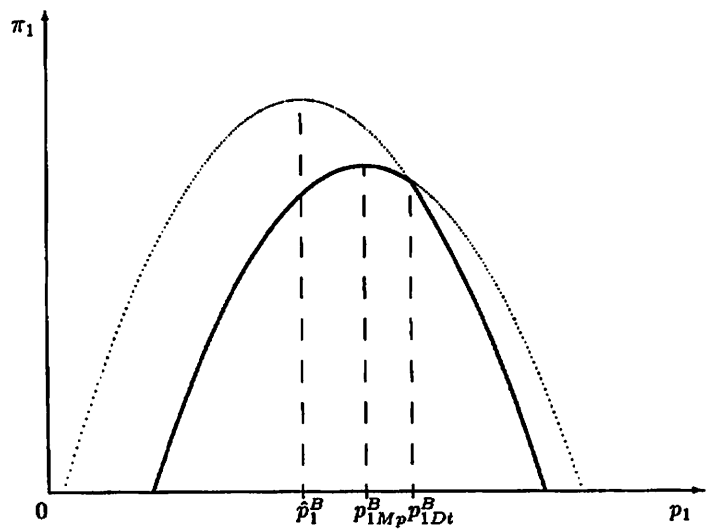

Figure 1b : *Firm 1's profit function for $\alpha > \alpha_b$ (bold line).*

Results of Lemma 3 are actually quite intuitive. If the stealing effect is sufficiently small ($\alpha < \alpha_a$), then both firms makes positive sales in equilibrium. Anyway, beyond the critical value α_a, the advertising firm finds it convenient to charge a limit price such that its competitor is (endogenously) squeezed out of the market.[8] In particular, if $\alpha_a \leqslant \alpha < \alpha_b$ then firm 1 charges the highest limit price, while if the stealing effect is really strong ($\alpha \geqslant \alpha_b$), firm 1 is able to take the all market by setting a kind of "monopoly" price p^B_{1Mp}. It is interesting to note that each firm's best reply is continuous with respect to α (as well as with respect to the other parameters), and the same applies to equilibrium profits, due to the continuity of demand.

For future reference, we write the equilibrium profits of firms in the three subcases considered :

- $\alpha < \alpha_a \Longrightarrow p^B_1 = p^B_{1Ac}, p^B_2 = p^B_{2Ac}$ and equilibrium profits π^B_1, π^B_2 are given by (11) and (12);
- $\alpha_a \leqslant \alpha < \alpha_b \Longrightarrow p^B_1 = p^B_{1Dt}, p^B_2 = 0$ and equilibrium profits are:

$$\pi^B_1 = \pi^B_{1Dt} = \frac{(\alpha c - \gamma - \underline{a})\left[2c\left(\underline{a} + \gamma\right) - b\left(\alpha c - \gamma - \underline{a}\right)\right]}{c^2} - 1, \tag{19}$$

$$\pi^B_2 = \pi^B_{2Dt} = 0;$$

- $\alpha \geqslant \alpha_b \Longrightarrow p^B_1 = p^B_{1Mp}, p^B_2 = 0$ and equilibrium profits are:

$$\pi^B_1 = \pi^B_{1Mp} = \frac{\left(\underline{a} + \gamma\right)^2}{b} - 1, \tag{20}$$

$$\pi^B_2 = \pi^B_{2Mp} = 0.$$

Obviously, due to the symmetric structure of the game, in the case where only firm 2 invests in advertising, we obtain the reversed equilibrium prices and payoffs.

3.3 Case C : Both firms invest

We finally consider the (symmetric) case where both firms decide to invests in advertising. The demand curves of each firm are (with $I_1 = I_2 = 1$) :

$$q_1 = \underline{a} + \frac{3}{2}\gamma - b p_1 + c\left(p_2 - p_1\right) \tag{21}$$

$$q_2 = \underline{a} + \frac{3}{2}\gamma - b p_2 + c\left(p_1 - p_2\right).$$

Now only the market enlargement effect appears, while the strategic effect is reciprocally cancelled out by the investment of the two firms. Profits are

[8] Amir (2000) found conditions leading to endogenous exit in a two-period symmetric Cournot duopoly with R&D returns to process innovation.

given by:

$$\pi_1 = p_1 q_1 = p_1 \left[\underline{a} + \frac{3}{2}\gamma - b p_1 + c(p_2 - p_1) \right] - 1 \tag{22}$$

$$\pi_2 = p_2 q_2 = p_2 \left[\underline{a} + \frac{3}{2}\gamma - b p_2 + c(p_1 - p_2) \right] - 1.$$

By first-order conditions we get prices:

$$p_1^C = p_2^C = \frac{2\underline{a} + 3\gamma}{4b + 2c} > 0 \tag{23}$$

The corresponding demands are always positive and the equilibrium profits are:

$$\pi_1^C = \pi_2^C = \frac{(b+c)\left(2\underline{a} + 3\gamma\right)^2}{4(2b+c)^2} - 1 \tag{24}$$

Following again Claim 1, the pair $\{p_1^C, p_2^C\}$ represents the unique subgame Nash equilibrium characterized by both firms making positive sales. Furthermore, it is easy to check that we are here in the same situation as for case A, and then:

Lemma 4: *In the subgame when both firm invest, there is a unique Nash equilibrium in pure strategies given by* $\{p_1^C, p_2^C\}$.

4 The advertising game

In the last section, we have dealt with the equilibria of the different price subgames. We have seen that, for every parameter value a unique (in term of payoffs) NE exists.[9] Now, given the binary nature of the advertising choice, we can solve backward the first stage with a simple 2x2 matrix containing equilibrium payoffs from the second stage. Uniqueness in the latter payoffs makes it possible to have a unique representation of this matrix, that we show in Table 1:

		firm 2 0		*firm 2* 1	
firm 1	0	$\pi_1^A = \pi_2^A$		π_2^B	π_1^B
	1	π_1^B	π_2^B	$\pi_1^C = \pi_2^C$	

Table 1

[9] From now on, if not elsewhere specified, we refer to uniqueness in terms of payoffs.

We can first remark that, due to the symmetric structure of the payoffs, at least one "SPE" will always exist. Suppose, on the contrary, that no "SPE" exists. For $(0,0)$ not to be a SPE we need $\pi_i^B > \pi_i^A$. Assume now that also $(1,1)$ is not a "SPE", hence $\pi_2^B > \pi_i^C$. But when these two inequalities on profits hold simultaneously, then $(1,0)$ and $(0,1)$ are "SPE" and this contradicts the claim that no "SPE" exists.

Before going into a detailed analysis of our "SPE", we compare the payoffs appearing in the principal diagonal. This will shed light on the Pareto efficiency of the NE from firms' standpoint as well as on the qualitative nature of the game. One may easily check that

$$\pi_i^C \geqslant \pi_i^A \text{ iff } \gamma \geqslant \gamma_1, \text{ where } \gamma_1 = \frac{2}{3}\left[\frac{\sqrt{\underline{a}^2(b+c)+(2b+c)^2}}{\sqrt{b+c}} - \underline{a}\right] > 0. \quad (25)$$

Obviously, both firms gain in investing when the enlargement of the market due to advertising is big enough. In this case, the demand coming from new consumers boosts firms' demand and allows them to recoup their initial investment. Interestingly, it may be the case that the two firms invest in equilibrium while it would have been better not to invest, or the other way round, thus giving rise to *prisoner dilemma* outcomes. This can be simply assessed by means of parameter γ_1, which will therefore play a fundamental role throughout the following analysis.

The simple structure of the game is such that we can quite easily characterize all possible situations. We can in fact encounter just four outcomes. Omitting cases of weak inequalities, we know that $(0,0)$ is a "SPE" iff $\pi_i^A > \pi_1^B$, while $(1,1)$ is a "SPE" iff $\pi_i^C > \pi_2^B$, and thus combining the two we get :

1. When only $\pi_i^A > \pi_1^B$ holds, then $(0,0)$ is the unique "SPE" of the game. The two firms do not invest in advertising and, depending on the value of γ, we could possibly obtain a *prisoner dilemma* game.
2. When only $\pi_i^C > \pi_2^B$ holds, then $(1,1)$ turns out to be the unique "SPE" of the game. Again, depending on γ, we may have or not a *prisoner dilemma*.
3. If both conditions hold together, we obtain a *coordination game* with two "SPE" along the principal diagonal.
4. Lastly, if both these conditions are not satisfied, we get a *chicken game* with two asymmetric "SPE" along the secondary diagonal characterized by only one firm investing in advertising.

To link equilibrium profits with the parameters of the model, we have to consider three different expressions associated to π_1^B, depending on the value taken by α. Bearing this in mind, we can give necessary and sufficient conditions on (α, γ) for $(0,0)$ to be a "SPE" :

Proposition 1 $(0,0)$ *is a "SPE" for sufficiently low combinations between the values of* α *and the ones of* γ*. In particular : (i) when* $\alpha \geqslant \alpha_b$*, we need*

$\gamma \leqslant \gamma_2$; (ii) when $\alpha_a \leqslant \alpha < \alpha_b$, we need either $\gamma \leqslant \gamma_2$ or, if $\gamma > \gamma_2$, then $\alpha \leqslant \alpha_c$ ($< \alpha_b$); (iii) when $0 < \alpha < \alpha_a$, we need either $\gamma \leqslant \gamma_3$ ($> \gamma_2$) or, if $\gamma > \gamma_3$, then $\alpha \leqslant \alpha_d$ ($< \alpha_a$). When $\gamma \geqslant \gamma_4$ ($> \gamma_3$), $(0,0)$ is never an equilibrium, independently of α. Moreover, this Nash Equilibrium, when it exists, turns out to be Pareto dominant from firms' standpoint for sufficiently low values of γ ($\gamma < \gamma_1$), otherwise the game is of a prisoner dilemma type.

Proof see Appendix A.2. □

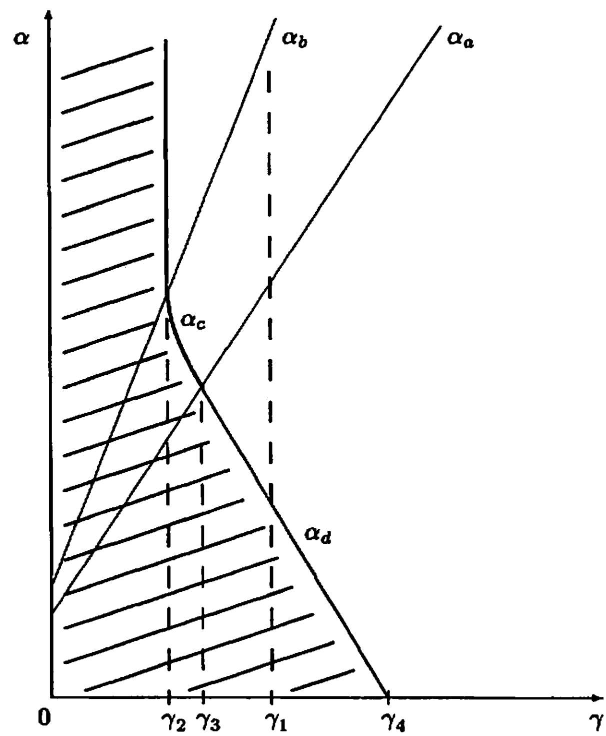

Figure 2 : *The equilibrium* $(0,0)$

The dashed area in Figure 2 indicates those values that sustain $(0,0)$ as a "SPE" in the (α,γ) space.[10] As one can see, both firms decide not to invest when the combination of the market size effect and the strategic stealing effect is weak enough. There is, indeed, a certain degree of substitution in the two effects. A strong stealing gain α could be compensated with a weakening of the market enlargement in order for $(0,0)$ to be a "SPE". However, for very high values of γ ($\gamma \geqslant \gamma_4$), whatever α is, $(0,0)$ is never a "SPE". Furthermore, when it exists as an equilibrium, $(0,0)$ is Pareto

[10] Figure 2 has been depicted using $c = 1$, $b = 1$, and $\underline{a} = 0.3$.

dominant for firms only when the market size effect is sufficiently weak, i.e. when $\gamma < \gamma_1$. As introduced above, Pareto efficiency is evaluated by the parameter γ_1, that compares the two symmetric outcomes of the game, π_i^A *vs* π_i^C.

Interestingly, a *prisoner dilemma*, indicated by the portion of the dashed area on the right of γ_1, appears when the perspectives of market enlargement are quite favorable, but the stealing gain is limited. One firm alone has no advantage to invest in advertising since it does not steal that much from the other firm which, by contrast, would enjoy from the expansion of the market without paying any cost for it. Although firms would be better off by both investing and enlarge substantially the market, they refrain from doing so.

Let us now consider the equilibrium $(1,1)$. By evaluating π_i^C *vs* π_2^B, and taking into account the restrictions on both profit functions, we can conclude that in the parameter space (α, γ) :

Proposition 2 $(1,1)$ *is a "SPE" for sufficiently high combinations between the values of* α *and those of* γ. *In particular, we need at least that* $\gamma \geqslant \gamma_5$ *and either* $\alpha \geqslant \alpha_a$, *or, when* $\alpha < \alpha_a$, $\alpha \geqslant \alpha_e$ $(< \alpha_a)$. *When* $\gamma \leqslant \gamma_5$, $(1,1)$ *is never an equilibrium, independently of* α. *On the contrary, if* $\gamma \geqslant \gamma_6$ $(> \gamma_5)$, $(1,1)$ *is always a "SPE". Such a solution represents a Pareto dominant strategy for firms for sufficiently high values of* γ $(\gamma > \gamma_1)$, *otherwise it gives rise to a prisoner dilemma.*

Proof see Appendix A.3. □

The dotted area in Figure 3 describes the equilibrium conditions in the (α, γ) space.[11] Contrary to before, both firms invest in equilibrium when the combination of the two effects is strong enough. There is, again, a certain degree of substitution between α and γ. When the stealing effect is weak, $(1,1)$ constitutes a "SPE" of the game only if the market expansion translates into a considerable increase in firms' profits. However, if γ is big enough $(\gamma > \gamma_6)$, then, whatever is α, $(1,1)$ is always a "SPE".

Turning to Pareto efficiency and evaluating π_i^C *vs* π_i^A, from (25) we know that $(1,1)$ is Pareto dominant from firms' standpoint only whenever $\gamma > \gamma_1$. On the contrary, a *prisoner dilemma* situation arises when $\gamma < \gamma_1$ and it is indicated in Figure 3 by the portion of the dotted area on the left of γ_1. Its nature is the mirror image of the one found in the previous case, given that it implies a sufficiently strong stealing gain. Even if both firms would be better off without doing advertising because the market expansions possibilities are quite limited $(\gamma < \gamma_1)$, they decide to advertise at equilibrium because they are fully aware of the substantial gain (loss) of being the only advertising (non-advertising) firm.

Combining Proposition 1 and 2, we can fully characterize the four possible outcomes of the model in terms of the parameters. In the next section

[11] Figure 3 has also been drawn using $c = 1$, $b = 1$, and $\underline{a} = 0.3$.

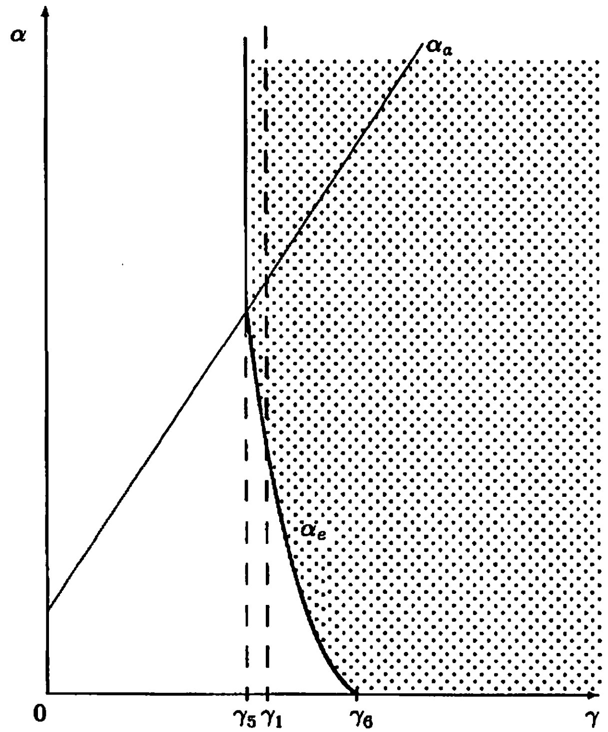

Figure 3 : *The equilibrium* (1, 1)

we will give some insights on how these equilibria configurations react to changes in parameters as well as their underlying economic interpretation.

5 Further results and economic interpretations

In the previous section we gave necessary and sufficient conditions on the two-dimensional parametric space (α, γ) such that $(0,0)$ and $(1,1)$ are "SPE". Following Propositions 1 and 2, we reasonably expect to find that, when there are small incentives for firms to advertise (i.e. low values of α and γ), $(0,0)$ is the only equilibrium of the game. By contrast, for sufficiently high values of α and γ, we expect $(1,1)$ to be the only outcome. Now, what is not clear is what happens in intermediate situations. Both a *coordination* and a *chicken game* will be possible, but conditions leading to each outcome still remain unknown at this stage.

In order to shed some light on the forces underpinning the game, we resort to comparative statics analysis. This task turns out to be extremely difficult because equilibria are characterized in the two-dimensional space

(α, γ) and we need to consider simultaneously all the different threshold values appearing in Propositions 1 and 2. Let for the moment focus on parameter γ : threshold values γ_2, γ_3 and γ_4 serve to identify the relative position of the α curves that define the region where $(0,0)$ is an equilibrium of the game, while γ_5 and γ_6 play an analogous role for the α curves that determine equilibrium $(1,1)$. Lastly, remind that γ_1 specifies Pareto efficiency. Hence, the ranking of such threshold values of γ allows for a complete characterization of the equilibria of the game. After tedious calculations, one can show that :

$$\gamma_6 > \gamma_1 > \gamma_5 \qquad \text{and} \qquad \gamma_4 > \gamma_1 > \gamma_3 > \gamma_2. \tag{26}$$

Unfortunately, we cannot directly order γ_6 *vs* γ_4 and γ_5 *vs* γ_3, but we use the initial size of the market, parameterized by $\underline{a}$, to discriminate between such threshold values of γ. In particular, depending on the relative position of $\underline{a}$ with respect to two critical values $\underline{a}_1$ and $\underline{a}_2$ (with $0 < \underline{a}_1 < \underline{a}_2$), three different situations will appear : [12]

1. $\underline{a} < \underline{a}_1 \Longrightarrow \gamma_4 > \gamma_6 > \gamma_1 > \gamma_5 > \gamma_3 > \gamma_2 > 0$;
2. $\underline{a}_1 < \underline{a} < \underline{a}_2 \Longrightarrow \gamma_6 > \gamma_4 > \gamma_1 > \gamma_5 > \gamma_3 > \gamma_2 > 0$;
3. $\underline{a}_2 < \underline{a} \Longrightarrow \gamma_6 > \gamma_4 > \gamma_1 > 0 > \gamma_5, \gamma_2, \gamma_3$.

Figure 4 describes the whole situation in the first case (i.e. $\underline{a} < \underline{a}_1$)[13]. We can now clearly identify both a *chicken game* and a *coordination game*. In the white area, neither the conditions of Proposition 1 nor those of Proposition 2 hold, and so $(1,0)$ and $(0,1)$ are the (unique) equilibria. This scenario appears when the possibility of enlarging the market is neither too limited nor too excessive (otherwise either $(0,0)$ or $(1,1)$ would respectively be the only outcomes), and the strategic stealing gain is sufficient for the investing firm to profitably cover the investment costs. Moreover, as we know from Lemma 3, if $\alpha < \alpha_a$, then the firm which is not investing is still selling a positive amount of product, while for $\alpha > \alpha_a$ it is endogenously driven out of the market. In particular, when $\alpha_a \leqslant \alpha < \alpha_b$ the investing firm finds it convenient to set a limit price, while above α_b it has such a big advantage that it can charge a kind of monopoly price.

On the other hand, where the dotted and dashed areas overlap, we have an interesting *coordination game*. Both $(0,0)$ and $(1,1)$ can be simultaneously equilibria and this happens again for intermediate values of γ, but now coupled with a weak strategic effect. When α is small, a firm that invests alone must bear all costs of advertising, while its gain comes almost entirely from the enlargement of the market. At the same time, the other firm gains more or less the same, without paying anything for it. If the return on advertising is quite good (in terms of γ), then the non-investing firm could find it profitable to devote resources to adverting too. On the

12 Calculations available upon request.

13 Figure 4 merges figures 2 and 3. It has in fact been drawn using $c = 1$, $b = 1$, and $\underline{a} = 0.3 < \underline{a}_1 = 0.43$.

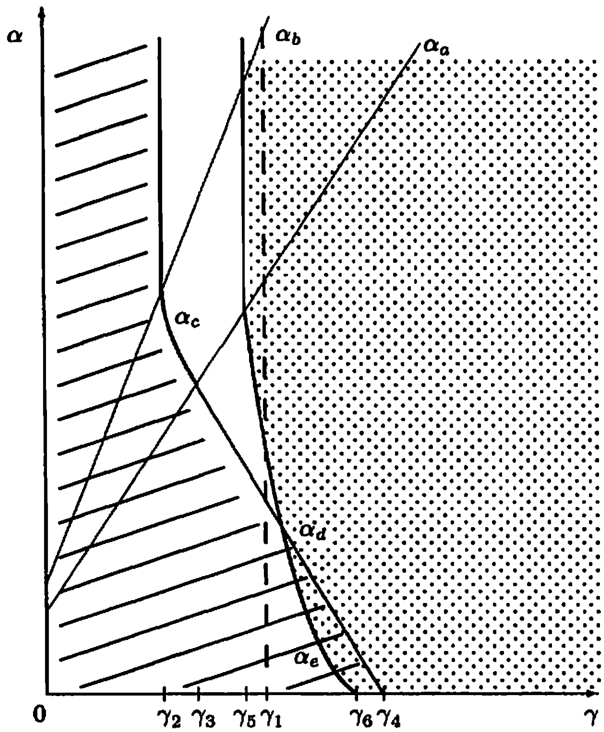

Figure 4 : *Analysis of equilibria : Case 1*

contrary, if this return is not too high, then the other firm can reasonably reconsider its investment decision. Therefore, for intermediate values of γ and low α, both kinds of deviations are plausible and we have a problem of coordination. Interestingly, as the figure shows, when such a problem arises the Pareto optimal equilibrium for our firms is the one with investment.

Figure 5 and 6 depict respectively the all set of equilibrium conditions for the cases where $\underline{a}_1 < \underline{a} < \underline{a}_2$ and $\underline{a}_2 < \underline{a}$.[14] There are two main differences with respect to Figure 4. First, the dashed area shrinks indicating that the equilibrium $(0, 0)$ is less and less likely to occur. This is due to the fact that, when the initial size of the market $\underline{a}$ increases, then firms are, *ceteris paribus*, more capable to cover the fixed costs of advertising. This reasonably makes firms more willing to invest in advertising. Second, the coordination game disappears. This happens for the same reasons that cause the dashed area to reduce. Rising $\underline{a}$, it is less likely that a firm cannot cover the fixed cost of advertising, even if it invests alone.

Further intuitions could be drawn by the relative dimension of c, the parameter which measures the degree of substitutability between the pro-

[14] Figure 5 and 6 have been drawn still taking $c = 1$, and $b = 1$, but while the former refers to $\underline{a} = 1$ (which is in between $\underline{a}_1 = 0.43$ and $\underline{a}_2 = 2.12$), the latter uses $\underline{a} = 2.5 > \underline{a}_2 = 2.12$.

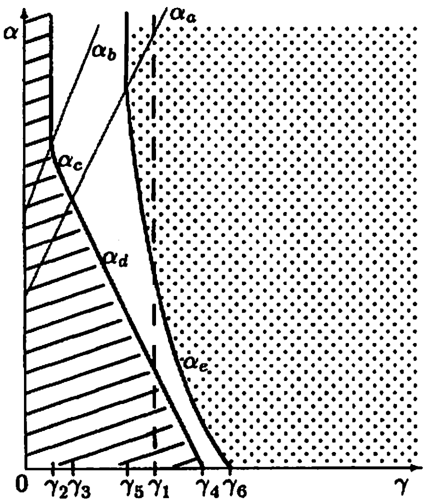

Figure 5 : *Analysis of equilibria : Case 2*

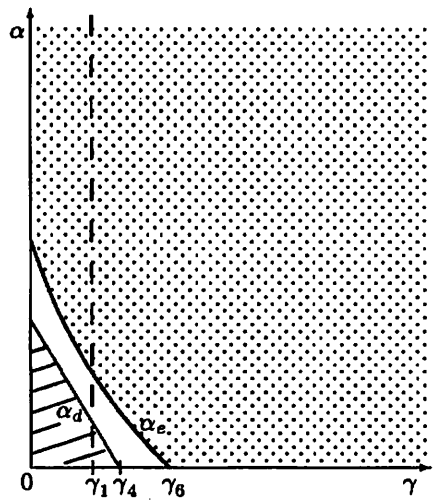

Figure 6 : *Analysis of equilibria : Case 3*

ducts of the two firms.[15] For high values of c, we end up in a situation like the one depicted in Figure 4 where, as we pointed out so far, the dashed area expands while the dotted one shrinks with respect to Figures 5 and 6. In other words firms are, everything else equal, more reluctant to invest. Intuitively, when products are close substitutes (high c), then competition in prices turns out to be very fierce. Consequently, equilibrium profits decrease (and at the limit they tend to zero) and firms are then less willing to advertise given that such an activity requires a fixed cost. This behavior is consistent with the findings of Grossman and Shapiro (1984) *inter alia.*

Turning to the asymmetric situation where just one firm advertises, the net effect of a change in c is instead quite ambiguous. Indeed, the impact of the strategic effect on demands, given in equation (2) by $c\alpha$, would be stronger, giving a relative advantage to the investing firm. On the other hand, the increase in competition in the goods market lowers profits, dampening the incentive to advertise. Consequently, the size of the white area may either increase or decrease. In our simulations, it actually increases from Figure 6 to 5, while decreasing when moving from 5 to 4. Put it differently, as products become more differentiated, it is not necessarily the case that a firm finds it profitable to invest in advertising if the other does not. Taking into account asymmetric outcomes thus leads to discover this somehow counter-intuitive relation between the equilibrium level of advertising and the degree of product differentiation. A similar ambiguous relation has been highlighted, although in a different framework, by von der Fehr and Stevik (1998).

[15] In fact, we can alternatively review the ranking of the γ's in term of c. As a function of c, both $\underline{a}_1$ and $\underline{a}_2$ vary from values close to zero to infinity and their first derivatives are strictly positive. Therefore, for a given $\underline{a}$, we can always find values of c sufficiently high to have $\underline{a} < \underline{a}_1$, and then decreasing c we pass to the other two situations $\underline{a}_1 < \underline{a} < \underline{a}_2$ and $\underline{a}_2 < \underline{a}$.

Finally, coming back to the *coordination game*, we can observe that it arises only for high values of c (Figure 4). In other words, coordination becomes an issue when the degree of interdependence among agents, captured here by price competition, is strong enough. This indeed conforms with intuition, but we can further prove that in such a case investing is the best choice:

Proposition 3 *The coordination game could arise only for low values of* $\underline{a}$ $(\underline{a} < \underline{a}_1)$ *or, equivalently, when product are highly substitutes (big c). Furthermore, when both* $(0,0)$ *and* $(1,1)$ *are "SPE", then the latter is always Pareto dominant from firms' standpoint.*

Proof see Appendix A.4. □

The latter result may be interpreted in terms of externalities. The positive effect of market enlargement is not fully internalized by an investing firm because it spills over into the other firm's demand. Therefore, equilibria may be characterized by underinvestment in advertising. This is actually the case of $(0,0)$ in our coordination game. On the other hand, for a coordination problem to arise we need a weak strategic effect (low α), otherwise it would always be convenient to advertise. Consequently, the first externality dominates the latter leaving room for equilibria in which firms refrain from investing even if it is not a Pareto optimal choice.

6 Conclusions

In this paper we considered a two-stage duopoly model with differentiated products where firms decide whether to invest in advertising or not and then they compete in prices. Contrary to the standard approach of considering advertising as either cooperative or predatory, we dealt with two opposite effects: a market enlargement and a stealing effect. This separation allows to better understand the forces underpinning the game, as well as to characterize in a richer way its equilibria. Particular attention has been paid to the specific role of product differentiation in advertising decisions as well as to the assessment of their Pareto optimality from firms' standpoint.

Depending on the values taken by the parameters, both symmetric and asymmetric equilibria appear. Among them, two outcomes are of particular interest: a *coordination game* in which both investing and non-investing are simultaneously equilibria; a *chicken game* in which only one firm invests in equilibrium with the second one possibly driven (endogenously) out of the market.

The *coordination game* arises only when products are strongly substitutes, suggesting that coordination matters when the degree of interdependence among agents, captured by price competition, is sufficiently high. Interestingly, when such a problem of coordination appears, the investment strategy leads in our model to the Pareto optimum.

Turning to the *chicken game*, we actually found two very interesting results. First, there exists a parameter region that supports a limit pricing behavior by the investor with the rival being *endogenously* squeezed out of the market. Furthermore, if the stealing advantage for the investing firm is strong enough, then it is able to take the entire market just by setting a kind of monopoly price. Second, in this asymmetric case the impact of product substitutability on the advertising efforts is ambiguous. This result contradicts the common view of a positive relationship between product differentiation and the equilibrium level of advertising and comes from the interplay between two opposite forces at work in the asymmetric equilibrium : the stealing gain that depends negatively on differentiation, and the strength of price competition that is instead relaxed by a decrease in product substitutability.

Starting from a very simple framework, we obtained quite interesting results. However, one of the main limitations of our approach stands in the use of a binary strategy set for investment decisions. Ideally, it would be better to use a more sophisticated relationship between investment in advertising and demand changes. On the other hand, as many other forms of investment, advertising has a strong discrete nature in the sense that it is sometimes more important to decide whether to invest or not rather than the exact amount to be spent on it. Furthermore, our plain structure turns out to be extremely flexible in the sense that it allows us to treat a large number of parameters (without resorting to normalization) and to completely characterize the game. However, it is true that asymmetric outcomes (represented by our *chicken game*) would have probably not occurred in a continuous framework. Nevertheless, if one thinks of advertising effort as having increasing returns for small investments, which does not sound as an implausible hypothesis, the asymmetric equilibria may reappear.

The same forces at work in the present paper could also be translated into a dynamic setting. Every firm could in fact be endowed with a stock of advertising that summarizes the effects of past advertising efforts. As a consequence, apart form current advertising, both the enlargement of the market and the consumers' shift from one firm to the other would depend on the stock of accumulated goodwill.

A Appendix

A.1 The demand structure

Consider demand functions q_i and q_{-i} as given by equation (1). This analytic formulation is clearly meaningful as long as price strategies are such that the implied q_i and q_{-i} are non-negative. Our goal here is to show how the demand system (2) can be obtained from equation (1) using continuity arguments. In the limit case in which p_i and p_{-i} are such that q_{-i}, as computed from (1), exactly equals zero, solving the equation $q_{-i} = a\left(I_i, I_{-i}\right) - bp_{-i} + c\left[p_i - p_{-i} + \alpha_{-i}\left(I_{-i}\right) - \alpha_i\left(I_i\right)\right] = 0$ for p_{-i} and plugging the solution into the equation of q_i one gets (after rearranging terms) :

$$q_i = 2a(I_i, I_{-i}) - bp_i - b\varphi(p_i) \tag{A1}$$

which is precisely the demand of firm i when the other firm gets zero sales ($p_i \leqslant p_i^l$). Equation (A1) is certainly correct for any couple of prices p_i and p_{-i} such that $q_{-i} = 0$ in (1). What remains to prove is that this holds for all prices p_i and p_{-i} that drive firm $-i$ out of the market, i.e. for lower p_i and/or greater p_{-i}. Consider for example a higher p_{-i}. Since firm $-i$ is already out of the market, it cannot ameliorate its position by increasing the price. Demands should thus be invariant to such increase in p_{-i} and, by continuity, q_i equals (A1), which is in fact a function of p_i only. On the other hand, if firm i charged a lower price, firm $-i$ would again stay out of the market, and we would have $q_{-i} < 0$ in (1). Following the above reasoning, demand of firm i should not depend on p_{-i}, as long as q_{-i} computed with (1) is non-positive. Everything thus works as if firm $-i$ charged a new price p_{-i} that would make q_{-i} exactly equal to zero, leading back to formulation (A1). Finally, since the price p_{-i} that solves $q_{-i} = 0$ cannot be negative, we have $\varphi(p_i) = \max\left\{\dfrac{a(I_i + I_{-i}) + c[\alpha_{-i}(I_{-i}) - \alpha_i(I_i)]}{b+c} + \dfrac{c}{b+c}p_i, 0\right\}$.

Let us now turn to price responsiveness of demand system (2). As long as firm $-i$ is active on the market, the appropriate demand curve is $q_i = a(I_i, I_{-i}) - bp_i + c[p_{-i} - p_i + \alpha_i(I_i) - \alpha_{-i}(I_{-i})]$ and its derivative with respect to p_i is simply $-(b+c)$. If price p_i goes below the limit price p_i^l, the demand function becomes (A1) and its slope can be either $-\dfrac{b(b+2c)}{b+c} > -(b+c)$ or $-b > -\dfrac{b(b+2c)}{b+c}$, if p_i is low enough to hit the constrain $\varphi(p_i) = 0$. As a conclusion, when p_i decreases demand q_i becomes less and less sensitive to price changes.

Finally, the limit price p_i^l is simply the non-negative solution (if it exist) to equation $q_{-i} = a(I_i, I_{-i}) - bp_{-i} + c[p_i - p_{-i} + \alpha_{-i}(I_{-i}) - \alpha_i(I_i)] = 0$ with respect to p_i :

$$p_i^l = \alpha_i(I_i) - \alpha_{-i}(I_{-i}) + \frac{-a(I_i, I_{-i}) + (b+c)p_{-i}}{c}. \tag{A2}$$

It is worth noting that it depends on p_{-i} (as expected) and, more important, that it can be negative, meaning that a limit price does not always exist.

A.2 Proof of Proposition 1

The necessary and sufficient condition for $(0,0)$ to be an equilibrium is that none of the two firms has an incentive to advertise alone. The profit accruing to both firms in case of no investments ($I_1 = I_2 = 0$) is given by (6). By symmetry, we can consider indifferently the deviation of one of the two firms. Suppose that firm 1 deviates ($I_1 = 1$) and invests in advertising; its equilibrium profits in the second stage price game is that of case B. As we have seen, although this payoff is (for each and every given value of the parameters) unique, its analytic expression changes in the parameters space. In fact, profit of firm 1 is given by: *(i)* π_{1Ac}^{B} when $\alpha < \alpha_a$, *(ii)* π_{1Dt}^{B} when $\alpha_a \leqslant \alpha < \alpha_b$ and *(iii)* π_{1Mp}^{B} when $\alpha \geqslant \alpha_b$. For each of the three cases we have to compare the profit that firm 1 gets when invests with the one that it gets without investing in advertising. As long as the latter is greater or equal to the former, $(0,0)$ will be a "SPE" of the reduced form of the game.

We begin with the case where $\alpha \geqslant \alpha_b$ and compare π_1^A with π_{1Mp}^{B} as they respectively appear in (6) and (20). As long as $\pi_1^A \geqslant \pi_{1Mp}^{B}$, $(0,0)$ will be an equilibrium. The equation $\pi_{1Mp}^{B} - \pi_1^A = 0$ is a convex parabola in γ with a negative (uninteresting) real root and a possibly positive real one:

$$\gamma_2 = \frac{\sqrt{b} + \sqrt{\underline{a}^2(b+c) + (2b+c)^2}}{2b+c} - \underline{a}. \tag{A3}$$

Therefore, since $\gamma \in (0, \infty)$, the necessary and sufficient condition we need is simply $\gamma \leqslant \gamma_2$. Clearly, if γ_2 is negative, there is no acceptable value of γ that makes $(0,0)$ an equilibrium in such a case ($\alpha \geqslant \alpha_b$).

The second situation is characterized by $\alpha_a \leqslant \alpha < \alpha_b$, where the relevant equilibrium profit to be compared with π_1^A is given by π_{1Dt}^{B}, whose analytic expression is given by (19). Contrary to before, π_{1Dt}^{B} now depends on α, reflecting the fact that firm 1 is not a "real" monopolist anymore and it has to charge a limit price. This explains why firm 1 is still sensitive to the extent of the "strategic effect" α. The equation $\pi_{1Dt}^{B} - \pi_1^A = 0$ is a concave parabola in α with two roots. When $\gamma < \gamma_2$ the two roots are complex conjugate and $(0,0)$ is an equilibrium because $\pi_{1Dt}^{B} - \pi_1^A < 0$ for any α. When $\gamma \geqslant \gamma_2$ the two roots are real; in particular, they both coincide with α_b for $\gamma = \gamma_2$ (this is consistent with the findings of the previous case where $\alpha \geqslant \alpha_b$). As we deal with a concave parabola, we are interested in the external solutions of the equation $\pi_{1Dt}^{B} - \pi_1^A = 0$ that are compatible with the interval of analysis ($\alpha_a \leqslant \alpha < \alpha_b$). Since the difference between one of these roots and α_b is increasing in γ (as revealed by the sign of the first derivative) we can neglect it because, whenever this root is a real number ($\gamma \geqslant \gamma_2$), it is greater or equal to α_b and then it lays outside the interval.

The other (smaller) root is given by:

$$\alpha_c = \frac{1}{b}\left[\frac{(b+c)(\underline{a}+\gamma)}{c} - \frac{\sqrt{\underline{a}^2(3b^2+3bc+c^2)+2\underline{a}(2b+c)^2\gamma-(2b+c)^2(b-\gamma^2)}}{2b+c}\right], \tag{A4}$$

which is decreasing in γ and meets α_a in:

$$\gamma_3 = \frac{\sqrt{\underline{a}^2(b+c)+(2b+c)^2}}{2\sqrt{b+c}} - \underline{a}. \tag{A5}$$

It is immediate to verify $\gamma_3 > \gamma_2$. As a consequence, for $\gamma \geqslant \gamma_2$ the condition $\pi^B_{1Dt} - \pi^A_1 \leqslant 0$ is equivalent to $\alpha \leqslant \alpha_c$.

Finally, when $0 < \alpha < \alpha_a$ we have to compare the usual π^A_1 with π^B_{1Ac} as it appears in (11). The profit difference $\pi^B_{1Ac} - \pi^A_1 = 0$ is in this case a convex parabola in α with a negative (uninteresting) real root and a (possibly) positive real one, which is always greater than the other, given by:

$$\alpha_d = \frac{(2b+3c)\left[\sqrt{b+c}\sqrt{\underline{a}^2(b+c)+(2b+c)^2} - (b+c)(\underline{a}+\gamma)\right]}{c(b+c)(2b+c)}. \tag{A6}$$

We look for internal solutions that are compatible with the interval of analysis ($0 < \alpha < \alpha_a$). It is easy to check that α_d is decreasing in γ and that $\alpha_d = \alpha_a$ when $\gamma = \gamma_3$. Consequently, when $\gamma \leqslant \gamma_3$, all $\alpha \in (0, \alpha_a)$ are solutions to the inequality $\pi^B_{1Ac} - \pi^A_1 \leqslant 0$ and so $(0,0)$ is certainly an equilibrium. On the other hand, when $\gamma > \gamma_3$, then $\alpha_d < \alpha_a$ and we need $\alpha \leqslant \alpha_d$ for a deviation to be unprofitable. Furthermore, $\alpha_d = 0$ in $\gamma = \gamma_4$, where:

$$\gamma_4 = \frac{\sqrt{\underline{a}^2(b+c)+(2b+c)^2}}{\sqrt{b+c}} - \underline{a}. \tag{A7}$$

Since $\gamma_4 > \gamma_3$, we have that $(0,0)$ cannot be an equilibrium for $\gamma \geqslant \gamma_4$ because it does not exist a positive internal solution in α for $\pi^B_{1Ac} - \pi^A_1 = 0$.

A.3 Proof of Proposition 2

The necessary and sufficient condition for $(1,1)$ to be an equilibrium requires that each firm takes no advantage in reconsidering its investment decision. Whenever both firms invest in advertising ($I_1 = I_2 = 1$), profits are simply equal to $\pi^C_1 = \pi^C_2$ as in (24) and they are non-negative iff $\gamma \geqslant \gamma_5$, with

$$\gamma_5 = \frac{2}{3}\left(\frac{2b+c}{\sqrt{b+c}} - \underline{a}\right). \tag{A8}$$

Imagine that firm 2 deviates ($I_2 = 0$); its equilibrium profit in the second stage price game is that of case B. As we have seen, although such payoff is unique, its analytic expression changes in the parameters space. Nonetheless, in the present case we are left with two scenarios and the profit of firm 2 amounts to : *(i)* π_{2Ac}^B when $0 < \alpha < \alpha_a$; *(ii)* zero when $\alpha \geqslant \alpha_a$ (for both $\alpha_a \leqslant \alpha < \alpha_b$ and $\alpha \geqslant \alpha_b$ firm 2 gets zero profits).

Let us begin with $\alpha \geqslant \alpha_a$, where we just need to compare π_2^C with $\pi_2^B = 0$. The equilibrium condition $\pi_2^C \geqslant \pi_2^B$ only requires that π_2^C is non-negative and we already know that this holds when $\gamma \geqslant \gamma_5$.

The other case, where $0 < \alpha < \alpha_a$, turns out to be more cumbersome. Relevant profits are given by π_{2Ac}^B as in (12), which is now a strictly positive number, and π_2^C. The equation $\pi_{2Ac}^B - \pi_2^C = 0$ is a convex parabola in α with two roots. When $\gamma < \gamma_5$ the two roots are complex conjugate and $(1,1)$ is never an equilibrium because $\pi_{2Ac}^B - \pi_2^C > 0$ for any α. On the contrary, when $\gamma \geqslant \gamma_5$ the two roots are real and they both coincide with α_a for $\gamma = \gamma_5$ (consistently with the findings of the case where $\alpha \geqslant \alpha_a$). As we consider a convex parabola, we look for internal solutions of the equation $\pi_{2Ac}^B - \pi_2^C = 0$ which are compatible with the interval under study ($0 < \alpha < \alpha_a$). A close inspection at the first derivative (with respect γ) of the difference between one of these root and α_a reveals that such derivative is positive as long as $\gamma \geqslant \gamma_5$, so that we can neglect it : whenever this root is a real number ($\gamma \geqslant \gamma_5$), it is greater or equal to α_a and so out of the interval we are analyzing. The other (smaller) root is :

$$\alpha_e = \frac{1}{2c^2(b+c)(2b+c)^2}\{2c(b+c)(2b+c)(2b+3c)(\underline{a}+\gamma)+ \\ -\sqrt{c^2(b+c)(2b+c)^2(2b+3c)^2\left[4\underline{a}^2(b+c)-4(2b+c)^2+12\underline{a}(b+c)\gamma+9(b+c)\gamma^2\right]}\}, \tag{A9}$$

Notice that the quantity $\alpha_e - \alpha_a$ is decreasing in γ; in particular, α_e reaches 0 for $\gamma = \gamma_6$, where :

$$\gamma_6 = \frac{2}{5}\left(\frac{\sqrt{\underline{a}^2(b+c)+5(2b+c)^2}}{\sqrt{b+c}} - \underline{a}\right). \tag{A10}$$

It is easy to verify that $\gamma_6 > \gamma_5$. As a consequence, the equilibrium condition $\pi_{2Ac}^B - \pi_2^C \leqslant 0$ is satisfied by the (internal) solution $\alpha \geqslant \alpha_e$. In particular, when $\gamma \geqslant \gamma_6$, then $(1,1)$ is always an equilibrium because no positive internal solution α exists for the equation $\pi_{2Ac}^B - \pi_2^C = 0$.

A.4 Proof of Proposition 3

Consider the case $\underline{a} < \underline{a}_1$, that entails the ranking : $0 < \gamma_2 < \gamma_3 < \gamma_5 < \gamma_1 < \gamma_6 < \gamma_4$. Starting from $0 < \gamma < \gamma_5$, we know from Proposition 2 that $(1,1)$ will never be a "SPE". We rule out this situation given that we look for intervals where *both* $(0,0)$ *and* $(1,1)$ hold simultaneously as equilibria of the game. In $\gamma_5 < \gamma < \gamma_4$ the outcome $(0,0)$ is a "SPE" if $\alpha \leqslant \alpha_d$ (see Proposition 1), while $(1,1)$ requires $\alpha \geqslant \alpha_e$ (see Proposition 2). Moreover, from Appendix A.2 we know that: *(i)* both α_d and α_e are decreasing functions of γ; *(ii)* α_d starts from $\gamma = \gamma_3$ and reaches 0 in $\gamma = \gamma_4$, while α_e starts from $\gamma = \gamma_5$ and reaches 0 in $\gamma = \gamma_6$. Given the above ranking of γ, it immediately follows that the two curves cross. In particular, it is possible to demonstrate that they meet twice. However, one of these two roots can be disregarded because it implies negative values for the parameter α and it is then economically meaningless. In the admissible region of parameters, thus, α_d meets α_e only in $\gamma = \gamma_7$, where

$$\gamma_7 = \frac{1}{7(b+c)^{3/2}}\left\{8(b+c)\sqrt{\underline{a}^2(b+c)+(2b+c)^2} - 10\underline{a}^2(b+c)^{3/2} + \right. \tag{A11}$$

$$\left. -2\sqrt{(b+c)^2\left[13\underline{a}^2(b+c)+2(2b+c)^2-12\underline{a}\sqrt{(b+c)\left[\underline{a}^2(b+c)+(2b+c)^2\right]}\right]}\right\}.$$

We can easily rank also this last threshold value of γ and we find that $\gamma_1 < \gamma_7 < \gamma_6$. Hence, $\alpha_e > \alpha_d$ for $\gamma_5 < \gamma < \gamma_7$ and $\alpha_e < \alpha_d$ for $\gamma_7 < \gamma < \gamma_4$, as we can see in Figure 4. It follows that *a coordination game appears when both* $\gamma_7 < \gamma < \gamma_4$ *and* $\alpha_e < \alpha < \alpha_d$. In the remaining interval, i.e. in $\gamma_4 < \gamma$, only $(1,1)$ can be a "SPE" given that we know from Proposition 1 that $(0,0)$ is never an equilibrium of the game.

To complete our demonstration we need to prove that the coordination game does not emerge when we consider higher values of $\underline{a}$, i.e $\underline{a}_1 < \underline{a}$. Let us first examine the case where $\underline{a}_1 < \underline{a} < \underline{a}_2$. The main variation with respect to the previous case is that $\gamma_4 < \gamma_6$. In the two "lateral" intervals, $0 < \gamma < \gamma_5$ and $\gamma_4 < \gamma$, a coordination game will never arise, exactly as before. Furthermore, when we consider the "intermediate" interval $\gamma_5 < \gamma < \gamma_4$, we see that α_d and α_e do not cross anymore, at least in the admissible region of parameters. This is obvious given that γ_4 and γ_6 are inversely positioned with respect to the previous situation. When the two curves exist, $\alpha_e > \alpha_d$ for every given value of γ, as we can see in Figure 5. It is not possible thus to find a region where $\alpha > \alpha_e$ and $\alpha < \alpha_d$ and, in turn, to sustain at the same time $(0,0)$ and $(1,1)$ as "SPE". The same reasoning applies to the interval in which $\underline{a}_2 < \underline{a}$, with the only difference that α_d and α_e are only partly represented given that they start from negative values of γ, as one can find in Figure 6.

We have then proved the first part of Proposition 3, showing that a coordination game only appears for $\underline{a} < \underline{a}_1$. In particular, this happens when $\gamma_7 < \gamma < \gamma_4$ and $\alpha_e < \alpha < \alpha_d$. The second part of Proposition 3 can be easily demonstrated given that $(1,1)$ and $(0,0)$ are both "SPE" only for $\gamma_7 < \gamma < \gamma_4$ and we proved before that $\gamma_1 < \gamma_7$. Remembering that the threshold value for Pareto efficiency is γ_1 (see 25), a coordination game may emerge only in the region where $(1,1)$ Pareto dominates $(0,0)$.

The last part of this proof deals with the possibility of using c instead of $\underline{a}$ to discern the case where the coordination game could arise. Unfortunately, a complete characterization of the game is not obtainable because we cannot find values of c that rank the threshold values of γ. We consider then the limit values for γ_4 and γ_6 when c going to infinity. The former goes to infinity, while the latter tends to a finite number. For high values of c, it becomes clear that $\gamma_4 > \gamma_6$ and we come back to the situation where α_d and α_e cross, giving thus rise to the possibility that a coordination game exists. Moreover, it is easy to prove that $\frac{\partial \underline{a}_1}{\partial c} > 0$ and $\frac{\partial \underline{a}_2}{\partial c} > 0$. Hence, when c increases, it is more likely that we find ourselves in $\underline{a} < \underline{a}_1$, the region where the coordination game may come out.

References

Amir R. (2000), "R&D Returns, Market Structure, and Research Joint Venture", *Journal of Institutional and Theoretical Economics*, 4, pp. 583-598.

Bagwell K. and G. Ramey (1988), "Advertising and Limit Pricing", *Rand Journal of Economics*, 19, pp. 59-71.

Bagwell K. and G. Ramey (1990), "Advertising and Pricing to Deter or Accommodate Entry When Demand is Unknown", *International Journal of Industrial Organization*, 8, pp. 93-113.

Butters G. (1977), "Equilibrium distribution of prices and advertising", *Review of Economic Studies*, 44, pp. 465-491.

Dixit A. and V. Norman (1978), "Advertising and Welfare", *The Bell Journal of Economics*, 9, pp. 1-17.

Friedman J.W. (1983), "Advertising and Oligopolistic Equilibrium", *Bell Journal of Economics*, 14, pp. 464-473.

Grossman G. M. and C. Shapiro (1984), "Informative advertising with differentiated products", *Review of Economic Studies*, 51, pp. 63-81.

Ishigaki H. (2000), "Informative advertising and entry deterrence: a Bertrand model", *Economics Letters*, 67, pp. 337-343.

Kaldor N. (1950), "The Economic Aspects of Advertising", *Review of Economic Studies*, 18, pp. 1-27.

Martin S. (2002), *Advanced Industrial Economics, second edition*, Oxford, Blackwell.

Milgrom P. and J. Roberts (1986), "Price and Advertising Signals of Product Quality", *Jounal of Political Economy*, 94, pp. 796-821.

Muehling D. Stoltman J. and S. Grossbart (1990), "The Impact of Comparative Advertising on Levels of Message Involvement", *Journal of Advertising*, 19, pp. 41-50.

Nelson P. (1974), "Advertising as Information", *Journal of Political Economy*, 82, pp. 729-754.

Piga C. (1998), "A Dynamic Model of Advertising and Product Differentiation", *Review of Industrial Organization*, 13, pp. 509-522.

Schmalensee R. (1978), "A model of Advertising and Product Quality", *Journal of Political Economy*, 86, pp. 486-503.

Schmalensee R. (1983), "Advertising and entry deterrence : an exploratory model", *Journal of Political Economy*, 91, pp. 636-653.

Sutton J. (1991), *Sunk Costs and Market Structure*, MIT Press.

Shubik M. and R. Levitan (1980), *Market Structure and Behavior*, Harvard University Press, Cambridge, Mass.

von der Fehr N.-H. and K. Stevik (1998), "Persuasive Advertising and product Differentiation", *Southern Economic Journal*, 65(1), pp. 113-126.

Wolinsky A. (1984), "Product differentiation with imperfect information", *Review of Economic Studies*, 51, pp. 53-61.