Do stock prices and interest rates possess a common trend?

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1 Introduction

Recently several studies have demonstrated that because of technological progress and financial deregulation, markets across the nations are interconnected. The philosophy of deregulation is becoming, as Mario Rutten observes (2002), widespread in some of the Asian economies where many financial institutions are being encouraged to experiment and test the limits of governmental rules and regulations\(^1\). There is good evidence about the integration of bond markets. Kasa (1992), Campbell and Cochrane (1999), Jagannathan (1997) and Hansen, Heaton and Luttmer (1995) presented evidence of the presence of a common stochastic trend underlying the co-movement of the stock markets in the UK, USA, Canada, Japan and Germany. But empirical studies on the relationships between the stock markets and the interest rates are scanty\(^1\). Rahman and Mustafa (1997) have recently analyzed the direction of causality between interest rates and stock prices for US markets\(^2\). Rahman and Mustafa (1997), along with an earlier study by Nozar and Taylor (1988) could not find any conclusive evidence as to the direction of causality. The purpose of this paper is to empirically capture the interrelationships between the stock markets and interest rates,

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\(^1\) Banks in India are edging their way into insurance stock brokerage and underwriting. The result of the regulatory erosion has increased the motivation for change and the encouragement for innovation throughout the financial services industry. Were there no spirit of deregulation, as in Bangladesh, such innovations would probably be introduced a little hesitantly.

\(^2\) For a theoretical approach to the interrelationship between stock prices and interest rates, see Blanchard (1981). In general, it suggests that a rise in the interest leads to a fall in stock prices and vice versa.
not for any Western but for a set of Asian markets, by means of a new technique called co-dependence. We will use a set of data for three countries in Asia – India, Pakistan and Bangladesh, over a time period spanning from 1985 to 2003.

The plan of the paper is as follows: Section 2 deals with the methodology on the basis of which we will test for common trends and common cycles. Section 3 will present the test results. The paper concludes in section 4.

2 Methodology

To determine the long-run equilibrium relationship between two time series variables, the most usual method used is cointegration, as developed by Engle and Granger (1987) and Johansen (1988). To ascertain whether common cycles exist, the method recently developed by Engle and Kozicki (1993) and Vahid and Engle (1993), termed codependence, is used. The statistical model is the unobserved-components model. Two economic time series, \( y_{1t} \) and \( y_{2t} \), might be generated by the model

\[
\begin{bmatrix}
y_{1t} \\
y_{2t}
\end{bmatrix} = \begin{bmatrix}
\lambda \\
1
\end{bmatrix} \omega_t + \begin{bmatrix}
\epsilon_{1t} \\
\epsilon_{2t}
\end{bmatrix}
\] (1)

where \( \omega \) is serially correlated but the vector \( \epsilon \) is not. In this case, \( y_{1t} - \lambda y_{2t} \) will be serially uncorrelated. Notice that no assumption is made about the contemporaneous covariances among the components. In (1) it is clear that dynamics of the two series are common. This interpretation recognizes that the cycles can be common to both countries whether the shocks \( \epsilon_{1t}, \epsilon_{2t} \), are contemporaneously correlated or not. This statistical model in (1) does not contain a statement about the process of \( \omega \). If it has any detectable feature, such as trends and seasonality, then this feature will be common and can be eliminated by taking a particular linear combination. As Engel and Kozicki (1993) put it, “the main difference between these two techniques is that cointegration involves co-movement between a set of non-stationary variables and co-dependence represents co-movement among a set of stationary variables.”

In general, cointegration between two non-stationary time series, stock prices \( (p_t) \) and interest rates \( (i) \), where \( p \) is expressed in logarithms, indicates the two series exhibit a shared stochastic trend:

\[
p_t = \beta i_t = \mu_t
\] (1)

If \( \mu_t \) is stationary, then \( i_t \) and \( p_t \) are said to be cointegrated, where \( \beta \) is the cointegrating vector. In contrast, a form of co-dependence between two stationary time series, say \( \Delta p_t \) and \( \Delta i_t \), is defined as a serial correlation.
common feature, where a linear combination of the stationary variables removes all past correlation such that it is completely unpredictable with respect to any information. According to Vahid and Engle (1993), Elements of $\Delta p_t$ have a serial correlation common feature if there exists a linear combination of them, which is an innovation with respect to all observed information prior to time $t$. That is to say that $\Delta p_t$ and $\Delta i_t$ possess a common cycle if

$$\Delta p_t - \omega \Delta i_t = \zeta_t$$

(2)

where $\omega$ is a parameter and $\zeta_t$ is a white noise error term.

**Definition**: A feature that is present in a group of series is said to be common to those series if there exists a nonzero linear combination of the series that does not have the feature.

To test for a serial correlation common feature$^3$, two-stage least squares is used, where the instruments are the lagged values of both variables, a constant and in the case of a cointegrating long-run relationship, the error correction term, that is;

$$\Delta p_t = \gamma_0 + \gamma_i \Delta i_t + + \psi_t$$

(3)

The instrument set is

$$1, \Delta p_{t-1}, \ldots, \Delta p_{t-k}, \Delta i_{t-1}, \ldots, \Delta i_{t-k}, E_{t-1}$$

$$1, \Delta p_{t-2}, \ldots, \Delta p_{t-k}, \Delta i_{t-2}, \ldots, \Delta i_{t-k}, E_{t-2}$$

When a set of variables are cointegrated, their differences are related to the past not only through the lagged differences but also the error correction terms. Therefore, when we try to estimate the explained part of their differences from the past information (the first stage of the two-stage least squares regression we include an error correction term, $E_{t-1}$ as well as their lagged differences.

In the case of monthly data, the instrument set typically includes twelve lags of the change in the stock price index and change in the interest rate. Because there is a cointegrating relationship between stock prices and interest rates, the lagged residual is also included among the regressors. The overidentifying restrictions are tested by the LM statistic, which has a chi-squared distribution and the degrees of freedom equal to the number of overidentifying restrictions$^4$. This is nothing but a test of linear restrictions on

$^3$ Granger causality tests were carried out using first-differenced variables and concluding an error correction term, where there was evidence of cointegration. The direction of causality was ambiguous, Shively (2001).

$^4$ Suppose that $y_t$ is an $n \times 1$ vector and $z_t$ is $k \times 1$ with $k \geq n)$. These series are related by the linear regression equation, $y_t = \Gamma z_t + c_t$ where $c_t$ satisfies $E(c_t|z_t) = 0$. The hypothesis of common-features is that there exists an $n \times 1$ common features vector $\delta$ such that the residual $\mu_t(\delta) = \delta^T y$ is uncorrelated with $z_t$. This equivalent to the reduced rank restriction $\Gamma = V \phi$, where $\phi$ is $n \times (n-1)$. These two expressions of the null hypothesis are related by the requirement that $\delta^T \phi = 0$. Here $H_0: 0$, no feature; $H_1: \delta^T \phi \neq 0$, feature provide a hypothesis-testing basis for detecting a feature in the series $\{y_t\}$. The test statistic is to test the orthogonality between $\mu_t(\delta)$ and $z_t$. Let $Y$ and $Z$ be the $T \times 1$ and $T \times k$ observation matrices for $y_t$ and $z_t$, respectively.
the coefficients of the lagged differences and the estimated error-correction terms which satisfy the 'relevant' past.

This test statistic is based on the Sargan (1958) test for legitimacy of the instruments, which is \( T x.R^2 = \chi^2_k \), where \( k \) is the number of restrictions\(^5\). Cointegration, as Engle and Issler (1995) stress, does not, however, prevent common serial correlation features occurring or imply that they do exist. Furthermore, if no cointegration is found, then it does not necessarily mean that common serial correlation features will not occur, although in this case the error correction term should not be included among the instruments.

**Theorem 1**: If \( y_t \) is a \( n \) dimensional vector of \( I(1) \) variables with \( r \) linearly independent cointegrating vectors \( r < n \), then if elements of \( y_t \) have common cycles, there can, at most, exist \( n - r \) linearly independent cofeature vectors that eliminate the common cycles. Moreover, these linear combinations must be linearly independent of the cointegration vectors.

**Proof**: Any linear combination of the first differences that is an innovation implies that the same combination of the levels must be a random walk and hence non-stationary. Therefore, a linear combination can not lie in the cointegration space. Moreover, since there can only be \( n \) linearly independent vectors in \( \mathbb{R}^n \), no more than \( n - r \) such combinations can exist\(^6\).

3 The data set and the test results

The data set consists of monthly observations, from January 1985 to January 2003 for three Asian Countries – India, Pakistan and Bangladesh. The long-term interest rates being unavailable, we have used the three-month treasury bill rate for each country; the stock market data consist of the market index\(^7\). Table 1 shows that the Augmented Dicky-Fuller (ADF) tests suggest that all the stock market indexes and interest rates are integrated of order one, \( I(1) \). The Johansen (1988) Maximum Likelihood Procedure (Table 2) was used to test for cointegration between stock prices and interest rates. We first used the prices and interest rates. We first used the Akaike

and \( z_t \). Let \( P_z = Z(Z'Z)^{-1}Z' \) be the projection matrix and let \( \mu(\delta) \) be the residual vector. Set

\[
\delta(\delta) = \frac{\mu(\delta)' P_z \mu(\delta)}{\sigma^2(\delta)}
\]

where \( \sigma^2(\delta) \) equals either \( \left( \frac{1}{n} \right) \mu(\delta)' \mu(\delta) \) for a Lagrangian multiplier (LM) type test statistic or \( \left( \frac{1}{n} \right) \mu(\delta)' (I - P_z) \mu(\delta) \) for a Wald - type of statistic.

\(^5\) For a formal proof of the test for co-dependence, see Engle and Kozicki (1993)

\(^6\) This rather trivial theorem has interesting implications for the first-order cointegrated systems. See, for example, Engle and Yoo (1986)

\(^7\) All data come from International Financial Statistics
Information Criterion to determine the number of lags required for the test, although lags were not excluded if it introduced serial correlation. The trace results indicate that India produces a single cointegrating vector at the 5% level of significance, while Pakistan produces a single vector at the 10% level of significance.

Table 1 ADF unit root tests for stationarity

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>p</th>
<th>Δp</th>
<th>i</th>
<th>Δi</th>
</tr>
</thead>
<tbody>
<tr>
<td>INDIA</td>
<td>-0.231</td>
<td>-4.38</td>
<td>-1.97</td>
<td>-16.10</td>
</tr>
<tr>
<td>PAKISTAN</td>
<td>-1.220</td>
<td>-4.44</td>
<td>-1.14</td>
<td>-3.38</td>
</tr>
<tr>
<td>BANGLADESH</td>
<td>-1.171</td>
<td>-5.99</td>
<td>-0.84</td>
<td>-4.103</td>
</tr>
</tbody>
</table>

p is the stock market index and, i is the interest rate, Δ is the first-difference operator. The appropriate lag lengths were determined by the Akaike criteria; the critical value of the ADF statistic is -2.89 (5%).

Table 2 Johansen-Juselius maximum likelihood cointegrated tests

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>r = 0</th>
<th>r \leq 1</th>
<th>Lags</th>
<th>Long-run \beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>INDIA</td>
<td>21.146**</td>
<td>4.317</td>
<td>5</td>
<td>-0.145</td>
</tr>
<tr>
<td>PAKISTAN</td>
<td>19.120*</td>
<td>2.703</td>
<td>3</td>
<td>-0.340</td>
</tr>
<tr>
<td>BANGLADESH</td>
<td>11.313</td>
<td>2.682</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

5% 19.966 9.243
10% 17.851 7.535

The values at the bottom of the table are the critical values.
* Significant at the 10% level; ** Significant at the 5% level

The error correction models (ECM) for these countries (Table 3) are well specified, although the coefficients of the error correction terms are fairly small, pointing to the possibility of a slow speed of adjustment back to equilibrium following an exogenous disturbance. There is evidence at the 5 countries investigated. With the exception of India, the relationship between stock prices and interest rates is not significant with the coefficients being about .05 in India and .002 and .021 for the other two countries tested (Table 4).
Table 3 Estimated error correction model

INDIA
\[ \Delta p = 0.002 - 0.68 \Delta i + 1.18 \Delta p_{t-5} - 0.42 EC \]

\[ (0.08) \quad (-9.23) \quad (2.13) \quad (3.23) \]

Adj R^2 = 0.55, DW = 1.80, RESET = 0.19 (0.67)
t-ratios are in parenthesis

PAKISTAN
\[ \Delta p = 0.007 - 0.013 \Delta i + 0.030 \Delta p_{t-3} - 0.42 EC \]

\[ (0.08) \quad (-7.03) \quad (2.171) \quad (5.123) \]

Adj R^2 = 0.75, DW = 1.87, RESET = 0.39 (0.52)
t-ratios are in parenthesis

BANGLADESH
\[ \Delta p = 0.001 - 0.017 \Delta i + 0.01 \Delta p_{t-2} - 0.62 EC \]

\[ (0.01) \quad (-6.03) \quad (2.019) \quad (4.123) \]

Adj R^2 = 0.51, DW = 1.77, RESET = 0.49 (0.22)
t-ratios are in parenthesis

In the above equations EC are the residuals from the cointegrating tests. DW is the Durbin-Watson statistic; RESET is the Ramsey test for functional form with a critical value of 3.842 (5%). The lags which are significant are not reported in the equations.

Table 4 Tests for codependence

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>( \gamma_0 )</th>
<th>( \gamma_1 )</th>
<th>LM STATISTIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>INDIA</td>
<td>0.007 (2.999)</td>
<td>0.055 (2.712)</td>
<td>36.022</td>
</tr>
<tr>
<td>PAKISTAN</td>
<td>0.09 0 (4.111)</td>
<td>-0.002 (0.413)</td>
<td>12.655</td>
</tr>
<tr>
<td>BANGLADESH</td>
<td>0.003 (2.021)</td>
<td>-0.021 (0.644)</td>
<td>10.311</td>
</tr>
</tbody>
</table>

The t-statistics are in parentheses for the constant and the coefficients, based on the equation (3). For the Sargan LM test, which has 24 degrees of freedom, the 5% critical value is 21.414.

4 Conclusion

There is little evidence that stock prices and interest rates do possess a common trend with the exception of India. However, there is strong evidence
of common cycles for the other countries. These findings provide support
to the view that although bond markets and stock markets in these coun-
tries are linked, this may not be through a common trend, but through a
common cyclical pattern. So, while modeling the interaction between the
bond markets and stock markets, a dynamic error correction model may be
considered. From the policy point of view, if we know that two markets are
linked through a common cyclical pattern, we can take advantage of this
result in better forecasting or in the decomposition of stock price change
affected by bank interest rate change.

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