Show me the money: retained earnings and the real effects of monetary shocks

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1 Introduction

In recent years a large number of studies have used the identified-VAR methodology to assess the effects of monetary shocks. This literature documents that contractionary monetary shocks have a persistent negative effect on output and employment, and a persistent positive effect on interest rates. For example, Christiano, Eichenbaum, and Evans (1999) review a number of different approaches for identifying monetary shocks, and find that interest rates rise for at least six months after a contractionary monetary shock, whereas the negative effect on output lasts for well over a year. These conclusions are robust across most identification schemes for monetary shocks used in the literature1.

Explaining these findings is a challenge for economic theory. Frictionless models do not generate any real effects of monetary disturbances. In the recent theoretical literature there are two main classes of models which generate real effects of monetary shocks, the "liquidity" model and the "sticky-price" model (see Cole and Ohanian (2002) for a recent comparison of the two frameworks). Even though both models give rise to real effects of monetary shocks, they have trouble generating persistence. In the "liquidity" or "limited participation" model, households are unable to ad-

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1 An exception is Uhlig (2000). Uhlig uses a Bayesian approach to identify monetary shocks, and does not find that contractionary monetary shocks lower output.
just their asset holdings immediately when a monetary shock hits (see Lucas Jr. (1990) and Christiano and Eichenbaum (1992)). Firms and banks can react to monetary disturbances at once, whereas households react only with a delay of one period. The liquidity model generates real effects of monetary shocks. However, the effects are short-lived. Once households are able to adjust their asset position in the period following the shock, all real effects disappear. The frictions at the heart of the "sticky price" model are nominal rigidities generated by staggered price-setting (see Taylor (1980) and Blanchard (1991)). While in a sticky-price model real effects can last longer than one period, Chari, Kehoe, and McGrattan (2000) show that in a calibrated model with staggered price setting there is very little persistence unless the frequency of price adjustments is assumed to be unrealistically low.

This paper explores whether a model with a different friction, namely small adjustment costs for household asset transactions, can account for persistent effects of monetary shocks. I develop an otherwise standard cash-in-advance model in which households have checking and saving accounts and face a small adjustment cost for transfers between their accounts. This cost can be interpreted as banking fees, as well as the opportunity cost of the time which is used for carrying out the transactions (the "shoe-leather" cost of the Baumol-Tobin model). Notice that this setup is very similar to the liquidity model in spirit. In particular, the liquidity model can be interpreted as an adjustment-cost model where the cost for immediate transactions is infinite, but the cost for scheduled future transactions is zero.

I use a calibrated version of the adjustment-cost model to ask two questions. First, does the model generate real effects of monetary shocks that are similar to what is observed in the data? Second, how large do the adjustment costs have to be for the real effects to be sizable and persistent? The answer to the first question is a qualified "yes". The adjustment-cost model can generate real effects of monetary shocks, but only if the model is extended to allow a realistic representation of the flow of funds between firms and households. Specifically, it is necessary to allow for retained earnings in the business sector. Retained earnings serve to isolate the households from the direct impact of monetary disturbances. If we do not allow for retained earnings, persistence does not arise. This is a surprising outcome, in the sense that in existing models the specifics of the flow of funds between firms and households are not important for the transmission of monetary shocks. The answer to the second question is that once we allow for retained earnings, very small adjustment costs are sufficient to lead to sizable and persistent real effects of monetary policy. The adjustment cost is modeled as a time cost, and it can be quantified by comparing it to working time. In the baseline calibration, the realized adjustment cost never exceeds three seconds per quarter per person.

Our results stand in marked contrast to a number of existing papers that also introduce adjustment costs within the liquidity-constraint framework, and find that adjustment costs lead to persistent effects even without
requiring the accumulation of retained earnings (see, for example, Christiano and Eichenbaum (1992), Chari, Christiano, and Eichenbaum (1995), or Evans and Marshall (1998)). The reason for the different findings is that these authors rely on a specific asymmetric formulation for the adjustment cost. In particular, while adjusting savings is assumed to be costless, adjusting the amount of cash used for consumption expenditures is costly. As pointed out by Rotemberg (1995), this asymmetry is hard to justify from a microeconomic perspective. In the existing models, an asymmetric adjustment cost is essential to generate persistence, since the models require that savings react much more strongly to a monetary shock than cash holdings. If the adjustment cost also applied to savings or to transfers between cash and savings, monetary shocks would no longer have persistent real effects. This paper shows that if adjustment costs are introduced in a more realistic form, the presence of retained earnings is necessary for monetary shocks to have persistent effects.

Retained earnings matter for the transmission of monetary policy because they affect the overall balance between different uses of funds in the economy. In the model, funds can be used either for consumption expenditures or for savings. Since prices are flexible, the overall amount of funds that is initially available in the economy does not affect real variables. In this sense, money is neutral. On the other hand, the balance of the use of funds between consumption and savings does have real consequences.

In the model economy, households own funds in two different ways. First, they hold funds directly in their own checking and savings accounts. Second, households hold funds indirectly through the firms in the economy, which they own. It is through these indirect holdings that retained earnings matter. When a monetary shock hits, initially only the asset holdings of firms are affected. For example, an expansionary monetary shock increases the amount of funds held by the business sector. Since funds held by the business sector are not used for consumption, the economywide ratio of funds used for consumption and savings changes after such a shock. Without adjustment costs, households would then lower their own savings to re-establish the preferred ratio of consumption to savings. With adjustment costs, consumers adjust their asset holdings to a lesser degree, and the resulting imbalance affects real variables such as output and employment. If earnings in the business sector are retained, the imbalance and therefore the real effects of the monetary shock will persist.

The implications of the model with retained earnings for the flow of funds between households and the business sector are in line with empirical findings. In the next Section, we present evidence that shows that corporate profits react quickly to a monetary shock, whereas dividend payments adjust only after a considerable delay. Consistent with this finding, Christiano, Eichenbaum, and Evans (1996) report that the household sector does not adjust its financial assets and liabilities for several quarters after a monetary shock, while there is an immediate impact on the business sector.
In summary, our results suggest that portfolio adjustment costs are a promising avenue for explaining persistent real effects of monetary shocks. A key finding is that a better account of the flow of funds between the household and business sectors may be central for understanding the monetary transmission mechanism. This conclusion puts the adjustment-cost model in marked contrast to existing monetary models, which do not assign a major role to the flow of funds. The paper is organized as follows. The next section provides an empirical analysis of the relationship between monetary shocks and corporate profits and dividends. The model is introduced in Section 3. Section 4 discusses some theoretical results on the effects of monetary shocks, and shows how the decision problem of the household is modified in different versions of the model. In Section 5 the model is calibrated, and numerical experiments are carried out to assess the effects of monetary shocks in the adjustment-cost model. Section 6 concludes.

2 Empirical Evidence

This section examines the effects of monetary shocks on corporate profits and dividend payments. A central conjecture of this paper is that profits or losses in the corporate sector which arise from monetary shocks are transferred to households only with a delay. If this conjecture is true, the asset position of the household sector is insulated temporarily from the effects of monetary disturbances. The theoretical analysis in the remainder of the paper will show that this insulation has important implications for the real effects of monetary shocks.

Following the major part of the empirical literature on monetary shocks, we rely on the identified-VAR methodology to assess the reactions of aggregate economic variables to monetary disturbances. In particular, our specification is close to Christiano, Eichenbaum, and Evans (1996). The data set contains quarterly observations on U.S. economic and monetary aggregates from the first quarter of 1959 until the third quarter of 2002. The following variables were included in the analysis: Real GDP (Y), the GDP deflator (P), an index of commodity prices (PCOM), total reserves (TR), non-borrowed reserves (NBR), the federal funds rate (FF), real corporate profits (PR), and real corporate dividends (DIV). With the exception of the federal funds rate, all variables are in logs. Profits and dividends were deflated using the GDP deflator. The commodity price index was included to avoid the well known price puzzle. Without this measure, contractionary monetary policy shocks (defined as orthogonalized innovations to FF or NBR) lead to a prolonged rise in the price level (see Sims (1992)). As discussed in Christiano, Eichenbaum, and Evans (1996), this anomalous res-

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2 All data were extracted from the FRED data base at the Federal Reserve Bank of St. Louis. The identification codes for the series are: GDPC1, GDPDEF, PPICRM, TRARR, BOGNONBR, FEDFUNDS, CPROFIT, and DIVIDEND.
The usual interpretation is that commodity prices matter, because they contain information about future inflation that is available to the policy maker, but not contained in the remaining variables in the VAR.

Monetary shocks were identified by imposing a triangular structure on the variance-covariance matrix of the error term. In other words, the variables were ordered such that each variable can have an instantaneous effect only on variables lower in the order. The following ordering was employed: $Y$, $P$, $PCOM$, $FF$, $NBR$, $TR$, $RPR$, $RDIV$. This is the same ordering as in Christiano and Eichenbaum (1995), with the new variables specific to this analysis ($RPR$ and $RDIV$) ordered last. It appears plausible to order dividends after profits, since profits first have to exist before they can be distributed. Of course, the impulse response functions presented below depend to some degree on the specific ordering employed. In terms of the overall effect of monetary shocks, what appears to matter most is that $Y$ is ordered first. Our basic conclusions regarding the relationship between monetary shocks, corporate profits, and dividends are surprisingly robust with respect to the ordering of the Cholesky decomposition. In particular, the effects are virtually unchanged if dividends are ordered before profits.

I use orthogonalized shocks to $FF$ as the definition of a monetary policy shock. Using a shock to non-borrowed reserves as an alternative measure yields similar results. Figure 1 shows the impulse response of the main variables we are interested in to a one-standard-deviation contractionary shock.
to \( FF \). The dashed lines are two-standard error bands. Output starts to decline with a delay of two quarters after the shock, with the largest impact occurring after about two years. The reaction of profits has a similar shape as the reaction of output. Notice, however, that the reaction of profits is much stronger than the reaction of output in magnitude. A contractionary monetary policy shock therefore has a sizable negative impact on the profits of the corporate sector. Corporate dividends reflect these lower profits, but only with a delay. The impulse response function of dividends shows almost no reaction for the first five quarters after the monetary shock, and turns negative thereafter. The reactions of profits and dividends have a similar magnitude, but the reaction of dividends is delayed relative to profits. The results are consistent with the conjecture that the corporate sector transfers profits or losses to households only with a delay. Figure 2 repeats the same exercise with nominal profits and dividends, instead of inflation-adjusted figures. The results are very similar to Figure 1.

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Table 1: Reaction of Dividends to Profit Shock (Percent of Total Impact)

Figure 3 displays impulse response functions for shocks to GDP, real profits, and real dividends. Of particular interest here is the reaction of dividends to a change in profits. Notice that our ordering assumption allows dividends to adjust immediately after a shock to profits. The graph shows, however, that this does not happen. Instead, the reaction of dividends to a change in profits is hump-shaped, with almost no immediate reaction, and the maximum impact being reached only after five to six quarters. Thus, once again the results back up our assumption that the corporate sector retains earnings. This result is entirely plausible if we take into account how decisions on dividend payments are made in practice. While dividends are often paid quarterly, the vast majority of firms adjusts dividends at most once a year, based on profits in the preceding year. Additional delays arise because official profit figures are generally available only a few months after the end of a fiscal year, and shareholder meetings are held even later.

For the purposes of calibrating the theoretical model described below, it will be useful to look at the relationship of dividends and profits in more detail. Table 1 shows how the reaction of dividends to a shock to profits is spread out over time. The largest impact occurs five quarters after impact. Half of the total accumulated impact is reached between four and five quarters after impact, and the reaction fades out about three years after the innovation to profits.

In summary, the empirical evidence supports the conjecture that contractionary monetary shocks lower corporate profits, and that this change
affects households in the form of lower dividend payments only with a delay. Consistent with these findings, Christiano, Eichenbaum, and Evans (1996) report that the household sector does not adjust its financial assets and liabilities for several quarters after a monetary shock. The next section develops a model that demonstrates why these findings may be important for the transmission of monetary shocks.

![Graphs showing impulse response functions for GDP, profits, and dividends.](image)

**Figure 3: Impulse Response Functions, Real Profits**

### 3 The Model

The model is based on the standard cash-in-advance framework. The economy is populated by the monetary authority and a continuum of three types of competitive agents: households, firms, and banks. There is measure one of each type of agent, so that the model can be formulated in terms of a representative household, firm, and bank. Apart from the cash-in-advance constraint, there are two additional frictions present in the baseline model: a liquidity constraint and an adjustment cost. The liquidity constraint forces
consumers to make saving decisions before the monetary shock is revealed, while the adjustment cost penalizes changes in the stock of savings. The liquidity constraint can be interpreted as an infinite adjustment cost for immediate changes in savings. The baseline model incorporates the liquidity constraint to facilitate comparisons to earlier literature. Specifically, if adjustment costs are set to zero, the economy reduces to the baseline model in Christiano and Eichenbaum (1995). I also consider a version of the model without the liquidity constraint. If adjustment costs are set to zero in this version, the economy reduces to the stochastic cash-in-advance model considered by Cooley and Hansen (1989).

**Assets and the Monetary Authority**

In the model economy, consumers have access to two different assets, checking accounts and saving accounts. All transactions are settled using the checking account. Banks are subject to a 100 percent reserve requirement on checking accounts, which implies that these accounts carry no interest in equilibrium.3

There is a central bank which supplies currency to the economy. The money stock at the beginning of period $t$ is denoted by $M_t$. Since households and firms do not hold any cash (the only assets are checking and savings accounts), all the money is in the hands of the banks. The central bank carries out monetary policy by giving a cash injection $X_t$ to the bank at the beginning of period $t$. $X_t$ is a random variable, and monetary policy is the only source of uncertainty in the model. The money stock $M_t$ evolves according to:

$$M_{t+1} = M_t + X_t.$$ (1)

**Banks**

There is a competitive banking industry which accepts deposits from firms and households and makes loans to firms. Banks are owned by the households. At the beginning of the period, the assets of the bank consist of the money stock $M_t$ (recall that the entire money stock is held by the bank). The liabilities consist of the checking deposit $D_t$ and the saving deposit $S_t$ of the household. We will see later that the bank makes profits and transfers profits to the households (who own the bank) only with a delay. Consequently, there is an amount $\Pi_t$ of retained earnings that was carried over

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3 Even though cash is not used for transactions in the economy, results are the same as in an otherwise identical economy that uses only cash and no checking accounts at all. In other words, checking accounts could equivalently be labeled as cash. I prefer the checking-account terminology since most of M1 is made up of deposits. The important distinction is between non-interest-bearing assets that can be used for transactions, and interest-bearing assets such as saving accounts.
from earlier periods. Since assets have to equal liabilities, we have:

\[ M_t = D_t + S_t + \Pi_t. \tag{2} \]

The first event within the period is the realization of the monetary policy shock. The central bank hands out \( X_t \) dollars to the bank. After the arrival of the monetary injection \( X_t \), the bank gives a loan \( B_t \) to the firm, where the loan takes the form of a demand deposit made available to the firm. The bank has to observe the 100 percent reserve requirement on checking accounts, that is, the demand deposits have to be backed by cash:

\[ D_t + B_t \leq M_t + X_t. \tag{3} \]

The banking industry is competitive, so that the interest rate \( r_t \) is taken as given by the bank. The optimization problem of the bank is to maximize profits from giving the loan to the firm, subject to the reserve requirement. The bank therefore solves:

\[
\max_{B_t} \{ r_t (B_t - S_t) \}
\]

subject to (3). As long as the interest rate is non-negative (as will be assumed later), the problem of the bank has a trivial solution: the loan \( B_t \) will be the maximum possible given the reserve requirement, so that the reserve requirement holds with equality.

All transactions during the period are transfers between the demand deposits of the firm and the household. At the end of the period, after the firm pays back the loan, the bank credits fraction \( \lambda \) of retained earnings \( \Pi_t \) to the consumer's checking account. Here \( \lambda \) is an institutional parameter that represents the rate at which retained earnings flow from the business sector to households. Typically \( \lambda \) will be smaller than one, which reflects that businesses usually do not pay dividends whenever a cash flow occurs, but only in larger intervals. Since the consumers own the banks, in a frictionless model they would consider retained earnings as equivalent to their own savings, and the choice of \( \lambda \) would be irrelevant. In the adjustment-cost economy, in contrast, the value of \( \lambda \) is a key determinant of the effects of monetary shocks.

The current profits of the bank in period \( t \) are given by the sum of the monetary injection \( X_t \) and net interest income \( r_t (B_t - S_t) \). The law of motion for the retained earnings of the bank is therefore:

\[ \Pi_{t+1} = X_t + r_t(B_t - S_t) + (1 - \lambda)\Pi_t. \]

Using the accounting identity (2), the reserve requirement can be written as \( B_t \leq S_t + \Pi_t + X_t. \) Since the reserve requirement holds with equality in equilibrium, the law of motion for retained earnings can also be expressed as:

\[ \Pi_{t+1} = (1 + r_t)(B_t - S_t) - \lambda \Pi_t, \]

or:

\[ \Pi_{t+1} = (1 + r_t)(\Pi_t + X_t) - \lambda \Pi_t. \]
The last equation shows that profits arise from loaning existing retained earnings and the monetary injection to the firm. Notice that even though retained earnings evolve over time, the decision problem of the bank has a static solution which does not depend on intertemporal prices.

In terms of the application of the model, the bank profits should be interpreted as the profits arising in the entire corporate sector as a result of monetary injections. We assign all the profits to the bank merely for convenience; this convention allows us to keep the decision problem of production firms particularly simple.

Production

In the baseline economy, the production technology is linear and uses only labor. While capital accumulation will be introduced as an extension below, the labor-only case facilitates the analysis of the role of adjustment costs and retained earnings for the transmission of monetary shocks. The representative firm operates the technology $y_t = l_{1t}$ to produce the consumption good. The firm hires labor $l_{1t}$ from the household at wage $W_t$, and sells the consumption good to the household at price $P_t$. Following Christiano and Eichenbaum (1995), it is assumed that the wage bill has to be paid before payments for the consumption good are received. The firm therefore requires a loan $B_t$ from the bank to cover the wage bill $W_t l_{1t}$. The nominal interest rate for the loan is $r_t$. The firm operates in a competitive industry and takes prices as given. The firm's problem is:

$$\max_{l_{1t}, B_t} \{P_t l_{1t} - W_t l_{1t} - r_t B_t\}$$

subject to: $W_t l_{1t} \leq B_t$.

Since the decision problem is linear, firms make zero profits in equilibrium, and the scale of production is determined outside the production sector. Notice that since the firm has to borrow the wage bill, the real wage depends on the interest rate. This interdependence is necessary to generate real effects of monetary shocks.

Preferences

The representative household ranks streams of consumption and leisure according to the utility function:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t) \right\}. \quad (4)$$

The household enters period $t$ with a balance $D_t$ on the checking account and $S_t$ on the savings account. In the morning, after the monetary shock is
realized, the household buys consumption goods \( c_t \) subject to the "cash-in-advance" constraint \( P_t c_t \leq D_t \), that is, purchases cannot exceed the existing balance on the checking account. The household works \( l_{1t} \) hours for wage \( W_t \), gets interest \( r_t S_t \) on the balance on the savings account, and receives a dividend \( \lambda \Pi_t \) from the bank. After all other transactions are settled, the household makes a transfer \( I_t \) from the checking to the savings account. However, carrying out this transaction takes time and is therefore costly in terms of leisure. The time \( l_{2t} \) required for the transfer is determined by \( l_{2t} = g(I_t/M_t) \), where \( M_t \) is the money stock at the beginning of the period and \( g \) is a nonnegative convex function\(^4\).

The household’s problem is to maximize (4) subject to the following constraints:

\[
\begin{align*}
P_t c_t & \leq D_t, \quad (5) \\
D_{t+1} & = D_t + r_t S_t + \lambda \Pi_t + W_t l_{1t} - I_t - P_t c_t, \quad (6) \\
S_{t+1} & = S_t + I_t, \quad (7) \\
l_{2t} & = g(I_t/M_t), \quad (8) \\
l_t & = l_{1t} + l_{2t}. \quad (9)
\end{align*}
\]

Constraint (5) is the cash-in-advance constraint, equation (6) is the law of motion for the checking account, equation (7) is the law of motion for the savings account, equation (8) determines the adjustment cost, and equation (9) states that total non-leisure time is divided between work and adjusting the savings account.

**Equilibrium**

To close the model, the market clearing conditions for the goods and the asset markets need to be specified. The market clearing condition for the asset market is:

\[
M_{t+1} = D_{t+1} + S_{t+1} + \Pi_{t+1}, \quad (10)
\]

that is, the initial money stock in the next period has to equal retained earnings plus the demand for checking and savings deposits by the household. The goods market clearing constraint is:

\[
c_t = l_{1t}. \quad (11)
\]

Figure 4 summarizes the sequence of events in period \( t \). The first event of the day is the cash transfer \( X_t \) from the monetary authority to the bank, and the last event is the transfer \( I_t \) from the consumer’s checking account to the savings account. The cash-in-advance constraint is the requirement that

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\(^4\) The adjustment cost is defined relative to the money stock, so that adjustment costs are constant in a steady state with constant money growth.
the household cannot use receipts within the period to buy the consumption good, and the liquidity constraint requires that transfers between the checking and savings accounts are carried out only after all other transactions are settled.

It is convenient to redefine variables relative to the money stock at the beginning of the period. I will use lowercase letters to denote variables relative to the money stock, that is, \( p_t = P_t/M_t \), \( w_t = W_t/M_t \), \( x_t = X_t/M_t \), \( b_t = B_t/M_t \), \( i_t = I_t/M_t \), \( d_t = D_t/M_t \), \( s_t = S_t/M_t \), and \( \pi_t = \Pi_t/M_t \).

**Definition 1 (Equilibrium with Liquidity Timing)** Let \( \Omega_t \) denote the information that is known in period \( t \), i.e., the history of all monetary shocks up to and including period \( t \). An equilibrium given a money supply process \( x_t(\Omega_t) \) consists of allocation functions \( c_t(\Omega_t) \), \( l_t(\Omega_t) \), \( i_{1t}(\Omega_t) \), \( i_{2t}(\Omega_t) \), \( s_t(\Omega_t-1) \), \( \pi_t(\Omega_t-1) \), \( b_t(\Omega_t) \), \( i_t(\Omega_t) \) and price functions \( p_t(\Omega_t) \), \( w_t(\Omega_t) \), \( r_t(\Omega_t) \) such that:

1. The representative household solves:

\[
\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t) \right\} \quad \text{subject to:} \quad (12)
\]

\[
p_t c_t \leq d_t, \quad (13)
\]

\[
(1 + x_t) d_{t+1} = d_t + r_t s_t + \lambda \pi_t + w_t l_{1t} - i_t - p_t c_t, \quad (14)
\]

\[
(1 + x_t) s_{t+1} = s_t + i_t, \quad (15)
\]

\[
l_{2t} = g(i_t), \quad (16)
\]

\[
l_t = l_{1t} + l_{2t}. \quad (17)
\]

2. The representative firm solves:

\[
\max \{ p_t l_{1t} - w_t l_{1t} - r_t b_t \} \quad \text{subject to:} \quad (18)
\]

\[
w_t l_{1t} \leq b_t. \quad (18)
\]

3. The bank solves:

\[
\max \{ r_t (b_t - s_t) \} \quad \text{subject to:} \quad (19)
\]

\[
b_t \leq 1 + x_t - d_t, \quad (20)
\]

\[
(1 + x_t) \pi_{t+1} = (1 + r_t)(b_t - s_t) - \lambda \pi_t. \quad (21)
\]
4. The market-clearing conditions are satisfied:

\[ c_t = l_{1t}, \]
\[ 1 = d_{t+1} + s_{t+1} + \pi_{t+1}. \]  

Equilibrium with Cash-in-Advance Timing

The liquidity constraint and the adjustment cost serve a similar purpose in the model. In particular, the liquidity constraint can be interpreted as an infinite cost for immediate adjustments in savings. To disentangle the role of the liquidity constraint and the adjustment cost, I also consider a version of the model without the liquidity constraint. In this version of the model, timing is as in the usual cash-in-advance model, that is, consumers are allowed to adjust their savings after the monetary shock hits and before purchasing consumption goods. Figure 5 displays the time line for this version of the model. The definition of an equilibrium changes in two places.

Definition 2 (Equilibrium with Cash-in-Advance Timing) The definition of an equilibrium with cash-in-advance timing is identical to Definition 1 with the exception of two changes:

- The cash-in-advance constraint (13) is replaced by:
  \[ p_t c_t \leq d_t - i_t. \]  

- The reserve requirement (20) becomes:
  \[ b_t \leq 1 + x_t - (d_t - i_t). \]  

Equation (24) reflects the fact that only the balance on the checking account after adjusting savings can be used for consumption purchases, since the transfer to the checking account is made at the beginning of the period. Similarly, the reserve requirement changes to (25) since the balance on the consumer's checking account changes before the loan is given.
Equilibrium with Capital Accumulation

So far, we have concentrated on a model with labor as the only factor of production. The relative simplicity of this framework will be useful to work out how a monetary shock affects private households through the flow of assets between households and the corporate sector in isolation. A number of authors argue (see, for example, Dow Jr. (1995)), however, than endogenous accumulation of capital is an important channel for the transmission of monetary shocks. This section extends the basic model by introducing physical capital as a factor of production. This will allow us to assess whether the presence of additional assets leads to major modifications of our basic results.

The main choice that has to be taken when introducing capital in the model is whether households or firms hold capital. This is a choice of convenience only, since both versions lead to the same results. In our model, it turns out to be easiest to hand the capital to consumers directly. The reason is that if firms or intermediaries make investment decisions, they have to take into account exactly how consumers value different dividend streams. This is complicated in our model, since the retained-earnings assumption implies that the earnings of any given period are paid out to consumers over many different periods. We therefore hand the investment decisions to the households, and leave the bank’s and the firm’s problem mostly intact.

The decisions problem for households in the version with physical capital is to maximize utility (4) subject to (5), (7), (9), and the following modified constraints:

\[ D_{t+1} = D_t + r_t S_t + R_{Kt} k_t + \lambda \Pi_t + W_t l_t - I_t - P_t z_t - P_t c_t, \] (26)

\[ k_{t+1} = (1 - \delta) k_t + z_t, \] (27)

\[ l_2 t = g(I_t/M_t, z_t). \] (28)

Here \( k_t \) is the capital stock in period \( t \), \( z_t \) is investment, and \( \delta \) is the depreciation rate. There are two different interest rates: \( r_t \) is the interest paid on savings, and \( R_{Kt} \) is the return paid on physical capital. In a deterministic model, if both \( S_t \) and \( k_t \) are positive there would be an arbitrage condition requiring that \([1 - \delta] P_{t+1} + R_{Kt} \]/\( P_t = 1 + r_t \) holds. In the stochastic model, this condition holds only in expected value, since monetary shocks affect the two returns differently. Also notice that the adjustment cost function (28) now also depends on investment \( z_t \), which allows us to treat investment in physical capital (or equity) \( z_t \) symmetrically with investment in financial assets \( I_t \).

The central bank and the banking sector work just as before. The production sector now operates a constant-returns technology employing capital and labor. I assume that the technology is Cobb-Douglas with capital share \( \alpha \). The firm rents capital and labor from households, and needs to borrow the wage bill as well as the current cost of capital from the bank.
The optimization problem of the representative firm is:

$$\max_{l_{tt}, k_t, B_t} \left\{ P_t k_t^{\alpha} l_{tt}^{1-\alpha} - W_t l_{tt} - R_k t k_t - r_t B_t \right\} \text{ subject to:}$$

$$W_t l_{tt} + R_k t k_t \leq B_t.$$

Since the production function exhibits constant returns, firms make zero profits in equilibrium.

Once again, we will redefine variables relative to the money stock at the beginning of the period. The only additional variable that needs to be transformed is the return on capital: \( r_{K_t} = R_{K_t}/P_t \).

**Definition 3 (Equilibrium with Capital Accumulation)** An equilibrium given a money supply process \( x_t(\Omega_t) \) and initial capital level \( k_0 \) consists of allocation functions \( c_t(\Omega_t), l_t(\Omega_t), l_{tt}(\Omega_t), l_{tt}(\Omega_t), d_t(\Omega_t), s_t(\Omega_t), i_t(\Omega_t), k_t(\Omega_t), z_t(\Omega_t), \pi_t(\Omega_{t-1}), b_t(\Omega_t) \), and price functions \( p_t(\Omega_t), w_t(\Omega_t), r_t(\Omega_t), r_{K_t}(\Omega_t) \), such that:

1. The representative household maximizes utility (4) subject to (13), (15), (17), and the following constraints:

\[
(1 + x_t) d_{t+1} = d_t + r_t s_t + r_{K_t} k_t + \lambda \pi_t + w_t l_{tt} - i_t - p_t (z_t + c_t),
\]

\[
k_{t+1} = (1 - \delta) k_t + z_t,
\]

\[
l_{2t} = g(i_t, z_t).
\]

2. The representative firm solves:

$$\max \left\{ P_t k_t^{\alpha} l_{tt}^{1-\alpha} - W_t l_{tt} - r_{K_t} k_t - r_t b_t \right\} \text{ subject to:}$$

$$w_t l_{tt} + r_{K_t} k_t \leq b_t.$$  \hspace{1cm} (29)

3. The bank maximizes profits (19) subject to (20) and (21).

4. The market-clearing conditions (23) and:

\[
c_t + z_t = k_t^{\alpha} l_{tt}^{1-\alpha}
\]

are satisfied.

4 What Happens After a Monetary Shock?

This section takes a closer look at the decision problems of banks, firms, and households to analyze the effects of a monetary shock in the model economy. Throughout the analysis, money supply is assumed to be a first-order Markov process. I limit attention to stationary equilibria, so that decisions are functions of the state variables and the money supply shock only, and
for analytical convenience it is assumed that the cash-in-advance constraint holds with equality. The money supply process can always be chosen such that this condition is fulfilled in equilibrium, and in the computational part below the process is chosen accordingly. I will also concentrate on the version of the model without capital accumulation; the effects of introducing capital will be discussed in Section 5.

In the model without capital, both the firm and the bank have static problems which have simple solutions. Since households cannot react immediately to a monetary shock, it is therefore straightforward to determine the initial effects of a monetary shock by analyzing the firm and the bank in isolation. As long as the nominal interest rate is nonnegative, it is optimal for the firm to borrow exactly as much money as is required to pay wages. The first-order condition for the firm's problem (18) then implies:

\[ p_t = (1 + r_t)w_t. \]  

(30)

The only decision variable for the bank is the size of the loan \( b_t \) offered to the firm. If the interest rate is positive, it is optimal to lend the maximum amount possible. Then (20) holds with equality, and since \( b_t = w_t l_{1t} \) from the firm's problem, we have:

\[ w_t l_{1t} = 1 + x_t - d_t. \]  

(31)

The cash-in-advance constraint (13) and the goods market clearing constraint (22) imply that \( l_{1t} = d_t / p_t \), which allows us to rewrite (31) as:

\[ \frac{w_t}{p_t} = \frac{1 + x_t - d_t}{d_t}. \]  

(32)

Equation (32) links the real wage to the monetary shock and the balance on the checking account. From (30) we have \( 1 + r_t = p_t / w_t \), so that the interest rate is determined by:

\[ 1 + r_t = \frac{d_t}{1 + x_t - d_t}. \]  

(33)

Equations (32) and (33) give us some first results about the effects of monetary policy shocks. A contractionary money supply shock corresponds to a low realization of \( x_t \). Since \( d_t \) cannot be adjusted in response to the shock, equations (32) and (33) imply that this shock leads to an immediate drop in the real wage and rise in the interest rate. This is a standard outcome in models with a liquidity constraint. Because less funds are available in the credit market after a contractionary shock, the interest rate increases to clear the market. This in turn increases the wedge between price and wage, so that the real wage falls. Since the household’s labor supply depends on the real wage, output declines as well.
So far, we have established that a contractionary shock has the “right” effects on impact, i.e., interest rates rise and output falls. What does it take to make these effects persistent? Since the underlying source of the effects is a lack of funds in the credit market, the effects will be persistent if the fraction of funds used for purchases $d_t$ (the checking-account balance) is above its steady-state value in subsequent periods. Formally, notice that the interest rate is strictly increasing in $d_t$ in (33). If a large fraction of funds is used for purchases, once again less funds are available in the credit market, which reiterates the original effect of the contractionary shock.

To see how this relates to savings (and ultimately the adjustment cost), we can use the market-clearing condition $1 = d_t + s_t + \pi_t$ to rewrite (33) as:

$$1 + r_t = \frac{d_t}{x_t + s_t + \pi_t}.$$ (34)

(34) shows that the impact of a contractionary shock on the interest rate (which in turn affects output) is persistent if $d_t$ increases relative to $s_t + \pi_t$ after the shock. Notice that since bank profits depend on the size of the cash injection, $\pi_t$ is below steady state in the periods after a contractionary shock. Therefore, even if $s_t$ does not change, the fall in retained earnings will induce a rise in $d_t$ relative to $s_t + \pi_t$, which propagates the lack of funds in the credit market. In a frictionless model, households would consider the bank’s retained earnings as equivalent to their own savings, since the households own the bank. When retained earnings fall, households would increase their own savings to reach the preferred level of overall savings. With the portfolio adjustment cost, households change $s_t$ only slowly and do not offset the decrease in retained earnings entirely. Therefore the lack of funds in the credit market is propagated, and the effects of the original shock are persistent.

Notice that portfolio adjustment costs would not generate persistence if we did not allow for retained earnings. In the standard limited-participation model of Christiano and Eichenbaum (1995) all funds, including bank profits, flow to the household at the end of every period. In this model, even with an adjustment cost the share of savings in total funds does not decrease after a contractionary shock. In fact, numerical experiments in Section 4 will show that without retained earnings the long-run effects of a monetary shock are the exact opposite of the initial effect.

To build further intuition for the results, it is useful to examine how the decision problem of the household is modified by the various frictions in the model economy. I will start with the cash-in-advance version of the model, which allows households to adjust their savings after the monetary shock. The first-order conditions for the maximization problem of the household can be written as:

$$u_2(c_t, 1 - l_t)/w_t = \beta E_t \left\{ \frac{u_1(c_{t+1}, 1 - l_{t+1})}{(1 + x_t)p_{t+1}} \right\},$$ (35)
\[ u_2(c_t, 1 - l_t)g'(i_t) + u_1(c_t, 1 - l_t)/p_t = \beta E_t \left\{ \frac{u_1(c_{t+1}, 1 - l_{t+1})}{(1 + x_t)p_{t+1}} + \frac{u_2(c_{t+1}, 1 - l_{t+1})}{1 + x_t} \left[ \frac{r_{t+1}}{w_{t+1}} + g'(i_{t+1}) \right] \right\}. \]  

Condition (35) reflects the tradeoff between leisure today and consumption tomorrow. The cash-in-advance constraint implies that current labor income can be used for purchasing consumption goods only in the next period. The presence of \(1 + x_t\) in this equation drives a wedge between the marginal utility from leisure and the marginal utility from consumption. In the cash-in-advance model of Cooley and Hansen (1989) this wedge is the only source of real effects of monetary policy shocks. If the money supply process is autocorrelated, the current realization of the shock influences the expectation on the right hand side, which in turn affects incentives to work in the current period. This expectational effect does not help to explain why output falls after a contractionary shock. If a contractionary shock signals lower inflation in the future (implying lower \(x_t\) and a smaller value for the wedge), output would in fact increase, which is the opposite of what is observed in the data.

Condition (36) comes into play when the household decides on savings. If there is no adjustment cost and \(g(i_t)\) is zero, (36) simplifies to:

\[ u_1(c_t, 1 - l_t)/p_t = \beta E_t \left\{ \frac{u_1(c_{t+1}, 1 - l_{t+1})}{(1 + x_t)p_{t+1}} + \frac{r_{t+1}u_2(c_{t+1}, 1 - l_{t+1})}{(1 + x_t)w_{t+1}} \right\}. \]  

(37) can be further simplified if the money supply process is i.i.d. Since we are considering stationary equilibria, with i.i.d. shocks the expectations in (35) and (37) are constants, which implies that consumption and labor are constant, and the price and wage are inversely proportional to \(1 + x_t\). Hence, with zero adjustment cost, cash-in-advance timing, and i.i.d. shocks, there are no real effects of monetary shocks.

Let us now see how the decision problem of the consumer is modified if we introduce the liquidity constraint, so that the household cannot adjust savings immediately after a monetary shock. With liquidity timing, the first-order conditions are (35) and:

\[ u_2(c_t, 1 - l_t) \left[ \frac{1}{w_t} + g'(i_t) \right] = \beta E_t \left\{ \frac{u_2(c_{t+1}, 1 - l_{t+1})}{1 + x_t} \left[ \frac{1 + r_{t+1}}{w_{t+1}} + g'(i_{t+1}) \right] \right\}. \]  

(38) If \(g(i_t)\) is identically zero, (38) simplifies to:

\[ u_2(c_t, 1 - l_t)/w_t = \beta E_t \left\{ \frac{u_2(c_{t+1}, 1 - l_{t+1})}{(1 + x_t)w_{t+1}} \right\}. \]

Using (35), applying the law of iterated expectations, and shifting one period backwards yields:

\[ E_{t-1} \{u_1(c_t, 1 - l_t)/p_t\} = \beta E_{t-1} \left\{ (1 + r_t) \frac{u_1(c_{t+1}, 1 - l_{t+1})}{(1 + x_t)p_{t+1}} \right\}. \]
This equation is similar to (37), but notice that the time index on the expectation operator has shifted back one period. The household still faces the same incentives with regards to savings, but since savings cannot be adjusted immediately, the first-order condition only holds in expected value. Within the period, no adjustment is possible, and real effects arise due to the lack or abundance of funds in the credit market.

We therefore see that the liquidity constraint as such without adjustment cost leaves the optimality constraint intact, in the sense that it still holds in expected value one period ahead. The agents can be surprised, but just for one period. Therefore, real effects of monetary shocks do not persist. However, if adjustment costs are not zero, the presence of the adjustment-cost function in (38) introduces further modifications to the optimality conditions. Given sufficiently high adjustment costs, the conditions can be changed in almost arbitrary ways. The interesting question then becomes whether realistically small adjustment costs are sufficient to generate persistence. This question is addressed in the next section.

5 Small Adjustment Costs and Persistence

In this section I discuss the quantitative properties of the model. Simulations require the choice of specific functional forms and parameter values. The parameterization of preferences and technology follows Christiano and Eichenbaum (1995). The utility function is:

\[ u(c_t, 1 - l_t) = \psi \ln (c_t) + (1 - \psi) \ln (1 - l_t). \]

The model period is chosen to be one quarter. The utility parameters were set to \( \beta = 0.99 \) (corresponding to a risk-free real interest rate of 4 percent p.a.) and \( \psi = 0.24 \). The adjustment-cost function is quadratic:

\[ g(i_t) = a(i_t + b)^2 = a((1 + x_t)s_{t+1} - s_t + b)^2. \]

The parameter \( b \) in the adjustment cost function was set such that adjustment costs are zero in the deterministic steady state. The parameter \( a \) is left unspecified for now. The dividend-payment parameter \( \lambda \) was set to match the evidence on the effects of profits on dividends in Table 1. The table shows that 50 percent of the total cumulative impact on dividend payments is reached between four and five quarters after a shock to corporate profits. To match the half-life of retained earnings in the model to this observation, \( \lambda \) is chosen such that \((1 - \lambda)^{4.5} = 0.5\), which results in \( \lambda = 0.14 \).

The process for money supply is given by:

\[ x_{t+1} = \bar{x} + \rho x_t + \sigma \epsilon_{t+1}, \]
where \( \varepsilon_{t+1} \) is i.i.d. normal with mean zero and unit variance. \( \bar{x} \) was chosen to result in an average inflation rate of one percent per quarter or four percent per year, which matches average inflation in the U.S. from 1959 to 2002 in the data set used in Section 2. The parameter \( \sigma \) governs the standard deviation of (quarterly) inflation, and was set to \( \sigma = 0.006 \) to match the empirical counterpart. Different values for \( \rho \) were used to explore the impact of persistence in the money supply process on the economic consequences of monetary shocks. Specifically, we will compare outcomes under i.i.d money shocks (\( \rho = 0 \)) and outcomes with a correlation coefficient of \( \rho = 0.5 \). The i.i.d case has the advantage that it isolates the effects of the liquidity constraint and adjustment costs from the anticipated-inflation effects discussed by Cooley and Hansen.

In a stationary equilibrium the decisions of households, firms, and the bank are functions of the state variables \( d_t, s_t, \) and \( x_t \) only. Retained earnings are given by \( \tau_t = 1 - d_t - s_t \) and therefore do not need to be included as a separate state variable. The problems of the firm and the bank are static and can be computed directly. The household's decisions can be summarized by policy functions that map the state vector into household decisions. Since analytical solutions are not available, numerical approximations to the true decision rules were computed. Given the policy functions, I simulated the economy to derive the effects of a monetary shock. Specifically, I computed impulse response functions for a contractionary money supply shock for different versions of the model. The graphs in Figures 6 to 12 show the response of a number of variables to a shock over a period of twelve quarters. The economy starts out in the steady state in the first quarter. In the second quarter a one-standard-deviation negative money supply shock hits, which implies that the money shock grows by 0.4 percent instead of one percent.

The Role of Adjustment Costs and Retained Earnings

Figure 6 shows the results for the model with liquidity timing and zero adjustment cost, i.e., \( a = 0 \). To isolate the effect of the liquidity constraint from the expected-inflation effect, we set the autocorrelation of money supply to zero for now, i.e., \( \rho = 0 \). As discussed in the last section, in this case the real effects of the monetary shock last only one period. On impact of the shock, output, labor supply, and consumption (which are all equal in the labor-only model) fall, while the interest rate rises in response to the lack of funds in the credit market. At the end of the period, the household adjusts savings, and all real variables return to their steady-state value. Notice how the household adjusts savings to exactly offset the change in retained earnings in order to keep overall savings at the desired level. The inflation rate increases on impact of the shock, and declines only in the following period. The negative monetary shock cannot lower prices immediately, because the

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5 More specifically, a perturbation method was used. I computed second-order approximations to the policy functions, following the approach of Judd and Guu (1997) and Schmitt-Grohe and Uribe (2004).
amount of funds $d_t$ available in the goods market is fixed, and the higher interest rate raises the cost of production from the perspective of the firm. If the process for money supply were positively autocorrelated instead of i.i.d., there would be persistent effects, albeit in the wrong direction. A negative monetary shock would lower expected inflation and therefore increase the incentive to work, leading to higher future consumption.

To gauge which range of adjustment costs is reasonable, we can use the results displayed in Figure 6 to compute which adjustment costs the household would face for any given $a$ if we hold constant the optimal path of savings at no adjustment cost. If the adjustment cost parameter were $a = 10$ instead of zero, but the household still followed the same path of savings, the adjustment cost would peak at 0.02 percent of time worked. Assuming that an average worker works about 500 hours in a quarter, this translates into a maximum adjustment time of about six minutes per quarter. Even this maximum cost appears moderate relative to the actual time people spend on financial transactions. Note that if the adjustment cost is imposed, the actual cost will be even lower, because the consumer can modify the saving decision.

Figure 7 shows results for the same model with an adjustment cost parameter of $a = 10$. The fall in output is more pronounced than in the model without adjustment costs, and persists for about three quarters after the shock. The effect on the interest rate is also persistent. By comparing the graphs for savings in Figures 6 and 7, we can see that the rise in savings is much smaller with the adjustment cost. The household still makes up for part of the decrease in retained earnings, but the increase is not offset completely, and the checking-account balance increases relative to the money stock. Compared to the cost that would accrue with the path of savings in Figure 1, adjustment costs are miniscule. The maximum adjustment cost makes up about 0.0002 percent of time worked, which translates into less than three seconds per quarter.

Thus, Figure 7 shows that even small adjustment costs can have a large impact on the behavior of the model. The intuition for this result is the familiar "the hill is flat at the top" argument. The household aims to achieve an optimal balance between consumption and leisure. It follows from the first-order conditions that near the optimum substituting consumption for leisure has only small effects on utility. In other words, at the optimum the consumer is locally indifferent with respect to reallocations of consumption and leisure. Therefore even small adjustment costs can have sizable effects on the optimal allocation.

The size and persistence of the real effects can be increased by choosing a higher adjustment cost parameter $a$ and a lower retained-earnings parameter $\lambda$. As $a$ increases, the consumer adjusts savings less and less, and as $\lambda$ decreases, retained earnings flow more slowly to the household. However, the results with $a = 10$ and $\lambda = 0.14$ displayed in Figure 7 are close in magnitude to the maximum that can be achieved. Increasing $\lambda$ adds at
Figure 6: Impulse Response with Liquidity Timing, $a = 0, \lambda = 0.14, \rho = 0$

Figure 7: Impulse Response with Liquidity Timing, $a = 10, \lambda = 0.14, \rho = 0$

Figure 8: Impulse Response with Liquidity Timing, $a = 10, \lambda = 1, \rho = 0$
most two quarters to the duration of the effects. Even if we increase $a$ to infinity (i.e., savings are fixed at the steady-state level, the consumer does not adjust at all), the economy returns to the steady state at the rate at which retained earnings are handed out to the consumer.

Figure 8 shows what happens if we do not allow for retained earnings. Here we set $\lambda = 1$, so that all profits are credited to the consumer in one piece, although given our timing assumptions there is still a lag of one period. While the effects on impact of the shock are similar, there is no persistence at all. To the contrary, output falls only in the impact period, and is above steady state starting in the second period after the shock. What is happening is that in the period after the shock the low profits of the bank are reflected in a lower balance on the consumer’s checking account. Earnings are credited all at once, and after the negative shock the transfer from the bank is much lower than usual. To re-balance assets, the consumer now would have to transfer funds from the savings to the checking account. This adjustment is slowed down by the adjustment cost, however, and the household is stuck with savings that are too high relative to other assets for a few periods. Therefore an overabundance of funds in the credit market arises, and the initial effects of the monetary shock are reversed. Hence, we see that both the adjustment cost and retained earnings are necessary to generate persistent real effects.

**Autocorrelated Money Shocks**

So far, we have only considered the case of i.i.d. money shocks. While this proved useful to illustrate the effect of the liquidity constraint, we know that in the real world money shocks are autocorrelated. Autocorrelation in money supply may offset the negative effect on output of a contractionary monetary shock, since in models without adjustment costs, a lower expected inflation rate eases monetary distortions and tends to raise output. We will see, however, that in the adjustment-cost model the expected-inflation effect is more than offset by other factors. Figure 9 shows outcomes for the same parameters as in Figure 7, but with a positive autocorrelation in money supply of $\rho = 0.5$. The initial effect on output and prices is very similar to the case of i.i.d. shocks. Subsequently, however, we see that the effects of the money shock are considerably more persistent when money supply is autocorrelated. While the expected-inflation effect is present, it is offset by additional liquidity effects arising in the periods after the initial shock. The initial contractionary shock signals that further negative shocks to money supply will follow in future periods. Due to the presence of adjustment costs, the household does not fully offset these future shocks in advance, so that the original liquidity effect is repeated on a smaller scale. Compared to this additional liquidity effect, the lower expected inflation rate has only a minor positive impact. Autocorrelation in money supply therefore amplifies the effects of monetary shocks relative to the i.i.d. model. Due to the larger
required adjustments, the total adjustment cost is higher than in the i.i.d. case, but still extremely small. The maximum adjustment cost now makes up about 0.0012 percent of time worked, which translates into less than 23 seconds per quarter.

In the models considered so far, there are in effect two adjustment costs present: an infinite adjustment cost for immediate changes to savings (the liquidity constraint), and a lower adjustment cost for later reallocations. Figure 10 shows results with the same parameters that were used for Figure 9, but this time the model with cash-in-advance timing is used. In other words, the consumer can adjust savings right after the monetary shock is revealed, so that only one source of adjustment cost is present. Since this model has a different steady state, the outcomes are shifted relative to Figure 9. Apart from the shift, it is remarkable how similar the outcomes are. The effect of the monetary shock on output and prices is virtually identical.

Figure 9: Impulse Response with Liquidity Timing, $a = 10$, $\lambda = 0.14$, $\rho = 0.5$

Figure 10: Impulse Response with Cash-in-Advance Timing, $a = 10$, $\lambda = 0.14$, $\rho = 0.5$
in the two versions of the model. This implies that once adjustment costs are introduced into the model, the liquidity constraint is no longer required to generate real effects of monetary shocks.

**Capital Accumulation**

Yet another potential source of propagation of monetary shocks is endogenous capital accumulation. To make results comparable, the calibration of the capital model is identical to the labor-only model wherever possible. The adjustment cost function now also depends on investment $z_t$ and is specified as:

$$g(i_t, z_t) = a(i_t + b)^2 + a_K(i_t + b_K)^2,$$

where $b_K$ is chosen to result in zero adjustment costs in the deterministic steady state. The capital share is set to $\alpha = 0.33$, which matches its empirical counterpart in the U.S., and the depreciation rate is $\delta = 0.02$ per quarter. Figure 11 shows results for the capital model with adjustment costs of $a = 10$ and $a_K = 1$, and an autocorrelation parameter of money supply of $\rho = 0.5$. The adjustment cost parameter for capital is chosen to be smaller than the one for savings, since investment is much higher than the transfer to the savings account in the steady state. The results for output, prices, and assets are very similar to the result of the corresponding model without capital (see Figure 9). Notice that the return on capital is temporarily lower than the interest rate, since the higher cost of financing lowers returns to capital owners. In the long run, the interest rate and the return on capital cannot move in different directions, since bank savings and capital are competing investments from the perspective of the household. An immediate adjustment does not take place, however, due to the presence of adjustment costs. The stock of capital declines slightly after the initial shock, amplifying the negative effect on output. Quantitatively, however, the decline in capital is very small.

Figure 12 shows outcomes for the same model without an adjustment cost for capital, $a_K = 0$, but keeping the adjustment cost for asset transfers at $a = 10$. This model leads to noticeably different results. Output still declines after the shock, and the effect is more persistent than before. Consumption, however, increases after the monetary shock, contrary to what happened in the other models. In this model, consumption growth is driven by the return on capital, which is not subject to adjustment cost. As before, the return on capital falls after the monetary shock, partly because labor supply falls, and partly because the financing cost increases. A lower return on capital implies that the household desires a lower consumption growth rate, which increases current consumption at the expense of future consumption. The household therefore consumes more and invests less. A few periods down the road, however, the by then lower capital stock pushes consumption below its steady-state value. The immediate jump in consumption also
Figure 11: Impulse Response with Capital Accumulation, $a = 10$, $a_K = 1$,
$\lambda = 0.14$, $\rho = 0.5$

Figure 12: Impulse Response with Capital Accumulation, $a = 10$, $a_K = 0$,
$\lambda = 0.14$, $\rho = 0$
leads to a lower initial price level, through the cash-in-advance constraint. Thus, the behavior of inflation is also different in this model.

We therefore see that the behavior of the capital model depends crucially on the adjustment cost for capital. With a cost, the outcomes are basically the same as in the labor-only model. Without an adjustment cost, there is still a persistent decline in output, but the reaction of consumption changes significantly. Given that the model period is only one quarter, the case with an adjustment cost is probably more realistic, especially considering that time-to-build or time-to-plan frictions may also be present.

<table>
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Table 2: The Autocorrelation of Output

The Autocorrelation of Output

To summarize the effects of adjustment costs and retained earnings on persistence, Table 2 displays the autocorrelation function of output in different versions of the model. With i.i.d. money shocks and no adjustment costs (not shown), the autocorrelation function is zero at all lags. With i.i.d. shocks and an adjustment cost of $a = 10$, autocorrelation is positive, but fades out after three quarters. With an adjustment cost but without retained earnings ($\lambda = 1$), the autocorrelation is negative from the second quarter on, due to the reversal in the reaction of output displayed in Figure 8. If the money shock is persistent, output inherits this quality. With $\lambda = 0.14$ and $\rho = 0.5$, autocorrelation increases at all lags, and fades out more slowly. Introducing capital does not change the results, as long as an adjustment cost for capital is present. Finally, in the model with freely adjustable capital, the autocorrelation of output decays more slowly than in any of the other models.
6 Conclusions

Our quantitative results show that portfolio adjustment costs are a promising explanation for the persistent effects of monetary policy shocks observed in the data. In the baseline calibration, the effect of a contractionary shock on output lasts about a year. At the same time, the realized adjustment cost never exceeds the equivalent of three seconds per quarter. The results also show that institutional features concerning the flow of funds between households and the business sector may be an important determinant of the monetary transmission mechanism. In the model, banks retain earnings and credit profits to the households only with a delay. While the timing of dividends does not have any consequences in a frictionless model, in the adjustment-cost model accounting for retained earnings turns out to be essential for generating persistence.

The results suggest two promising areas for future research. First, in the present model retained earnings are introduced as an exogenously given feature of the economy. The economic determinants of these arrangements should be investigated more closely. Also, more information is needed on the exact quantity and timing of the flow of funds between firms and households. In the language of the model, measuring $\lambda$ is as important as measuring the adjustment cost.

Second, a limitation of the current framework is that the adjustment-cost function is assumed to be convex. This assumption was necessary to generate continuous policy functions within a representative-agent framework. From a microeconomic perspective, a more realistic formulation for the adjustment cost would be a fixed cost for each transaction that is independent of the size of the transaction. Once the consumer has walked to the bank or called the broker on the phone, the size of the ensuing transaction should no longer matter. With a fixed cost, however, the representative-agent framework reaches its limits. In a fixed-cost model, the effects of monetary policy shocks are a function of the fraction of agents who adjust their portfolio in a given period. This fraction is restricted to be either zero or one in the representative-agent model.

Unfortunately, models with heterogeneous agents and fixed costs are difficult to analyze. Alvarez, Atkeson, and Kehoe (2002) present a model with heterogenous agents, fixed costs for portfolio adjustments, and monetary shocks, but to simplify the analysis they consider an endowment economy, so that output and employment cannot react to to shocks. Solving a heterogenous-agent model with fixed costs that allows for real effects of monetary shocks would constitute substantial progress towards understanding the monetary transmission mechanism.
References


