Education supply, economic growth and the dynamics of skills*

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1 Introduction

This paper explores the link between social mobility and growth through investment in human capital. Our main interest is how the supply of high-skilled workers evolves over time in the short-term as well as in the long-term, how it is affected by education policies, and how it relates to the rate of growth.

In most countries we observe during the last decades a significant increasing proportion of skilled individuals among the population[1]. But simultaneously some developing countries (like most of sub-saharian countries) remain with a very large proportion of low-skilled workers.

Macroeconomic models related to education, such as Lucas (1988), cannot account satisfactory for these "education traps". Azariadis and Drazen (1990) provide an explanation for economic development based on increasing returns to education. Their model explains why economies with similar technologies will converge to different balanced growth paths. Unfortunately, their assumption of increasing returns has no microeconomic foundations. Moreover their framework cannot account for distributional effects. Galor and Zeira (1993) stressed that the existence of a fixed cost

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[1] In the United States, the student population increased on average by 4.4% per year from 1970 to 1980, and by 2.6% per year since the beginning of the 1990's. As a consequence, the supply of skills has been deeply modified. The proportion of workers with less than high-school education fell from 36.1% in 1970 to 11.1% in 1994 (see Phelps and Zoega, 1996). They also find similar patterns for the other G7 countries.

to enter education creates increasing returns to education if credit markets are imperfect. For any initial distribution of wealth, this induces long-term segregation between educated and non-educated families. A feature of their model, however, is that the long-term proportion of educated individuals is always below the initial one and always decreases during the transition path[2]. This seems inconsistent with stylized facts in most countries.

We propose a particular, but natural, assumption for the financing of schooling. Altruistic parents finance (or not) university for their children. More precisely, they may finance the part of the university cost which is not publicly funded. Since wealth is correlated with the education level, high-skilled (HS) parents are more likely to pay higher education for their children than low-skilled (LS) parents. However, the model provides an original result since, for some ranges of the distribution, complete intergenerational immobility occurs[3].

Our second main assumption concerns the human capital accumulation mechanism at school. This accumulation depends on the "quality of schooling". Several empirical offer cautious support for the position that education resources, such as the teacher-pupil ratio, matter. These resources tend to improve, on average, student's test scores, graduation rates and/or student's market outcomes (see for example Card and Krueger, 1992, or Sander, 1993)[4]. This assumption is not new in the macroeconomic literature. Glomm and Ravikumar (1992) as well as Bénabou (1996) use the education expenditures per student as a proxy for the quality of education. This formulation has two main shortcomings. First, it neglects the opportunity cost of teaching : teachers are withdrawed from the production of the final goods. Eicher (1996) shows that this opportunity cost has major implications for the relative supply of skilled workers. Second, for non-compulsory schooling, not only the number of teachers but also that of students is endogenous. Consequently the quality of education, as measured by the teacher-pupil ratio, is a function of a number of model parameters including the initial distribution of income. We can then argue that there is a "congestion effect" in education as the number of students (endogenously) increases.

Our framework is consistent with the different dynamics observed in the structures of skills across countries. As in other models with indivisibilities in education spending and imperfect credit markets, we show that the initial level of skills in a family affects the skills of their offsprings. The presence of congestion effects also has major implications for the possible dynamics of skills in the economy and, consequently, for the long-term effects of education policies. Similar education policies may have different macroeconomic implications depending on the distribution of skills. Contrary to Eckstein and Zilcha (1994), an increase in public education expenditure

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2 We are indebted to Cecilia García-Peñalosa for this remark.

3 Obviously, this result may not hold if agents also differ in their abilities to acquire human capital. See on this point, Loury (1981) as well as Chiu (1998).

4 The debate on this subject is highly controversial. For example, Hanushek and *al.* (1996) advocates that there is little evidence that the level of school resources has a statistically significant effect on test scores.

may not have long-term positive effects on growth. In the short-term, such a policy could stimulate enrollment at university, but the resulting aggravation of the congestion effect may discourage a durable increase in the number of students. Such economies will remain in a trap. As in Azariadis and Drazen (1990), this endogenous emergence of education traps results from threshold effects. But these thresholds depend here on educational policies and on the human capital technology in university formation. There are also explained by historical structures of skills. We say an economy is captured in a low-skill trap when education policies are unable to (permanently) increase the supply of skills. Yet these traps may occur at widely varying levels of equilibrium human capital. Thus our model complements the literature on "development traps". Becker, Murphy and Tamura (1990) argue that poverty traps are due to endogenous fertlity decisions. Eicher and García-Peñalosa (1999) explain traps as a result of an interdependance between the supply and the demand for skilled labor, when technical change is skill-biased. Acemoglu (1996) and Redding (1996) claim that coordination problems between firms and workers may also explain such traps. Our explanation is based on technological assumptions but does not necessary involve technical change[5].

The paper is organized as follows. The following section presents the basic model. Section 3 provides an analysis of the short-term equilibrium. In section 4, we present the dynamics. The study of the effects of the proportion of high skilled is provided in section 5. In section 6, we study development paths and assess the effects of educational policies. The last section contains some concluding remarks.

2 The model

We consider a three period lived overlapping generations model. Each period t, there is a continuum of agents born at t of weight 1 which are identical, but their parents are not.

Each agent born at t receives a basic formation, and a proportion of them receives an additional formation (say at university) during their first period of life. In period $t+1$, when adult, he works, consumes, saves and may pay the fee (finance the university) for his child. In his last period of life $t+2$, he is retired and consumes the gross return of his savings.

Parents in period t are either high skilled (HS) or low skilled (LS). The p_t HS workers are those who benefited the additional formation in period $t-1$. Their human capital level is h_t^s and their wage income is $w_t h_t^s$, where w_t is the wage per unit of efficient labor. The human capital of the $1-p_t$ LS parents (those who only received the basic formation in period $t-1$) is

[5] See Azariadis (1996) for a survey on the literature on poverty traps.

h_t^o and their wage income is $w_t h_t^o$. Parents choose to pay or not to pay the fee (cost of additional formation) for their child.

2.1 The educational system

The basic formation in period t applies to all agents and it is financed publicly. It leads to a human capital level

$$h_{t+1}^o = \Phi\left(\alpha_t^s p_t, \alpha_t^o (1 - p_t)\right) \bar{h}_t \tag{1}$$

which depends on the number $\alpha_t^s p_t$ of HS teachers (α_t^s is the proportion of the p_t HS workers who are teachers in the basic school), on the number $\alpha_t^o(1 - p_t)$ of LS teachers and on the mean human capital level (of adults) in the economy at date t :

$$\bar{h}_t = p_t h_t^s + (1 - p_t)\, h_t^o \tag{2}$$

Assumption 1 : Φ is an increasing function of its arguments: $\Phi_1 > 0$, $\Phi_2 > 0$.

Assumption 1 follows empirical studies which conclude to a positive, significant correlation between the number of teachers per student and the market outcomes of students (Card and Krueger (1992)). In the compulsory formation, an increase in the number of teachers improves the teacher-pupil ratio.

The cost of the basic education is the total wage cost of teachers[6] :

$$\alpha_t^s p_t w_t h_t^s + \alpha_t^o (1 - p_t)\, w_t h_t^o$$

This cost will be paid by the government through taxes on the wage income.

The additional formation in period t only applies to p_{t+1} children, whose parents accept to pay a fee, γ_t. The main cost of education is given by the teacher's wages, and higher education is provided mainly by HS teachers. In order to capture this effect, we make the simplifying assumption that the fee is proportional to the current HS wage :

$$e_t^s = \gamma_t w_t h_t^s \tag{3}$$

A low skill parent has to spend a higher share of his income than a high skill parent in order to finance university to his child. The production

[6] Workers and teachers wages are the same for an equal qualification (non arbitrage property). A simple interpretation of the two types of teachers is the following. To teach the basic formation it is sufficient to have had it, *i.e.* both LS and HS can do it. But HS are more efficient and there is some complementarity.

of h^s depends on three arguments: the number of university professors the government finances $(\beta_t p_t)$, the fee, γ_t, and the number of students p_{t+1}. Specifically:

$$h^s_{t+1} = \Psi\left(\beta_t p_t, \gamma_t, p_{t+1}\right) h^o_{t+1} \tag{4}$$

Assumption 2 : Ψ is an increasing function of $\beta_t p_t$ and γ_t, but a decreasing function of the number of students p_{t+1}. Moreover $\Psi(0, 0, p) = 1$ for all $p \geqslant 0$.

The university formation increases the human capital level which results from the basic formation. Nonetheless, without spendings ($\beta_t = \gamma_t = 0$) there is no supplementary human capital accumulation. The number of students reduces the productivity of the university formation because students congest the education system. Indeed, our function Ψ may be viewed as the reduced form of a formulation stressing the incidence of the teacher-pupil ratio. At period t, there are $\beta_t p_t + \gamma_t p_{t+1}$ teachers and p_{t+1} students. If the productivity in the university formation depends on its teacher-pupil ratio, one obtains a function of $\frac{\beta_t p_t + \gamma_t p_{t+1}}{p_{t+1}} = \frac{\beta_t p_t}{p_{t+1}} + \gamma_t$. Such a specific function verifies assumption 2.

The total financing by the government (including the basic formation) is

$$g_t = \left(\alpha^s_t + \beta_t\right) p_t w_t h^s_t + \alpha^o_t \left(1 - p_t\right) w_t h^o_t \tag{5}$$

Assumption 3 : $\alpha^s_t > 0$, $\alpha^o_t > 0$, $\beta_t > 0$, $\gamma_t > 0$, $\alpha^s_t + \beta_t < 1$, $\alpha^o_t < 1$ for all t.

We assume positive and feasible proportions of all types of teachers as well as a positive fee. We are then concerned with mixed education systems, as they exist in most countries.

2.2 The consumers

An agent born in $t-1$ works in period t. His human capital h_t is either h^s_t or h^o_t according to he has received or not the university formation.

With a tax rate τ_t on his wage income, the after-tax income of an agent with a human capital h_t is $(1-\tau_t) w_t h_t$. He consumes c_t, saves s_t, and spends e_t for the university formation of his child. He chooses e_t either equal to e^s_t and the child receives the university formation, or he chooses $e_t = 0$ and his child does not receive this formation. Consequently:

$$\begin{cases} h_{t+1} = h^s_{t+1} & \text{if } e_t = e^s_t \\ h_{t+1} = h^o_{t+1} & \text{if } e_t = 0 \end{cases}$$

He maximizes his life-cycle utility which depends on the human capital level of his child. For reasons of tractability, we assume a log-linear utility function[7] :

$$U_{t-1} = (1-a)\ln c_t + a\ln d_{t+1} + b\ln h_{t+1} \tag{6}$$

subject to the budget constraints:

$$(1-\tau_t)\, w_t h_t = c_t + s_t + e_t \text{ and } d_{t+1} = (1+r_{t+1})\, s_t \tag{7}$$

a and b verify : $0 < a < 1$ and $b > 0$; r_{t+1} is the interest rate on savings and d_{t+1} the consumption in period $t+1$.

Whatever the choice of e_t, the optimal decisions of consumption and savings are:

$$c_t = (1-a)(\Omega_t - e_t)\,, s_t = a(\Omega_t - e_t)\,, d_{t+1} = (1+r_{t+1})\, s_t \tag{8}$$

where $\Omega_t = (1-\tau_t)\, w_t h_t$ is the after-tax income.

The difference between the two life-cycle utilities U^s_{t-1} and U^o_{t-1} corresponding the choice of $e_t = e^s_t$ and $e_t = 0$ is

$$\begin{aligned} U^s_{t-1} - U^o_{t-1} &= \ln(\Omega_t - e^s_t) + b\ln h^s_{t+1} - \left(\ln\Omega_t + b\ln h^o_{t+1}\right) \\ &= \ln\left(\frac{(\Omega_t - e^s_t)\,\Psi^b_{t+1}}{\Omega_t}\right) \end{aligned}$$

where $\Psi_{t+1} = h^s_{t+1}/h^o_{t+1} = \Psi(\beta_t p_t, \gamma_t, p_{t+1})$ is the ratio of the two possible human capital levels for children.

As a consequence, at date t, the optimal choice of an agent with an after-tax income Ω_t is

$$\begin{aligned} e_t = e^s_t \text{ if } \Omega_t &> \frac{e^s_t}{1-\Psi^{-b}_{t+1}} \\ e_t = 0 \text{ if } \Omega_t &< \frac{e^s_t}{1-\Psi^{-b}_{t+1}} \end{aligned} \tag{9}$$

He is indifferent between the two choices when $\Omega_t = \frac{e^s_t}{1-\Psi^{-b}_{t+1}}$ and may choose $e_t = e^s_t$ or $e_t = 0$.

2.3 The firms

There is in each period one representative firm producing one good with two inputs, physical capital K_t and human capital (or efficient labor) H_t. The production function $Y_t = F(K_t, H_t)$ is homogenous of degree one, increasing

[7] This utility may depend on the net income of the child $(1-\tau_{t+1})w_{t+1}h_{t+1}$: this would not modify the decisions since the utility function is assumed to be log-linear.

and strictly concave with respect to each of the two production factors. The firm maximizes its profits : $F(K_t, H_t) - w_t H_t - (\delta + r_t)K_t$. w_t is the wage, δ the depreciation rate of capital and r_t the interest rate. At equilibrium the wage is equal to the marginal productivity of labor :

$$w_t = F'_L(K_t, H_t) = F'_L(k_t, 1) \tag{10}$$

where $k_t = K_t/H_t$ is the ratio of the capital stock and the human capital level used in production. The interest rate is equal to the marginal productivity of capital net of depreciation :

$$r_t = F'_K(K_t, H_t) - \delta = F'_K(k_t, 1) - \delta \tag{11}$$

3 Short-term equilibrium

At date t, the capital stock K_t is given and results from the savings decisions made in period $t-1$. The proportions p_t of HS and $1 - p_t$ of LS workers are given and result from the education decisions taken by parents in period $t-1$. The corresponding human capital levels h_t^s and h_t^o are known.

3.1 The tax rate

Given educational policies α_t^o, α_t^s, β_t, the government spendings are given by the relation (5). The tax rate τ_t is assumed to be chosen in order to balance the government constraint. Since all workers (including teachers) pay the same tax rate, the tax income $\tau_t(p_t w_t h_t^s + (1 - p_t)w_t h_t^o)$ should be equal to the government spendings g_t. Using the ratio $\Psi_t = h_t^s/h_t^o$, we obtain :

$$\tau_t = \frac{(\alpha_t^s + \beta_t)p_t\Psi_t + \alpha_t^o(1 - p_t)}{1 - p_t + p_t\Psi_t} \tag{12}$$

Note that $\tau_t \leqslant \max\{\alpha_t^s + \beta_t, \alpha_t^o\}$ which implies $\tau_t < 1$.

3.2 The number of students at equilibrium

The productivity of the university formation $\Psi(\beta_t p_t, \gamma_t, p_{t+1})$ depends on the number p_{t+1} of students. Since the after-tax income of HS workers is larger than the LS one, there are three possibilities :

a) all HS pay the university cost for their child, and no LS does it.

b) at least some LS pay it, and then all HS pay it.

c) at leat some HS do not pay it, and then no LS pay it.

Let us define, for fixed values of $\beta_t \geqslant 0$, p_t, $0 \leqslant p_t \leqslant 1$ and $\gamma_t > 0$,

$$\rho_t(p_{t+1}) = \frac{\gamma_t}{1 - \Psi\left(\beta_t p_t, \gamma_t, p_{t+1}\right)^{-b}} \tag{13}$$

$\rho_t(p_{t+1}) w_t h_t^s$ is the minimum after-tax income for which a parent accepts to pay the university cost for his child; $\rho_t(p_{t+1})$ is a measure of the ratio of costs and benefits of the additional education. The function $\rho_t(p_{t+1})$ is well defined and increasing for all $p_{t+1} \geqslant 0$, since the denominator does not vanish, $\Psi(\beta_t p_t, \gamma_t, p_{t+1}) \geqslant \Psi(0, \gamma_t, 1) > 1$.

We first consider interior values $0 < p_t < 1$. The three possibilities are the following:

Case a. The students are the children of the HS workers, and $p_{t+1} = p_t$. For this to be an equilibrium with optimal choice of education by both types of parents, the following inequalities should be satisfied with $p_{t+1} = p_t$:

$$\Omega_t^s = (1 - \tau_t)\, w_t h_t^s \geqslant \rho_t(p_{t+1}) w_t h_t^s \text{ and } \Omega_t^o = \frac{1}{\Psi_t} \Omega_t^s \leqslant \rho_t(p_{t+1}) w_t h_t^s$$

or equivalently, the tax rate should satisfy :

$$\rho_t(p_t) \leqslant 1 - \tau_t \leqslant \Psi_t \rho_t(p_t) \text{ with } \Psi_t = h_t^s / h_t^o \tag{14}$$

Case b. At least some of the LS workers pay the university fee and $p_{t+1} > p_t$. There are two subcases :

- when all LS workers pay the cost, $p_{t+1} = 1$, their income Ω_t^o should at least be $\rho_t(1) w_t h_t^s$, *i.e.*

$$1 - \tau_t \geqslant \Psi_t \rho_t(1) \tag{15}$$

  and then of course the HS workers pay it : $1 - \tau_t > \rho_t(1)$.

- When some LS pay the cost, but not all, $p_{t+1} < 1$, then the LS workers are indifferent between paying or not paying it and

$$1 - \tau_t = \Psi_t \rho_t(p_{t+1}) \text{ with } p_t < p_{t+1} < 1 \tag{16}$$

Then of course all HS workers pay the fee : $1 - \tau_t > \rho_t(p_{t+1})$.

Case c. At least some HS workers do not pay the cost and thus $p_{t+1} < p_t$.

- When no HS worker pay the cost, the university formation disappears ($p_{t+1} = 0$). This occurs if

$$\rho_t(0) \geqslant 1 - \tau_t \tag{17}$$

- When some but not all HS workers pay the cost, then

$$\rho_t(p_{t+1}) = 1 - \tau_t \text{ with } 0 < p_{t+1} < p_t \tag{18}$$

In both subcases, LS workers choose optimally not to pay the cost: $1 - \tau_t < \Psi_t \rho_t(p_{t+1})$.

Let us consider now corner values for the number of HS workers. In the special case where $p_t = 0$, there are no HS workers in period t, and thus no teachers in the university. Then the fee does not exist (it can be assumed arbitrarely large) and necessarely $p_{t+1} = 0$. In the other special case, when all workers are HS, $p_t = 1$, then case b is excluded and there are the three possibilities:

if $\rho_t(1) \leqslant 1 - \tau_t$ (case a), then $p_{t+1} = 1$.

if $\rho_t(0) \geqslant 1 - \tau_t$, then $p_{t+1} = 0$.

if $\rho_t(0) < 1 - \tau_t < \rho_t(1)$, then there exists p_{t+1}, $0 < p_{t+1} < 1$, such that $\rho_t(p_{t+1}) = 1 - \tau_t$.

Proposition 1 *Given $\beta_t \geqslant 0$, $0 \leqslant p_t \leqslant 1$, $\gamma_t > 0$, and $0 < \tau_t < 1$, there exists a unique number of students p_{t+1}. It is increasing, decreasing or unchanged according to the value of $1-\tau_t$ is larger than $\Psi_t \rho_t(p_t)$, smaller than $\rho_t(p_t)$, or between these two values.*

Proof: See the appendix.

Let us consider any structure of skills among parents in an economy with both types of workers. The number of students at equilibrium depends on the decisions of parents. Since parents have identical preferences, wealth is the key determinant of parental decisions. HS parents are richer than LS parents and their relative income differential is given by $\Psi_t > 1$. Moreover, the cost and the benefits of university are the same for all families (see relation (13)). Thus, if at least one LS parent decides to enroll his child at university, all HS parents also enroll their children. For any policy, the structure of skills in the next period is thus necessarely unique. There may occur three exclusive equilibria according to the after-tax incomes of parents and the ratio of cost and benefits of university.

The cost of university is proportional to the wage of the university teachers. Since university teachers are HS agents, Ψ_t may be viewed as a "cost differential" between LS parents and HS parents. The other component of the university cost is given by the fee, that is the numerator of $\rho_t(p_t)$.

The benefits from education depend positively on the productivity of university $\Psi(\beta_t p_t, \gamma_t, p_{t+1})$, which is common to all students. The higher this productivity (the smaller $\rho_t(p_t)$), the more likely parents enroll their children.

When the after-tax income of LS parents and the productivity of education are high enough relatively to the cost of education, some LS parents (at least) decide to finance university for their children. In such a case, the number of students is increasing. On the contrary, when the income of HS parents and/or the productivity of university are too low relatively to the fee, some HS parents (at least) don't enroll their children at university. Thus the number of students is decreasing. Finally, the case of intergenerational immobility occurs when LS parents are too poor to finance the cost of university while HS parents receive a sufficient after-tax income to enroll their children at university.

3.3 Equilibrium prices

The number of LS workers who are not teachers in the basic formation system is $(1-\alpha_t^o)(1-p_t)$ which is non negative if $\alpha_t^o \leqslant 1$; but the number of the HS teachers (in both formations) $(\alpha_t^s+\beta_t)p_t+\gamma_t p_{t+1}$ should be less than the number p_t of HS agents. When this is the case, the total labor supply (in efficient units) made to the firm is:

$$H_t = (1-\alpha_t^o)(1-p_t)\,h_t^o + \left[(1-\alpha_t^s-\beta_t)\,p_t - \gamma_t p_{t+1}\right] h_t^s \qquad (19)$$

Then the labor market equilibrium determines the current wage and the current interest rate (since K_t is given) following (10) and (11).

In the case $p_t = 0$, the economy works without university formation and $p_{t+1} = 0$. In the case $p_t > 0$, there is a restriction on the fee and the number of students. For an optimal choice of $p_{t+1} \leqslant p_t$ (cases a and c), it is sufficient that $\gamma_t < 1-\alpha_t^s-\beta_t$; but in the case b, the condition $(1-\alpha_t^s-\beta_t)p_t > \gamma_t p_{t+1}$ for existence of an equilibrium is more restrictive. It is verified when $\gamma_t < (1-\alpha_t^s-\beta_t)p_t$.

Proposition 2 *Given positive parameters α_t^s, α_t^o, β_t and γ_t which verify: $\alpha_t^s+\beta_t < 1$ and $\alpha_t^o < 1$, and given the past variables $K_t > 0$, $p_t > 0$ and $\Psi_t = h_t^s/h_t^o > 1$, there exists a unique short-term equilibrium at period t if the unique solution p_{t+1} defined in proposition 1 verifies*

$$(1-\alpha_t^s-\beta_t)\,p_t \geqslant \gamma_t p_{t+1} \qquad (20)$$

Proof: See the appendix.

All agents save optimally a proportion a of their income net of tax and of fee. The total net income of the parents is

$$p_t\Omega_t^s + (1-p_t)\Omega_t^o - p_{t+1}e_t^s = w_t\left[p_t\,(1-\tau_t)\,h_t^s + (1-p_t)\,(1-\tau_t)\,h_t^o - p_{t+1}\gamma_t h_t^s\right]$$

The total net income is necessarely positive since agents do not pay the fee when it is larger than his income net of tax. Thus

$$K_{t+1} = a\left[(1-\tau_t)\,\bar{h}_t - p_t\gamma_t h_t^s\right] w_t \qquad (21)$$

With $\Psi_{t+1} = h_{t+1}^s/h_{t+1}^o = \Psi(\beta_t p_t, \gamma_t, p_{t+1})$, all equilibrium variables are determined.

4 Dynamics

In order to keep the dynamics tractable, we introduce the following assumption.

Assumption 4: The parameters α^s, α^o, β and γ are positive. Moreover, they verify $\alpha^s + \beta = \alpha^o$ and $0 < \alpha^s + \beta + \gamma < 1$.

We assume that the government finances the same proportion of teachers in each type of agents HS and LS. With this assumption, the tax rate is constant $\tau = \alpha^o$. The taxes paid by the LS agents, $(1 - p_t)\tau w_t h_t^o$ are then just the cost of the LS teachers in the basic formation $(1 - p_t)\alpha^o w_t h_t^o$. The taxes paid by the HS agents $p_t \tau w_t h_t^s$ are equal to the cost of the HS teachers in both formations which is publicly funded $p_t(\alpha^s + \beta)w_t h_t^s$.

The last part of assumption 1 implies that the fee is compatible with at least one constant proportion p of HS agents or, equivalently, that the education policy is driven so that the total proportion of high skilled teachers is feasible.

Under this assumption, three types of dynamics may occur. As shown in the former section, they may be studied in light of the proportion of HS agents. We first present the case of developing education which occured in most countries over long horizons. Then we briefly discuss other feasible dynamics.

4.1 Dynamics with an increasing proportion of HS

Let us define the cost-benefits ratio:

$$B(p, q) = \frac{\gamma}{1 - \Psi(\beta p, \gamma, q)^{-b}}$$

This function is decreasing with respect to p and increasing with respect to q.

Proposition 3 *With an initial distribution of skills, $p_0 > 0$, and a feasible fee, $\gamma < (1 - \tau)p_0$, the intertemporal equilibrium starting with p_0 exists and the sequence p_t is non-decreasing iff $\Psi_0 B(p_0, p_0) < 1 - \tau$.*

Proof: See the appendix.

In the initial period we assume an exogenous distribution of skills with $p_0 > 0$, and a human capital differential between HS and LS agents, $\Psi_0 = h_0^s / h_0^o$. As we have seen in case b, at least some LS parents will pay the fee if they consider university as a good investment. This occurs if they are sufficiently wealthy agents $(1 - \tau > \Psi_0 B(p_0, p_0))$. In this case, the student population rises since all HS parents, wealthier than LS ones, also enroll their children at university $(p_1 > p_0)$. Moreover the student population in the newborn generation is unique.

If $\Psi_0 B(p_0, 1) \leqslant 1-\tau$, all LS parents pay the fee to their children and all the young generation enters university ($p_1 = 1$). In this case, the congestion effect is maximal but not sufficient to undermine the productivity of the university. The university remains attractive despite it is highly crowded.

If $\Psi_0 B(p_0, 1) > 1 - \tau$, an overcrowded university wouldn't be attractive for all LS agents : its relative rate of return would be too low and alternative investments (such as savings) would be preferred. Proposition 1 states that only some LS parents will pay the fee ($1 > p_1 > p_0$).

In order to determine a short-term equilibrium in period $t = 1$, the following condition should be verified (condition (20) in proposition 2) :

$$\gamma p_1 < (1 - \tau)p_0$$

We assume a stronger, but sufficient, condition $\gamma < (1 - \tau)p_0$.

Let us notice that in the increasing case, the dynamics are (relation (16)) :

$$\Psi\left(\beta p_{t-1}, \gamma, p_t\right) B\left(p_t, p_{t+1}\right) = 1 - \tau$$

which is a second order difference equation in p_t. We show in the appendix that, when this equation applies, the dynamics of p_t is monotonic non-decreasing. This implies that, in the absence of exogenous shocks, an economy which initiates an increase in its HS population won't go back.

The non-decreasing sequence p_t necessarely converges to a limit $\bar{p} \leqslant 1$. In the long term the growth rate is :

$$G_\infty = \Phi\left(\alpha^s \bar{p}, \alpha^o\left(1 - \bar{p}\right)\right)\left[1 - \bar{p} + \bar{p}\Psi\left(\beta\bar{p}, \gamma, \bar{p}\right)\right]$$

Thus the ratio k_t of the K_t and H_t converges to a stable steady state of the dynamics

$$k_{t+1} = \frac{a}{G_\infty} F_L'\left(k_t, 1\right)$$

4.2 Other dynamics

The model is also consistent with cases both of social mobility and of a decrease in the student population over long periods.

Social immobility

Social immobility occurs when all children undertake the same type of formation as their parents. All HS agents pay the fee and no LS pay it (case a). The number p of students is constant. It occurs when the following condition applies (see (14)) :

$$\frac{\gamma}{1 - \Psi^{-b}} \leqslant 1 - \tau \leqslant \frac{\gamma\Psi}{1 - \Psi^{-b}} \tag{22}$$

where $\Psi = \Psi(\beta p, \gamma, p) > 1$. When $p > 0$, the condition for the existence of the short term equilibrium, $\gamma < 1 - \alpha^s - \beta = 1 - \tau$, is then necessarely satisfied.

Remark : *Under assumption 2, for any value of p_0, $0 < p_0 < 1$, there exists an interval of values for the fee $\gamma > 0$ and an interval of values for the proportion p which contains p_0 such that the equilibrium leads to a constant p.*

With such values of the parameters, all short-term equilibria exhibit a constant proportion p, and the two types of human capital accumulation verify for all t:

$$h^o_{t+1} = \Phi\left(\alpha^s p, \alpha^o(1-p)\right)\bar{h}_t \text{ and } h^s_{t+1} = \Psi(\beta p, \gamma, p) h^o_{t+1}$$

Thus the mean human capital level grows at a constant rate

$$\bar{h}_{t+1} = G\bar{h}_t \text{ with } G = \Phi\left(\alpha^s p, \alpha^o(1-p)\right)\left[1 - p + p\Psi(\beta p, \gamma, p)\right]$$

and the dynamics of k_t

$$k_{t+1} = \frac{a}{G} F'_L\left(k_t, 1\right) \tag{23}$$

are monotonic (since $dk_{t+1}/dk_t > 0$) and converge to some stable steady state k^* which is decreasing with respect to G.

Note that all educational parameters α^0, α^s, β, γ have a positive effect on the growth rate. But G has a negative effect on the dynamics of k_t (equation 23).

The decrease in the proportion of HS

Given the initial value p_0, $0 < p_0 \leqslant 1$, the number of HS decreases if and only if $B(p_0, p_0) > 1 - \tau$. Then either there exists some p, $0 < p < p_0$, verifying $B(p,p) \leqslant 1 - \tau$ and in this case the sequence of p_t converges to a steady state value $\check{p}$ where $B(\check{p}, \check{p}) = 1 - \tau$. Or for all p, $0 < p < p_0$, we have $B(p,p) > 1 - \tau$, and in this case the sequence p_t converges to 0; and it may reach 0 after a finite number of periods.

When $B(p_0, p_0) > 1 - \tau$, the initial cost is too large when all HS workers would pay the fee. Thus, only a fraction of them will pay it, and the number of HS decreases. But this will reduce the number of HS teachers in the following period who are paid by the government (the tax rate τ is the same but the tax basis is reduced). Then the decreasing sequence p_t necessarely converges to a limit $\check{p}$ which is a steady state of the proportion of HS, if positive. If the limit is zero, the university formation disappears in the long-term.

Since p_t converges to 0 or $\check{p} > 0$, the sequence of capital-efficient labor ratio also converges and its limits is a steady state of

$$k_{t+1} = \frac{a}{G_\infty} F'_L\left(k_t, 1\right)$$

where G_∞ is the limit growth rate of the mean human capital stock.

In the case where p_t converges to $\check{p} > 0$,

$$G_\infty = \Phi\left(\alpha^s \check{p}, \alpha^o(1-\check{p})\right)\left[1 - \check{p} + \check{p}\Psi(\beta\check{p}, \gamma, \check{p})\right]$$

In the case where p_t converges to $\check{p} = 0$, $G_\infty = \Phi(0, \alpha^o)$.

5 Effects of the proportion of high skilled agents

As shown by proposition 3, there is a continuum of different values of the proportion of HS leading (in general) to different growth rates in economies having the same educational parameters. Indeed, given an educational policy $(\alpha^s, \alpha^o, \beta, \gamma)$, the balanced growth rate is given by $G = \Phi\left(\alpha^s p, \alpha^o(1-p)\right)\left[1 - p + p\Psi(\beta p, \gamma, p)\right]$. There are three effects of the proportion of HS on growth. First, this proportion acts on the basic formation through Φ. Second, it influences the rate of human capital accumulation in the university, Ψ. Finally, there exists a skill effect in the economy : there are p HS having the university formation, that is a higher human capital level ($\Psi > 1$) than the $1-p$ LS. Let us study more precisely these three effects.

The structure of skills in the economy depends on the proportion of HS agents. An increase in the share of HS raises the (stationary) population undertaking the university formation. Some LS parents finance the university formation to their child. Next, their heir will also be HS. Since the human capital level of HS is always higher than the LS one ($\Psi > 1$), an increase in the proportion of HS induces a structural effect improving the average human capital level in the economy. Nonetheless, a modification in the proportion of the HS population also changes the productivity in both formations. Given educational policies, the proportion of HS determines the number of teachers/professors but also the number of students in the university. This in turn alterates the productivity in the basic formation as well as in the university. Moreover, the productivity in the basic formation is shaped partly by the presence of HS teachers.

The university formation is optional. Hence, the size of the student population corresponds to the proportion of the young whose parents finance university fees. Moreover, a fraction α^s of HS adults are teaching in the university. Consequently, in the long-term, an increase in the number of teachers has two effects that are opposite in sign. On one hand, it worsens the congestion effect in the educational system and then reduces its productivity. Given the number of professors, there are more students at university. The quantity and/or the quality of knowledge professors can transmit to their students declines. On the other hand, this increased proportion of HS raises the number of professors. Given the number of students, this improves the productivity in the university formation (for the opposite reason than previously). In the particular case where the quality of education only depends on the teacher-pupil ratio, there is no long-term effect of a change in the skills structure. Indeed, this ratio remains unchanged since the number of professors as well as students grow at the same rate. The two effects neutralize each other so that the productivity in the university formation does not depend on the proportion of HS. This efficiency only depends on the proportion of HS financed publicly (and on fees). Given the government budget constraint, this acts upon the ability of financing HS teachers in the basic formation.

The basic formation is compulsory. Since there is no population growth, the number of students is constant. On the contrary, the number of teachers of each type changes with the proportion of HS. The number of LS teachers cuts back with this proportion whereas the number of HS ones increases. Hence, the overall effect is ambiguous and depends on the relative productivity of each type of teachers. When the allocation of teachers is efficient, the marginal productivities are proportional to wages: $\Phi_1/\Phi_2 = w^s/w^o = \Psi$ and the derivative $\alpha^s\Phi_1 - \alpha^o\Phi_2$ has the sign of $\alpha^s\Psi - \alpha^o$. Under the assumption that the government finances the same proportion of teachers of each type ($\alpha^o = \alpha^s + \beta$), the proportion of LS teachers will always be higher than the HS one. The sign of the derivative is then ambiguous. In particular, a relatively high proportion of HS teachers implies a relatively low public financing effort in the university formation. Indeed, an increase in the number of HS teachers in the basic formation significatively raises the number of HS teachers who might have relatively few additional knowledge. Then, a very high proportion of HS doesn't guarantee a high productivity in the basic formation. Even if the efficiency of university teaching is large when the proportion of HS teachers is relatively low in the basic formation, the effect of an increasing proportion of HS is negative on Φ. The intuition of this result is very simple. If the basic formation is principally made by LS teachers and if their proportion in the LS workers is constant, a decrease in the number $1-p$ of LS workers cuts back the number of teachers and, consequently, a decrease in the basic human capital level.

Proposition 4 *The global effect of the proportion of high skilled on the growth rate results from three partial effects. First, a mean human capital effect which is positive. Second, the productivity of the university formation, which is positive when the effect of the number of teachers dominates the congestion effect (there is no effect when this efficiency only depends on the teacher-pupil ratio). Third, the effect on the basic formation which depends on the proportion of the two types of teachers. It is positive if and only if* $\alpha^s\Psi - \alpha^o > 0$.

Proof: See the appendix.

The effect of the proportion of HS on the growth rate combines two positive effetcs and an ambiguous one. We shall assume in the next section that the overall effect is positive.

6 Educational policies : an heuristic illustration

Let us consider two particular forms of the cost-benefits functions $B(p,p) - (1-\tau)$ and $\Psi(\beta p, \gamma, p)B(p,p) - (1-\tau)$ depicted respectively by the curves (C) and (D) in the figure below. They allow to describe the dynamics of the choices of HS and LS workers to pay the fee. The x-axis describes the dynamics of p which is contingent to its initial value, p_0.

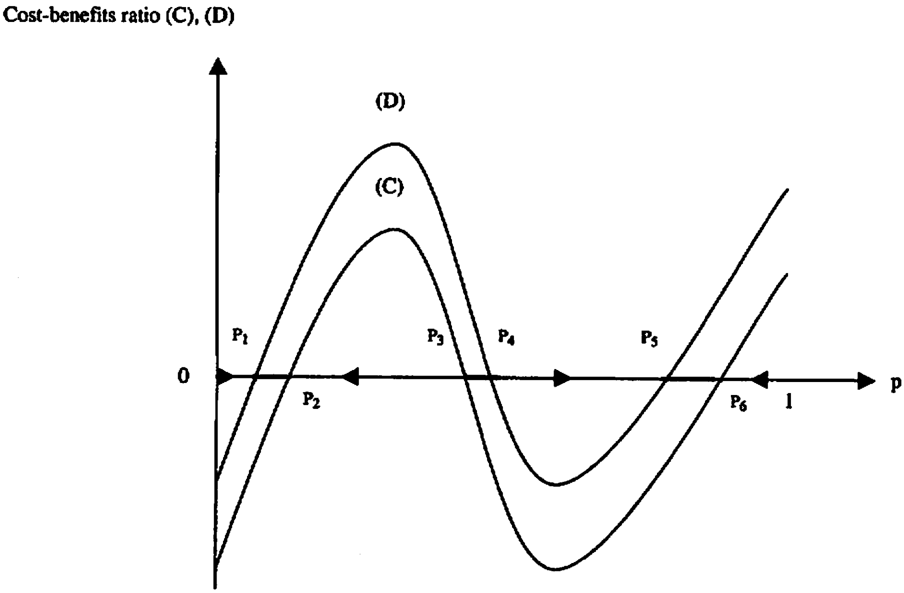

Figure 1 : *dynamics of p*

We can observe the three types of dynamics analysed in the previous sections.

The sets of p where $(C) \leqslant 0$ and $(D) \geqslant 0$, *i.e.* $[p_1, p_2] \cup [p_3, p_4] \cup [p_5, p_6]$, correspond to an invariant structure of skills in the economy. At each period $t \geqslant 0$ given the educational parameters, HS workers pay the fee and LS ones do not.

If p_0 lies in an interval where $(C) > 0$, *i.e.* $]p_2, p_3[\cup]p_6, 1]$, the sequence of p_t decreases.

In the other sets $]0, p_1[\cup]p_4, p_5[$ where $(D) < 0$, the sequence of p_t increases.

Note that an arbitrarely small initial value of p allows to reach the interval $[p_1, p_2]$. In an economy where university doesn't exist, an initial proportion of HS strictly positive could result from the existence of self-taught agents.

Every interior point in any interval where the proportion of HS workers is constant has a property of stability according to Lyapounov definition.

Consequently, there is no divergence following a local perturbation around such a point. Nonetheless, there exist two types of intervals differing in their boundary properties. "Stable extremities" intervals have the properties of a sink : if a shock initially moves the proportion of HS workers in the economy away from its boundary value (the proportions p_1, p_2, p_5 and p_6 in the figure), this proportion will converge to this bound. Thus, the shock has no permanent, long-term growth effect. "Unstable extremities" intervals are such that any shock leading the proportion of HS workers to leave the interval has long-term growth effects. Indeed, this proportion converges to a stable extremity of the closest "constant proportion" interval.

Let us consider an economy with no intergenerational mobility. In accordance on the discussion above, we can describe the consequences of a transitory educational policy. In particular, we consider pure redistributive policies which leave the tax rate unchanged ($\tau = \alpha^o = \alpha^s + \beta$). For example, an increase in the proportion of HS workers financed publicly (β) implies a decrease in the proportion of HS workers employed in the basic formation (α^s). If the economy lies in an "unstable extremities" interval[8], such a policy may allow to leave a trap with low growth and to reach a higher growth path. On the contrary, a policy promoting the basic formation leads to a lower growth path. If the economy lies in a "stable extremities" interval, these transitory educational policies won't have permanent growth effects. As in Azariadis and Drazen (1990), we show that little differences in human capital endowments, *i.e* here in the initial proportion of HS individuals, may lead to persistent differences in growth rates across countries. Consequently, some countries may initiate a higher growth path while others could stagnate. This result of multiple equilibria results from the existence of indivisibilities in the financing of the additional formation. These indivisibilities imply increasing returns to education, as in Galor and Zeira (1993). Nevertheless, we extend this result in two ways. First, we exhibit intervals with no social mobility. Countries may initially lie in a state of social immobility (an interval with constant p). Families with HS parents remain HS permanently while parents of LS families never pay university to their children. As a consequence, countries with identical educational policies but different initial proportions of HS have different growth rates. Second, transitory educational policies may have quite different long-term effects according to the importance of the HS population. Despite transitory educational policies promoting university formation, some countries can't jump from lower stages of the developement path to more rapid ones. They are trapped in a low development stage. For example, a temporary decrease in the fee or an increase in the proportion of teachers in the university formation[9], which increases the return to the additional formation, incites some LS parents to

8 At the bounds of such intervals, the cost-benefits curves are decreasing.

9 It is important to notice that these incentives have to be sufficient to leave an interval (which is characterized by no social mobility). If so, some LS parents will pay the additional formation to their children and there is an initial increase in p. Consequently, the magnitude of the educational policy has to be all the more important as the proportion of HS is low (in the interval).

finance university to their children. But, in subsequent periods, the university becomes too costly and some HS parents won't pay it to their children. In the long-term the proportion of HS workers converges to the upper bound of the initial interval. At the individual level, some families could benefit from the educational policy while others could suffer from it. But at an aggregate level, there is no social mobility since the proportion of HS remains constant (at least when the initial proportion of HS was at the upper bound of the interval).

On the contrary, temporary educational policies may help countries in reaching a new stage of development and in initiating a higher growth path. The initial incentives lead some LS parents to finance university to their children. In subsequent periods, the "return" to education is sufficiently high for new LS parents to invest. This is the case till the economy reaches the lower bound of the next interval, characterized by a higher growth rate. Hence, the cross-country differences in macroeconomic adjustements to educational policies can be attributed, among other factors, to differences in the proportion of HS workers. From this point of view, the model provides an explanation for the stages of development observed by Rostow (1960).

7 Conclusion

This paper analyzes the role of the distribution of skills in economic development through investment in human capital. The study demonstrates that, in the absence of a credit market for education, the distribution of skills significantly affects the aggregate economic activity. Furthermore, with indivisibilities in investment in human capital, these effects are carried to the long-term as well. Hence, growth is affected by the initial distribution of skills, or more specifically by the percentage of individuals who have the university formation level. A major consequence due to the congestion effect of the educational system lies in the potential existence of many constant growth paths with no social mobility. Moreover, similar educational policies have quite different long-term effects according to the proportion of high skilled agents. Indeed, the model provides an explanation for quite different development paths. On the one hand, economic development of many countries such as "dragons" which succeeded in promoting university and initiated a higher growth path. On the other hand, developing countries which failed to promote university and remain with a very large proportion of low skilled individuals. Such countries are trapped in a low development stage.

8 Appendix

A) Proof of proposition 1

We have seen that the special cases $p_t = 0$ and $p_t = 1$ lead to a unique value of p_{t+1}. Consider now the case $0 < p_t < 1$. When (14) is satisfied, then we also have for all $p_{t+1} > p_t$, $1 - \tau_t < \Psi_t \rho_t(p_{t+1})$ since ρ_t is increasing and case b is excluded; for $p_{t+1} < p_t$, we have $1 - \tau_t > \rho_t(p_{t+1})$ and case c is excluded. Thus when (14) is satisfied, $p_{t+1} = p_t$ is the unique solution.

Now assume (14) is not satisfied and first that $\Psi_t \rho_t(p_t) < 1-\tau_t$. Then either (15) is satisfied and $p_{t+1} = 1$ is the unique solution, or (15) is not satisfied: $\Psi_t \rho_t(p_t) < 1 - \tau_t < \Psi_t \rho_t(1)$ and then there exists a unique value of p_{t+1}, $p_t < p_{t+1} < 1$ which satisfies (16).

Similarly the last possibility $\rho_t(p_t) > 1-\tau_t$ implies either (17) and the disappearance of the university formation, or (18) defining a unique value of p_{t+1}, $0 < p_{t+1} < p_t$.

B) Proof of proposition 2

Given the parameters and the past variables, the equilibrium tax rate τ_t defined by (12) verifies $0 < \tau_t < 1$. There exists then a unique optimal number of students p_{t+1} (proposition 1); when condition (20) is satisfied, the labor supply to the firm H_t is determined by relation (19) and the equilibrium prices are given by (10) and (11).

C) Proof of proposition 3

The set of values of (p, γ) which verify the strict inequalities

$$\frac{\gamma}{1 - \Psi^{-b}} < 1 - \tau < \frac{\gamma \Psi}{1 - \Psi^{-b}} \tag{24}$$

is an open set, by continuity of Ψ. It is enough to show that (p_0, γ_0) belongs to this set for some $\gamma_0 > 0$. The function of γ:

$$\theta(\gamma) = \frac{\gamma \Psi\left(\beta p_0, \gamma, p_0\right)}{1 - \Psi\left(\beta p_0, \gamma, p_0\right)^{-b}}$$

is continuous on $[0, 1]$ since the denominator does not vanish (for $\gamma = 0$, $\Psi(\beta p_0, 0, p_0) > 1$ since $\beta > 0$). It verifies: $\theta(0) = 0$ and $\theta(1 - \tau) > 1 - \tau$ since $\Psi(\beta p_0, 1-\tau, p_0) > 1$. Thus there exists a largest value γ_1 of γ, $\gamma < 1-\tau$ such that $\theta(\gamma_1) = 1 - \tau$ For $\gamma > \gamma_1$, we have $\theta(\gamma) > 1 - \tau$. But we also have at γ_1 :

$$\frac{\gamma_1}{1 - \Psi\left(\beta p_0, \gamma_1, p_0\right)^{-b}} < \theta(\gamma_1) = 1 - \tau$$

Thus for $\gamma_0 > \gamma_1$, and γ_0 near γ_1, this inequality also holds by continuity. This shows that (p_0, γ_0) does belong to the open set. The proof is complete.

D) Proof of proposition 4

We have $\frac{\partial G}{\partial p} = (\alpha^s\Phi_1 - \alpha^o\Phi_2)(1-p+p\Psi) + \Phi[\Psi - 1 + p\beta\Psi_1 + p\Psi_3]$. When the efficiency of a formation only depends on the TPR, the human capital function in the university becomes $\Psi = \Psi\left(\frac{\gamma p_{t+1}+\beta p_t}{p_{t+1}}\right)$. In the long-term equilibrium, $p_t = p$. Then $\Psi = \Psi(\gamma+\beta)$ is independant from p. Since $\Psi > 1$, a sufficient condition to have $\frac{\partial G}{\partial p} > 0$ requires $\alpha^s\Phi_1 - \alpha^o\Phi_2 > 0$, that is $\alpha^s\Psi - \alpha^o > 0$ when the allocation of teachers in the basic formation is efficient.

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