Endogenous Timing and Quality Standards in a Vertically Differentiated Duopoly*

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1 Introduction

The regulation of industries where consumers are willing to pay higher prices for higher qualities takes often the form of minimum quality standards (MQSs), aiming at increasing social welfare through an increase in the average quality supplied in those industries. The rationale behind these interventions is that governments, either for paternalistic reasons or for the recognition of the presence of externalities, believe that the qualities offered by firms are too low (for a more detailed discussion, see Viscusi et al. (1995)).

In the case of oligopolistic markets, three main issues have been dealt with so far, namely (i) the introduction of MQSs and its consequences on market structure in a duopoly where quality improvements involve a fixed cost technology (Ronen (1991), Constantatos and Perrakis (1998), and Scarpa (1998)); (ii) the introduction of an MQS and its long-run competitive effects in a duopoly where quality improvements are obtained through an increase in variable costs, under full market coverage (Crampes and Hollander (1995), Ecchia and Lambertini (1997)); (iii) the effects of MQSs in

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an open economy with intraindustry trade (Motta and Thisse (1993), Boom (1995), Lutz (2000)).

In the aforementioned literature, the optimal MQS policy has been studied under the assumption that firms play à la Nash, which can be interpreted as a situation where firms are symmetric in terms of their relative market power. However, in many real-world oligopolistic markets, some firms enjoy dominant positions over competitors, either because of the past history of those markets, or because of endogenous strategic interaction. This poses two questions which we want to address in this paper. The first can be formulated as follows. What are the consequences of the introduction of an MQS on the distribution of market power across firms? This refers to a situation where the MQS may modify a status quo where a firm enjoys a dominant position. The second question is whether the regulator is able to increase social welfare through an MQS, irrespective of the endogenous distribution of market power across firms. Answering both questions could help to shed some new light on the effectiveness of the MQS as a policy instrument.

In order to address these issues, we model a vertically differentiated duopoly where we investigate the interplay between a regulator, choosing the MQS, and firms, choosing endogenously the timing of their respective moves. We adopt a two-stage model where firms set qualities in the first stage, and prices in the second, and all consumers in the market are served. We describe the endogenous timing of moves with respect to the choice of quality, that is, the outcomes generated by Nash and Stackelberg equilibria in the first stage of the game. This aspect summarises the possibility that firms have different market positions. As a benchmark, we initially study the equilibrium outcome characterising the unregulated market. Then, we introduce the problem of the regulator in setting the optimal MQS under endogenous timing.

In modelling the issue of endogenous timing, we follow d'Aspremont and Gérard-Varet (1980) and Hamilton and Slutsky (1990). They have shown that firms move sequentially whenever there exists at least one Stackelberg equilibrium which Pareto-dominates all the Nash equilibria. Otherwise, firms always play simultaneously. The intuition behind this result is as follows. Consider a one-shot duopoly game where firms can choose whether to move at the same time or scatter their respective decisions. If they decide to move simultaneously, no matter whether early or late, a Nash equilibrium obtains. If, conversely, they move sequentially, then a Stackelberg equilibrium is observed. The necessary condition for a Stackelberg equilibrium to obtain is that the leader's profits be higher than the Nash equilibrium profits. Otherwise, no firm would be willing to move first. Then, suppose that the follower's profits are lower than the Nash profits. If so, both firms decide to move at the same time in order to avoid playing the follower's

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1 The issue of choosing between Nash and Stackelberg equilibria has received a wide attention in oligopoly theory (see Gal-Or (1985), Dowrick (1986), Boyer and Moreaux (1987), inter alia)
role. The sufficient condition for firms to play sequentially, generating thus a Stackelberg equilibrium, is that both the leader’s and the follower’s profits are at least as high as the Nash profits.

Our main findings can be stated as follows. First, the timing game in the quality space has a unique equilibrium in pure strategies, involving simultaneous moves. The related optimal MQS is time consistent, although suboptimal from the viewpoint of the regulator. There exists, however, the possibility for the regulator to implement an optimal but time inconsistent policy driving firms towards a Stackelberg outcome with the high-quality firm in the leader’s role. Second, we prove that, when the low-quality firm is Stackelberg leader in the quality stage, the related MQS is ineffective. In summary, the MQS does not affect the relative market positions of firms, unless the regulator is time inconsistent. However, there exists a situation where the MQS cannot be used as a policy tool, namely, the setting where the low-quality firm has a dominant position. In this case, the ineffectiveness of the MQS is due to the fact that the low-quality firm aims at serving the average consumer, thus creating an upward bias in the average quality supplied in the market.

The paper is structured as follows. The duopoly model is laid out in section 2, whereas the unregulated market setting is presented in section 3. The optimal MQSs are derived in section 4. Concluding comments are in section 5.

2 The basic duopoly model

Here we describe a model of unregulated duopoly under complete information, presented in several contributions (Moorthy (1988), Cremer and Thisse (1994), Crampes and Hollander (1995), Lambertini (1996), Ecchia and Lambertini (1997)). Each firm $i \in \{H, L\}$ produces a vertically differentiated good characterised by quality $q_i$, with $q_H \geq q_L$, and then compete in prices against the rival. There exists a continuum of consumers indexed by their marginal willingness to pay for quality $\theta \in [\theta_0, \theta_1]$, with $\theta_0 = \theta_1 - 1$. The distribution of consumers is uniform, with density $f(\theta) = 1$, so that the total mass of consumers is also 1. We assume full market coverage, that is, each consumer buys one unit of the product that yields the highest net surplus $U = \theta q - p$. Production technology involves variable costs, which are convex in the quality level and linear in the output level:

$$C_i = q_i^2 x_i \quad i = H, L$$

The previous specification of the cost function has relevant implications as to the effects of a quality standard on market structure. In the

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2 Alternatively, quality improvements could hinge upon fixed costs, representing R&D efforts. This cost function would produce the well known finiteness property (Shaked and Sutton (1983)).
remainder, we will see that the risk of exit by the low-quality firm as a consequence of the introduction of a standard, which exists under fixed costs of quality improvements (Ronnen (1991), Constantatos and Perrakis (1998), Scarpa (1998)), is completely absent in the present setting. Firm $i$'s profit function is

$$\pi_i = (p_i - q_i^2)x_i$$

(2)

Competition between firms is fully noncooperative and takes place in two stages. In the first, firms set their respective quality levels; then, in the second, which is the proper market stage, they compete in prices. The solution concept applied is the subgame perfect equilibrium by backward induction.

3 The unregulated duopoly

In this section, we consider the setting without minimum quality standard. Given generic prices and qualities, the "location" of the consumer indifferent between the two varieties is $h = (p_H - p_L)/(q_H - q_L)$, so that market demands are $x_H = \theta_1 - h$ and $x_L = h - (\theta_1 - 1)$.

Consumer surplus in the two market segments is defined as follows:

$$CS_L = \int_{\theta_0}^{h} (\theta q_L - p_L)d\theta; \quad CS_H = \int_{h}^{\theta_1} (\theta q_H - p_H)d\theta;$$

(3)

social welfare corresponds to the sum of consumer surplus and firms' profits, $SW = CS_H + CS_L + \pi_H + \pi_L$.

As a benchmark, consider first the situation where qualities are chosen simultaneously. As this situation has been widely analysed in the literature (Cremer and Thisse (1994), Crampes and Hollander (1995), Ecchia and Lambertini (1997)), we can briefly summarise it. From the first order conditions (FOCs henceforth) at the second stage, the following equilibrium prices obtain:

$$p_H = \frac{(q_H - q_L)(\theta_1 + 1) + 2q_H^2 + q_L^2}{3}; \quad p_L = \frac{(q_H - q_L)(2 - \theta_1) + 2q_H^2 + q_H}{3}$$

(4)

Substituting and rearranging, we get the profit functions defined exclusively in terms of qualities, $\pi_i(q_H, q_L)$. The subgame perfect quality levels are

$$q_H = \frac{4\theta_1 + 1}{8}; \quad q_L = \frac{4\theta_1 - 5}{8},$$

(5)

which entails the general constraint $\theta_1 \geq 9/4$, in order for the poorest consumer to be in a position to buy the low-quality product. The corresponding equilibrium profits are $\pi_H^N = \pi_L^N = 3/16$, and equilibrium demands are
\( x_H = x_L = 1/2 \) (superscript \( N \) indicates that both stages are played simultaneously). The welfare level is \( SW(N) = (16\theta^2_1 - 16\theta_1 + 1)/64 \). Consumer surplus in each segment of the market is \( CS_H = (16\theta^2_1 - 8\theta_1 - 27)/128 \); and \( CS_L = (16\theta^2_1 - 24\theta_1 - 19)/128 \). Observe that the socially preferred qualities would be the first and third quartiles of the interval \( [\theta_1/2, \theta_1/2] \), which obtains from the calculation of the preferred varieties for the richest and the poorest consumer in the market, if such varieties were sold at marginal cost. This implies that (i) qualities are set, respectively, too low and too high as compared to the social optimum; and (ii) this model shares its general features with the model of spatial competition with quadratic transportation costs.

### 3.1 Quality leadership

We consider now the situation of quality leadership, i.e., the case where the quality stage is played sequentially, while the price stage is played simultaneously. Equilibrium prices at the market stage are defined by (4). We consider first the case where the high-quality firm is leader, solving the following problem:

\[
\max_{\pi_H} \pi_H = (p_H - q_H^2)x_H
\]

s.t. \[
\frac{\partial \pi_L}{\partial q_L} = \left( \frac{\theta_1 + q_H - 3q_L + 1)(q_L + q_H + 2 - \theta_1)}{9} \right) = 0
\]

Equilibrium qualities are \( q_H^l = (2\theta - 1)/4 \) and \( q_L^l = (2\theta - 3)/4 \), where superscripts \( l \) and \( f \) stand for quality leader and quality follower, respectively. Equilibrium profits and outputs are, respectively, \( \pi_H^l = 2/9 \) and \( \pi_L^l = 1/18 \); \( x_H^l = 2/3 \) and \( x_L^l = 1/3 \). The corresponding level of social welfare is \( SW(H^l) = (36\theta^2_1 - 36\theta_1 + 5)/144 \). The condition ensuring that the poorest consumer is served is \( \theta_1 \geq 2.21375 \).

If the low-quality firm is the leader, her problem consists in

\[
\max_{\pi_L} \pi_L = (p_L - q_L^2)x_L
\]

s.t. \[
\frac{\partial \pi_H}{\partial q_H} = \left( \frac{\theta_1^2 + 3q_H^2 - 4q_H(1 + \theta_1) + 2q_Hq_L - q_L^2 + 2\theta_1 + 1}{9} \right) = 0
\]

In duopoly, socially optimal qualities are (see Cremer and Thisse (1994)):

\[
q_H^* = \frac{4\theta_1 - 1}{8} ; \quad q_L^* = \frac{4\theta_1 - 3}{8}
\]

which are, respectively, lower and higher than \( q_H \) and \( q_L \) in (5).

It can be shown that the spatial model with quadratic transportation costs is actually a special case of a vertical differentiation model with quadratic costs of quality improvement (Cremer and Thisse (1991)).
The resulting equilibrium qualities are $q^L = (2\theta_1 + 1)/4$ and $q^H = (2\theta_1 - 1)/4$. The condition ensuring that the poorest consumer is served is $\theta_1 \geq 2.25831$. Due to the symmetry of the model, equilibrium profits and outputs are $\pi^L = 2/9$ and $\pi^H = 1/18; x^L = 2/3$ and $x^H = 1/3$. The corresponding level of social welfare is $SW(L) = (36\theta_1^2 - 36\theta_1 + 5)/144$.

In both cases, the quality leader locates in the middle of the interval of socially preferred qualities, defined by $[(\theta_1 - 1)/2, \theta_1/2]$, i.e., the leader produces the quality preferred by the median (and average) consumer. In relation to this, it is worth stressing that when the low-quality firm leads, the average quality is higher than in all other cases, and this will have some relevant bearings on the possibility of regulating such a market through an MQS.

3.2 Endogenous timing

Here, we confine our analysis to the range of $\theta_1$ wherein all the equilibria described above are admissible, i.e., $\theta_1 \geq 2.25831$. The relevant profits are represented in matrix I, where $F$ and $S$ stand for playing first and second, respectively.

\[
\begin{array}{ccc}
L \\
F & S \\
H & 3/16; 3/16 & 2/9; 1/18 \\
S & 1/18; 2/9 & 3/16; 3/16 \\
\end{array}
\]

Matrix I

Playing early ($F$) is a strictly dominant strategy for both firms, so that this game has a unique equilibrium, $(F, F)$ (see Lambertini (1996)).

Remark 1 The firms' timing decisions always yield simultaneous moves.

4 The regulated duopoly

In this section, we explicitly calculate the optimal levels of the MQS, as well as their consequences on the relevant equilibrium magnitudes. We consider the following game structure. In each of the following games, the policy

5 As it will become clear in the remainder, the condition $\theta_1 \geq 2.25831$, ensuring full market coverage in the unregulated duopoly, guarantees the admissibility of the following regulated games.
maker sets the optimal MQS mimicking to be in control of the low-quality firm at the quality stage, while firm \( L \) continues to set her price according to (4).

### 4.1 Simultaneous moves

The derivation of the optimal MQS when qualities are chosen simultaneously coincides with the analysis presented in Ecchia and Lambertini (1997). The resulting MQS is

\[
q^S_L = \frac{20\theta_1 - 34 + 9\sqrt{6}}{40}
\]

Superscript \( S \) denotes the presence of a minimum quality standard. Given \( q^S_L \) and its equilibrium price, full market coverage is possible if and only if \( \theta_1 \geq 2.23926 \). Observe that the introduction of the standard slightly loosens such a constraint as compared to the unregulated setting. The new level of the high quality is the best reply of the high-quality firm to the MQS:

\[
q^S_H = \frac{20\theta_1 + 2 + 3\sqrt{6}}{40}
\]

The new equilibrium profits are

\[
\pi^S_L = 0.22153; \quad \pi^S_H = 0.06714
\]

As a result of the adoption of the MQS, the degree of differentiation decreases (since both qualities increases, but the reaction of the high quality is weaker) and the demand for the high quality decreases while the demand for the low quality increases. This produces an increase in the low-quality firm's profits, and a reduction in the high-quality firm's profits (as in Ronnen (1991), and Crampes and Hollander (1995)). The net effect is negative, so that total industry profits are considerably decreased as compared to the unregulated equilibrium.

Social welfare amounts to \( SW^S(N) = [200\theta_1(\theta_1 - 1) + 18\sqrt{6} - 13]/800 \), which is obviously higher than that observed in the unregulated setting. The increase in welfare is due to two effects: (i) the increase in both quality levels; (ii) the increase in price competition, due to a reduced degree of product differentiation. However, the effect of the MQS on consumer surplus is not identical across consumers. The MQS increases the surplus of consumers purchasing the low quality for all acceptable values of \( \theta_1 \), while it decreases the surplus of consumers patronizing the high quality if \( \theta_1 \) is sufficiently high. Summing up, in this case it appears that the MQS policy, provided it is designed to maximize welfare regardless of its redistributive effects, trades off the losses suffered by the agents (firm and consumers) dealing with the high quality with the gains enjoyed by the other agents.
4.2 Quality leadership

Assume the price stage is simultaneous, equilibrium prices being given by (4). When qualities are chosen sequentially, two alternative cases arise. In the first, the high-quality firm is the leader. If so, the high-quality firm maximises profits under the constraint that the regulator chooses the MQS in order to maximise social welfare. In the second, the low-quality firm would lead: this implies that, in setting the MQS, the regulator maximises social welfare, taking into account the high-quality firm's best reply.

Case A: firm H leader. The leader's problem is

\[
\max_{q_H} \pi_H = \frac{(1 + \theta_1 - q_H - q_L)^2(q_H - q_L)}{9}
\]

\[
s.t. \quad \frac{\partial SW}{\partial q_L} = \frac{5q_H^2 - 15q_H^2 - 28q_L - 10q_Hq_L + 20\theta_1q_L - 5\theta_1^2 + 14\theta_1 - 8}{18} = 0
\]

Observe that (14) has the following solution:

\[
q_L = \frac{10\theta_1 - 5q_H - 14 \pm \sqrt{100q_H^2 + 140q_H - 100\theta_1q_H + 25\theta_1^2 - 70\theta_1 + 76}}{15}
\]

By checking the second order conditions, it is possible to verify that the regulator's best reply is given by the larger of the two solutions in (14). As a result, solving the leader's problem yields

\[
q_H^{SL} = \frac{\theta_1}{2} + 0.068811; \quad q_L^{SL} = \frac{\theta_1}{2} - 0.337644
\]

Notice that \(q_L^{SL} > q_L\), so that the MQS is binding. Obviously, equilibrium qualities are acceptable if the consumer at \(\theta_0\) is able to buy, i.e., \((\theta_1 - 1)q_L^{SL} - p_L^{SL} \geq 0\). This entails \(\theta_1 \geq 2.0206\). Equilibrium profits are \(\pi_H^{SL} = 0.072662\) and \(\pi_L^{SL} = 0.135004\). Output levels are \(x_H = 0.423178\) and \(x_L = 0.576822\). Social welfare amounts to \(SW^{SL}(HL) = 0.064768 + 0.01(\theta_0 - 1)/4\).

Case B: firm L leader. This amounts to consider the case where the regulator is the leader at the quality stage. He aims at

\[
\max_{q_L} SW = CS_H + CS_L + \pi_H + \pi_L
\]

\[
s.t. \quad \frac{\partial \pi_H}{\partial q_H} = \frac{3q_H^2 + 2q_Hq_L - q_L^2 - 4q_H(1 + \theta_1) + \theta_1^2 + 2\theta_1 + 1}{9} = 0
\]

The solutions to (18) are \(q_H = 1 + \theta_1 - q_L\), and \(q_H = (1 + \theta_1 + q_L)/3\). Taking into account strategic complementarity between qualities, the only
acceptable solution is the second. Solving the problem of the regulator as a leader, we get the equilibrium quality levels:

\[ q_{L}^{S} = \frac{40\theta_1 - 65 + 3\sqrt{145}}{80}; \quad q_{H}^{S} = \frac{40\theta_1 + 5 - \sqrt{145}}{80} \]  

The above qualities are acceptable if the poorest consumer is able to buy: this implies \( \theta_1 \geq 2.22258 \). Equilibrium profits are \( \pi_{H}^{S} = 0.0840355 \) and \( \pi_{L}^{S} = 0.218755 \). Output levels are \( x_{H} = 0.38264 \) and \( x_{L} = 0.61736 \). Social welfare amounts to \( SW^{S}(LqL) = 0.0406608 + \theta_1(\theta_1 - 1)/4 \). Notice that \( SW^{S}(Ll) > SW(Ll) \). However, \( q_{L}^{S} < q_{L}^{L} \), i.e., the standard is not binding. In the case where the low-quality firm leads, it would be socially desirable to decrease both quality levels. Yet, this cannot be achieved through a minimum quality standard, as the standard is not binding. We have thus proved the following lemma:

**Lemma 1** When the low-quality firm takes the lead in the quality stage, the MQS policy cannot improve social welfare.

Therefore, the MQS will not be adopted and the relevant payoffs for firms are those of the unregulated equilibrium.

Before investigating the issue of endogenous timing in the presence of a standard, it is worth stressing a few relevant results emerging from the analysis carried out so far:

**Proposition 1** Under variable costs of quality improvements,

- both firms survive in equilibrium after the introduction of the MQS;
- in the regulated equilibria, the low-quality (high-quality) firm’s profits are at least (most) as high as in the corresponding unregulated equilibria;
- in the regulated equilibria, the low-quality firm’s profits are always larger than the high-quality firm’s profits.

The first claim in the above proposition is in contrast with the conclusions reached in models where quality is the outcome of R&D activity (Ronnen (1991), Constantatos and Perrakis (1998), Scarpa (1998)), where introducing an MQS may bring about an undesirable increase in concentration. In our setting, the MQS never induces exit, as fixed costs are assumed away. The second claim states that the low-quality firm always benefits from the MQS, the only exception being the case where the same firm is quality leader. The intuition behind this is that the adoption of the MQS improves the position of the low-quality firm in the market (Crampes and Hollander (1995), Ecchia and Lambertini (1997)); if she is already acting as a leader in the product stage, then the MQS cannot increase her profits. The third statement establishes that it is optimal for the regulator to increase the market power of the low-quality firm up to a point where it is no longer convenient to be the high-quality seller.
4.3 Endogenous timing

Consider now the choice of timing w.r.t. to quality. In this case, the regulator needs to anticipate firms’ timing decisions in order to set the minimum quality standard. We establish the following

Proposition 2 The timing game in the quality space has a unique equilibrium in pure strategies, which entails simultaneous play.

Proof. The reduced form of the game is described by matrix II.

\[
\begin{array}{ccc}
L & F & S \\
H & F & 0.06714; 0.22153 & 0.07266; 0.1350 \\
S & 1/18; 2/9 & 0.06714; 0.22153 \\
\end{array}
\]

Matrix II

On the basis of lemma 1, we know that the MQS cannot be used by the regulator under the leadership of the low-quality firm. Hence, the payoffs in the south-west cell of matrix II are given by firms’ unregulated profits. It is immediate to check that, since for both firms playing \( F \) is a dominant strategy, the unique pure strategy equilibrium of the game is \( (F, F) \).

As a corollary to remark 1 and proposition 2, we have

Corollary 1 The firms’ choice of timing is unaffected by regulation.

As a consequence, we expect the regulator to introduce the MQS which is optimal under simultaneous moves. This produces the following relevant corollary:

Corollary 2 The MQS \( q^S_L \) is suboptimal from the regulator’s standpoint.

This follows immediately from the inequalities

\[
SW^S(\text{Hi}) > SW^S(N) > SW^S(\text{Li})
\]

Observe that there exists the possibility for the regulator to drive firms to \( (F, S) \), i.e., the situation where the high-quality firm takes the lead. To see this, notice that \( q^S_L > q^S_L \), that is, the optimal MQS under high-quality leadership is larger than the optimal MQS under simultaneous moves. This implies that the regulator can adopt \( q^S_L \), inducing the high-quality firm to play the leader’s role because she finds it convenient to do so. However, this policy is optimal from the regulator’s standpoint, but time inconsistent. A simple proof consists in checking that forward induction and backward induction do not coincide in this case. Consider first the backward induction
argument. This leads to simultaneous play on the part of firms, based on matrix II. Then, the regulator should set the MQS equal to $q^S_F$, taking the timing $(F,F)$ as given. Now, examine the forward induction argument. The regulator adopts $q^{SF}_L$, driving firms towards $(F,S)$. Hence, the two arguments are not consistent.

5 Concluding remarks

In the foregoing analysis, we have investigated the regulation through MQSs of a vertically differentiated duopoly where the timing of moves is endogenously chosen by firms. As a first and general result we have established that, in the present setting, the MQS involves no decrease in the intensity of competition and always favours the low-quality firm.

Concerning the timing of quality decisions, we have shown that the game has a unique equilibrium in pure strategies and the optimal MQS is time consistent when the policy maker takes firms' timing choice as given. However, the resulting equilibrium is socially suboptimal.

The previous analysis has addressed the issue of time consistency of regulatory policy in an oligopoly market. In this respect, we have shown that, whenever the decision to regulate an industry is taken, we need to evaluate the potential impact of regulatory measures on the structure of the oligopolistic game between firms. Our model indicates that the intervention of the regulator distorts the strategic interaction of firms in determining the endogenous distribution of roles, only if the regulator adopts a time inconsistent policy.

References


