Stochastic Nominal Wage Contracts in a Cash–in–Advance Model

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Introduction

Many economists agree that there exists empirical evidence for the existence of a positive and long–lasting effect of monetary shocks on output in the short-run. For instance, Sims and Zha (1995) or Christiano, Eichenbaum and Evans (1996) found such evidence for the US economy, and Sims (1992) and Bec and Hairault (1993) provide some empirical support for a positive and long–lasting hump–shaped response of output on US and European data. Any model that aims at explaining the Business Cycle should be able to account for these stylized facts. The basic cash–in–advance business cycle model (Cooley and Hansen (1989)) fails to account for these facts. In particular, it predicts an instantaneous fall in output following a positive money growth shock. Indeed, after a money injection, the purchasing power of all nominal balances is reduced, which leads households to increase leisure. This is the so–called inflation tax effect.

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Recently a class of limited participation or sluggish capital models (See e.g. Lucas (1990), Christiano (1991), Fuerst (1992) and Fuerst (1995)) has focused on liquidity effects that push interest rates down after a positive money growth shock and enhance a positive response of output. But, as noticed by Fuerst (1995), *"these models have one major empirical shortcoming: the inability to account for long-lasting effects of monetary stimulus"*. This actually goes back to a major shortcoming of most of general equilibrium models. As shown by Cogley and Nason (1995), a great variety of RBC models are unable to mimic two important stylized facts on US output dynamics: *(i)* GNP growth is positively autocorrelated over short horizons and *(ii)* GNP has an important trend-reverting component that has a hump-shaped impulse response function. According to Cogley and Nason, RBC models suffer from a lack of internal propagation mechanisms and cannot generate a satisfactory pattern of output dynamics unless they rely heavily on exogenous sources of dynamics. The inability of monetary models to account for a long-lasting hump-shaped effect of monetary disturbances on output also reflects their lack of internal propagation mechanism.

By allowing for real effects of monetary shocks along the business cycle, the literature on nominal contracts — and more specifically staggered contracts — seems to provide an promising avenue to circumvent this shortcoming. Blanchard (1983) showed that staggered price setting behavior can generate output persistence similar to that observed in the data. However, Chari, Kehoe and McGrattan (1996) found that this firms staggered price setting behavior fails to account for output persistence once plugged within a general equilibrium model[1]. In this paper we follow another line of research initiated by Taylor (1980) and Gray (1977) who analyzed the potentials of nominal wage contracts. Their aim was to show that the rational expectation hypothesis is not inconsistent with a real effect of monetary policy. The introduction of nominal wage contracts in a RBC framework aimed to account for the propagation of nominal shocks. Cho (1993) or Cho and Cooley (1995) assume that nominal wage contracts are set j periods in advance on the basis of the rational expectation of the wage that clears the labor market. In this paper, we assume, following Calvo (1983)[2], that the length of the contract is random. The choice of this modeling is motivated by the fact that it reinforces the internal persistence mechanism of the model. The duration of nominal wage contracts being stochastic, a monetary shock may affect the behavior of the real economy for a larger number of periods, depending on the mean duration of contracts. Ambler, Cardia and Phaneuf (1992) introduced this type of contracts in a general equilibrium framework. Nevertheless they do not explicitly describe the underlying optimizing behavior of agents. Recently Ambler, Guay and Phaneuf (1997) build a monetary

[1] Recently, a number of papers have been investigating whether extending monopolistic competition using real rigidities can generate the observed persistence. (See e.g. Kimball (1995), Kiley (1997) or Jeanne (1998))

[2] Calvo built a model with a continuum of price-setters and in which the probability of a given price-setter changing his price is constant at any point in time

business cycle model with a stochastic structure of contracts. Their result indicate that, when considered jointly with labor adjustment costs, these contracts can generate a long-lasting hump-shaped effect of money shock on output dynamics. Their contracts essentially rely on wage–wage emulation as in Taylor. We depart from this work in two ways: *(i)* our nominal wage anchor is based on Gray's (1976) expected market-clearing wage and *(ii)* no labor adjustment costs are introduced in the model. The latter assumption allows to evaluate the potential gains of stochastic nominal wage contracts *per se.*

The model is calibrated and evaluated for a European economy, namely the German economy which we consider to be representative. This choice is motivated by the fact that these economies are characterized by important nominal rigidities, especially on the labor market, and by the aforementioned empirical evidence for a long-lasting effect of money on output dynamics. In our framework the introduction of these stochastic nominal wage contracts leads to the abandon of the traditional assumption that the labor supply behavior is determined by intertemporal substitution motives. This allows us to weaken the negative effect of the inflation tax such that monetary shocks have a positive hump-shaped effect on output. The variance decomposition analysis suggests that monetary shocks explain up to 40% of the total variance of output in the first quarter and have a long lasting effect. Further, the model also mimics the correlation between output and inflation and real balances observed in Germany. It also lowers not only the standard deviation of inflation relative to output but also the persistence of inflation. We also propose an evaluation of the effects of variations in the mean duration of contracts on these indicators. Except for the cross-correlation of real money and output, the model does best for longer average length of contacts (2 years).

The remaining of the article proceeds as follows. The first section describes the competitive cash-in-advance model. Section 2 is devoted to the validation of the theoretical model. A last section offers some concluding remarks.

1 A Nominal Wage Contracts Model

This model relies on previous work by Cooley and Hansen (1989) and Hairault and Portier (1995). This section describes the arrangement of the markets as well as the behavior of the households, the firms and the monetary authorities.

1.1 The Representative Household

The economy is comprised of a unit mass continuum of identical infinitely lived agents. Each household has preferences on consumption and leisure represented by the following intertemporal utility function:

$$\mathcal{U} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, \ell_t) \right\} \tag{1}$$

where E_0 denotes the conditional expectation operator at time $t = 0$. β is the discount factor, C_t and ℓ_t denote respectively consumption and leisure.[3] Finally, $U(.,.)$ is the instantaneous utility function, satisfying the traditional Inada conditions.

The household enters period t with some nominal balances, M_t, that corresponds to its money demand at the end of period $t-1$, and a stock of capital, K_t. The household offers his labor on the labor market at the real wage rate W_t/P_t and rents out capital at the real interest rate, r_t. During the period the households gets a lump-sum transfer of cash equal to N_t. The intertemporal budget constraint of the household is thus given by:

$$M_{t+1} \leqslant M_t + N_t + P_t r_t K_t + W_t H_t - P_t C_t - P_t I_t \tag{2}$$

where P_t denotes the nominal price level, and N_t denotes a lump-sum money transfer from the monetary authorities. I_t denotes investment, which yields to the following law of motion of capital:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3}$$

where $\delta \in (0,1)$ is the constant depreciation rate of physical capital.

Further, each household is constrained on its consumption goods expenditures by the existing money balances at the beginning of the period:

$$C_t \leqslant \frac{M_t}{P_t} \tag{4}$$

Finally, each household has a unit time endowment that he allocates between leisure, ℓ_t and working time h_t:

$$\ell_t + h_t = 1 \tag{5}$$

[3] It should be clear to the reader that, for presentation purposes, we only report the program of a representative household without making clear reference to her index. For instance, C_t and h_t stand for $C_t(i)$ and $h_t(i)$, where i is the index of household i.

1.2 The Firm

All firms use the same technology with constant returns to scale in a perfect competition environment. All produce the same good using the constant returns to scale technology:

$$Y_t \leqslant A_t K_t^{\alpha} (X_{H,t} h_t)^{1-\alpha} \tag{6}$$

A_t follows an exogenous stochastic process:

$$\log(A_t) = \rho_A \log(A_{t-1}) + (1 - \rho_A) \log A + \epsilon_{A,t} \tag{7}$$

where $\epsilon_{A,t}$ is an i.i.d. gaussian process with mean zero and variance σ_A^2 and where $\mid \rho_A \mid < 1$. The total factor productivity A_t is a technological shock common to all firms of the economy and we assume that A_t is revealed to the agents at the beginning of the period t. $X_{H,t}$ grows at a constant rate $\gamma_{XN} > 1$ and denotes the Harrod neutral technological progress.

The labor index h_t is actually an aggregate of a continuum of labor types:

$$\log(h_t) = \int_0^1 \log(h_t(i)) \mathrm{d}i$$

where labor type $i \in (0, 1)$ is characterized by the period at which the labor contract has been signed.

We can treat this as a single firm that solves a period-by-period profit maximization problem:

$$\max_{K_t, h_t} \; Y_t - r_t K_t - W_t h_t$$

1.3 Money Supply

In each and every period, a central bank issues money, which is distributed to households according to a Friedmanian "helicopter drop" mechanism. The money supply is assumed to grow at a rate $(g_t - 1)$:

$$M_{t+1} = g_t M_t \tag{8}$$

g_t is assumed to follow an exogenous stochastic process of the following form:

$$\log(g_t) = \rho_g \log(g_{t-1}) + (1 - \rho_g) \log(\bar{g}) + \varepsilon_{g,t} \tag{9}$$

where $\varepsilon_{g,t}$ is an i.i.d. gaussian process with mean zero and variance σ_g^2. Finally, $\log(g_t)$ is a second order stationary process, imposing $|\rho_g| < 1$.

As suggested earlier we assume that the money created in period t is entirely distributed to households:

$$(g_t - 1) M_t = N_t \tag{10}$$

1.4 Nominal Wage Contracts

The introduction of nominal wage contracts has been widely studied in the RBC literature[4]. However, most of this work relied on fixed length contracts à la Gray (1977), where nominal wages are set k periods in advance such that $W_t = E_{t-k}W_t^*$, where W_t^* is the wage that would clear the labor market. Then this contract sets the level of nominal wage which remains fixed for the duration of the contract.

More recently, Ambler et al. (1997) or Benassy (1999) have studied the potential gains of introducing Calvo (1983) type of contracts. In this setting, contracts can remain unchanged with probability $(1-\pi)$ or end with probability π in each and every period. Otherwise stated, the mean duration of a contract is given by $1/\pi$. In the latter case, the contract is renegotiated on the basis of the available information at that time. It thus follows that $\pi(1-\pi)^j$ is the fraction of households having a wage contract of age j. We thus have contracts that last for several periods and end randomly. However, contracts introduced by Ambler et al. (1997) and Benassy (1999) essentially rely on wage–wage emulation as in Taylor[5].

In this paper we try to build a bridge between these two strands of literature, and build Calvo type contracts based on Gray's expected market-clearing wage rather than on wage–wage emulation. In such a framework, a contract signed at time t is just the sum of all expected walrasian wages, weighted by the surviving probability of the contract

$$\log(\chi_t) = \sum_{i=0}^{\infty} \pi(1-\pi)^i E_t \log\left(W_{t+i}^*\right) \tag{11}$$

where W_t^* denotes the Walrasian wage in period t. Therefore, a contract of age j at time t stipulates a fixed nominal wage agreed upon at time $t-j$, χ_{t-j}. Under such a contract the household agrees to supply as much labor as demanded by the firm[6].

Let W_t^c denote the average contractual wage, and $W_t^c(i)$ the contractual wage of household i. From the program of a firm, we get:

$$W_t^c(i) = (1-\alpha)\frac{Y_t}{h_t(i)}$$

A first implication of this formulation is that the nominal income on the labor market, $W_t^c(i)h_t(i)$, is the same across all households.[7] Then, taking

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4 See for example Cho (1993), Fairise (1995)

5 Another departure from these two papers is that we do not introduce labor adjustment costs, in order to focus on the potential gains of Gray–Calvo type of contracts.

6 We assume that the firm has the right to manage employment so as to avoid time inconsistency problems. Assume for example that a shock occurs in the economy and that the nominal wage contract and employment are bargained between the two agents, this could lead one of the agent to break the contract. In order to avoid this, once the contracts are signed, the firm has the right to freely adjust employment in order to smooth the effects of shocks.

7 Another implication of this property is that the dynamic behavior of all individuals will be the same.

the previous equation in logarithm and aggregating over labor types, we get :

$$\log(W_t^c) = \int_0^1 \log(W_t^c(i))\mathrm{d}i$$

As in each period, a fraction π of contracts end, there are $\pi(1-\pi)$ contracts surviving from period $t-1$, $\pi(1-\pi)^2$ from period $t-2$, and $\pi(1-\pi)^j$ from period $t-j$, so that the average wage rewrites as :

$$\log(W_t^c) = \sum_{i=0}^{\infty} \pi(1-\pi)^i E_t \log(\chi_{t-i}) \tag{12}$$

Using properties of lag operators, it is straightforward to show that (11) and (12) admit the following recursive form : [8]

$$\begin{cases} \log(\chi_t) = \pi \log(W_t^*) + (1-\pi)E_t \log(\chi_{t+1}) \\ \log(W_t^c) = \pi \log(\chi_t) + (1-\pi)\log(W_{t-1}^c) \end{cases}$$

This system describes the law of motion for nominal wages in a contractual framework. The existence of nominal wage contracts affects the behavior of the household. As the firm has the right to manage employment, the household is submitted to an additional constraint :

$$h_t^s = h_t^d \tag{13}$$

Otherwise, stated actual hours worked must equal labor demand whatever the state of Nature. In this framework, the optimal labor supply behavior for the household reduces to :

$$U_\ell(t) - \Lambda_t \frac{W_t^c}{P_t} = \psi_t$$

where ψ_t is the lagrangian multiplier associated to the constraint (13). It can then be interpreted as the cost, expressed in term of utility, of being constrained on the labor market. We assume that the contract is fixed by reference to the nominal wage that would minimize the cost of being constrained on the labor market, that is $\psi_t = 0$. This defines the wage target in (11). This amounts to setting the nominal wage target by reference to the nominal wage that would prevail in a walrasian framework, as in Cho (1993) and Cho and Cooley (1995). Thus, W_t^* is given by :

$$W_t^* = \frac{P_t U_\ell(t)}{\Lambda_t}$$

[8] See appendix B.

1.5 Equilibrium

This section presents the general equilibrium in two cases : the walrasian regime and the contractual regime.

1.5.1 The walrasian economy

The equilibrium of the walrasian economy is a sequence of prices $\{P_t, W_t, r_t\}_{t=0}^{\infty}$ and a sequence of quantities $\{C_t, h_t, K_{t+1}, M_{t+1}\}_{t=0}^{\infty}$ such that :

1. For a sequence of prices $\{P_t, W_t, r_t\}_{t=0}^{\infty}$, $\{C_t, h_t, K_{t+1}, M_{t+1}\}_{t=0}^{\infty}$ maximizes the representative household's utility subject to the intertemporal budget constraint and the cash–in–advance constraint.
2. For a sequence of prices $\{P_t, W_t, r_t\}_{t=0}^{\infty}$, $\{h_t, K_t\}_{t=0}^{\infty}$ maximizes the representative firm's profit.
3. For a sequence of quantities $\{C_t, h_t, K_{t+1}, M_{t+1}\}_{t=0}^{\infty}$, $\{P_t, W_t, r_t\}_{t=0}^{\infty}$ clears all markets, and in particular the good market :

$$Y_t = C_t + I_t$$

1.5.2 The contractual economy

The equilibrium of the contractual economy is a sequence of prices $\{P_t, W_t^c, r_t, \chi_t\}_{t=0}^{\infty}$ and a sequence of quantities $\{C_t, h_t, K_{t+1}, M_{t+1}\}_{t=0}^{\infty}$ such that :

1. For a sequence of prices $\{P_t, W_t^c, r_t, \chi_t\}_{t=0}^{\infty}$, $\{C_t, K_{t+1}, M_{t+1}\}_{t=0}^{\infty}$ maximizes the representative household's utility subject to the intertemporal budget constraint and the cash–in–advance constraint.
2. For a sequence of prices $\{P_t, W_t^c, r_t, \chi_t\}_{t=0}^{\infty}$, $\{h_t, K_t\}_{t=0}^{\infty}$ maximizes the representative firm's profit.
3. For a sequence of quantities $\{C_t, h_t, K_{t+1}, M_{t+1}\}_{t=0}^{\infty}$, $\{P_t, r_t\}_{t=0}^{\infty}$ clears capital and good markets.
4. The sequence $\{W_t^c, \chi_t\}_{t=0}^{\infty}$ is determined by equations (11) and (12).
5. For a sequence of prices $\{P_t, W_t^c, r_t, \chi_t\}_{t=0}^{\infty}$, each household supplies the exact amount of hours demanded by firms.

The rational expectations dynamic systems of equations that characterizes these equilibria admit no analytical solution. So each of them is log–linearized around the deterministic steady–state and solved using the method advocated in Farmer (1993). It is then possible to assess for the validity of the model and to characterize its qualitative properties.

2 Validation of the Model

This section is devoted to the analysis of the dynamic properties of the model in both model economies. We then assess for the ability of the model to mimic the main features of the German business cycle. But, since the model admits no analytical solution, we use a numerical approach that first necessitates the calibration of structural parameters.

2.1 Calibration

We calibrate the model for the German economy, from 1960 : 1 to 1989 : 4[9]. We use seasonally adjusted quarterly data. Most of the series are taken from the Deutsche Institut für Wirtschaftsforschung (DIW) just as in Brandner and Neusser (1992). We thank P. Brandner and K. Neusser for providing most of the German data series. Details on the data used can be found in appendix.

To calibrate the model we follow the method initiated by Kydland and Prescott (1982) and developed by Cooley and Prescott (1995). This calibration exercise consists in choosing parameter values such that the balanced growth path (steady state) of our model economy matches certain long-term features of the data.

2.1.1 The household

As mentioned earlier we consider the general parametric class of preferences of the form :

$$U(C_t, \ell_t) = \frac{\left(C_t^{\nu}\ell_t^{1-\nu}\right)^{1-\sigma} - 1}{1-\sigma}$$

We choose β such that the steady state annual real interest rate equals 3% ($\beta = 0.9926$). As there is, to our knowledge, no empirical evidence for σ on German data, we use the same value for σ as the one estimated in Eichenbaum, Hansen and Singleton (1988) for the U.S. — i.e. 4/3[10].

We normalize the total time available to work to one and calibrate the parameter ν such that the steady state number of hours is 0.36 which corresponds to the historical share of total disposable time endowment[11]. This yields a value for ν equal 0.41.

[9] This choice is related to the break in 1990 due the unification.

[10] This choice is not that critical as sensitivity analysis indicates that our results are robust to changes in the value of σ.

[11] The average working time in Germany is 40 hours a week, such that Germans work 40×13 hours a quarter. If we assume that each individual is left with 16 hours a day (allowing for 8 sleeping hours), the total time endowment is $16 \times 7 \times 13$ hours a quarter, such that the share of total disposable time endowment dedicated to labor is 36%.

Table 1 : *Household's Structural Parameters*

β	σ	ν
0.9926	4/3	0.41

2.1.2 The firm

The firm's production possibilities are summarized by the following Cobb-Douglas production function:

$$Y_t \leqslant A_t K_t^{\alpha} (X_t H_t)^{1-\alpha}$$

This functional form is suggested by the basic observation that capital and labor shares of output have been approximately constant over time. The parameter $1-\alpha$ is referred to as the labor share in output in a competitive environment: $1-\alpha = 0.66$, the historical share of labor income in real GNP.

We choose a benchmark value for capital depreciation rate ($\delta = 0.025$ as in Cooley and Hansen (1989)) and construct a capital series using the law of motion for capital (equation (3)). Then, we can compute a series for the Solow residual. Since, under perfect competition and constant returns to scale, the growth rate of the Solow residual represents a measure of the growth rate of technical progress augmenting the global productivity of factors, A_t is measured by:

$$\log(A_t) = \log(Y_t) - \alpha \log(K_t) - (1-\alpha)\log(H_t)$$

ρ_a and σ_a are then obtained by estimating a first-order autoregressive equation, AR(1), on A_t, after removing the growth component using a linear trend. We obtain a persistence for the technological shock of 0.96 and a standard deviation of 0.01.

2.1.3 Money Supply and nominal wage contracts

In order to calibrate the parameters of the money supply process, we estimate an AR(1) on the growth rate of the money aggregate M1. We use a seasonally adjusted (Census X-11) series for $M1$ taken from the Bundesbank (Seasonally adjusted business statistics). The persistence of the monetary shock is estimated to be 0.495 with a standard deviation of 0.011.

Table 2 : *Firm's Structural Parameters*

γ	δ	α	ρ_a	ϵ_a
1.0067	0.025	0.34	0.96	0.01

Table 3: *Money Supply Structural Parameters*

$\bar{g}$	ρ_g	ϵ_g
1.018	0.495	0.011

Since we have no insight on the value of π, the extinction probability (and the inverse of the average length of contracts), we will try several values.

2.2 Impulse Response Function Analysis

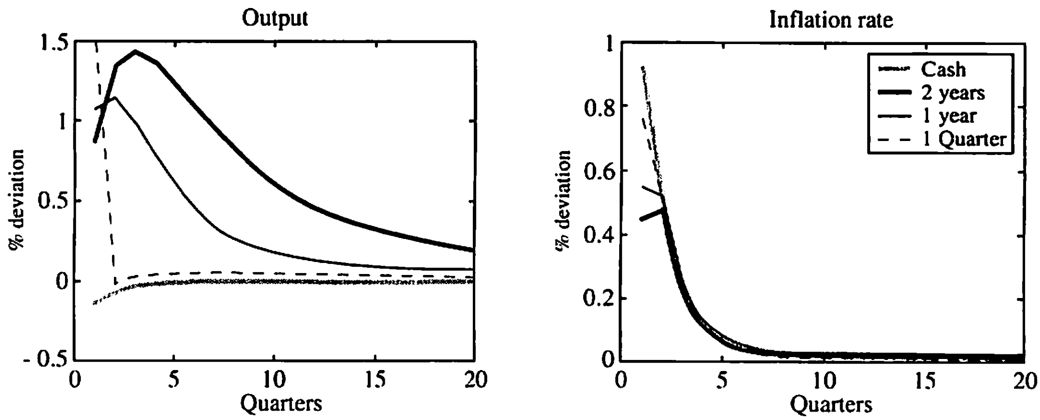
Impulse response functions (IRF) provide information on the response of the theoretical system to a stochastic shock in period t and gives the percentage deviation from the steady state of a given aggregate for different time horizons. It is worth noting that IRF are directly obtained from the theoretical model without using any detrending procedure[12]. They thus allow one to feature the intrinsic qualitative dynamic properties of the model economy with respect to an exogenous macroeconomic event.

In this model, the shape of IRF of the economy to a 1% technological shock does not differ from the benchmark RBC model. Thus, we only discuss the response of the system to a 1% monetary shock[13]. IRF are reported in figure 1 and 2. Instantaneously, inflation rises. Thus the purchasing power of the transferred money balances decreases, and so does consumption via the cash-in-advance constraint. This effect is common to all cash-in-advance models and is the so-called inflation tax phenomenon. In the simple cash-in-advance model, the household reports its consumption willingness to goods that do not bear the inflation tax, namely leisure and assets. So he transfers consumption to future periods, in which inflation tax will be lower. Saving increases, and so does investment. Further, the household will increase his demand for leisure so that hours fall in equilibrium. The decrease in hours worked implies that output will instantaneously be below its steady state level, since capital is predetermined. We thus find the traditional negative effect of money in the simple cash-in-advance model. In the nominal wage contract economy, the first effect that transits through investment remains. It is further reinforced by the fact that the household cannot freely adjust labor supply: labor is set by the firm. So the household can only respond to the inflation tax by transferring consumption to future periods. So, at general equilibrium, the response of investment is reinforced and with the

12 Indeed, it is much easier to obtain a hump-shaped response of output to a monetary shock using HP-filtered data, or quadratic detrended data. Hence the hump is due to the internal propagation mechanisms of the model and is not an *arte fact* of any detrending procedure.

13 As the model is log-linearized, all the responses are in terms of elasticities.

Figure 1 : *Impulse response functions to a monetary shock*



capital stock above its steady state level for future periods it will exert a positive effect on output.

Further, the nominal wage rate does not react instantaneously. Thus, the rise in inflation leads to a lowering in the real wage rate. *Ceteris paribus*, the demand for labor increases, and so do hours worked[14]. Given the predetermined capital stock, output rises above its steady state level. So in this model, because of nominal wage contracts, money injection exerts a positive effect. For the case of a one period non-stochastic contract, the effects of a monetary shock fade away very quickly.

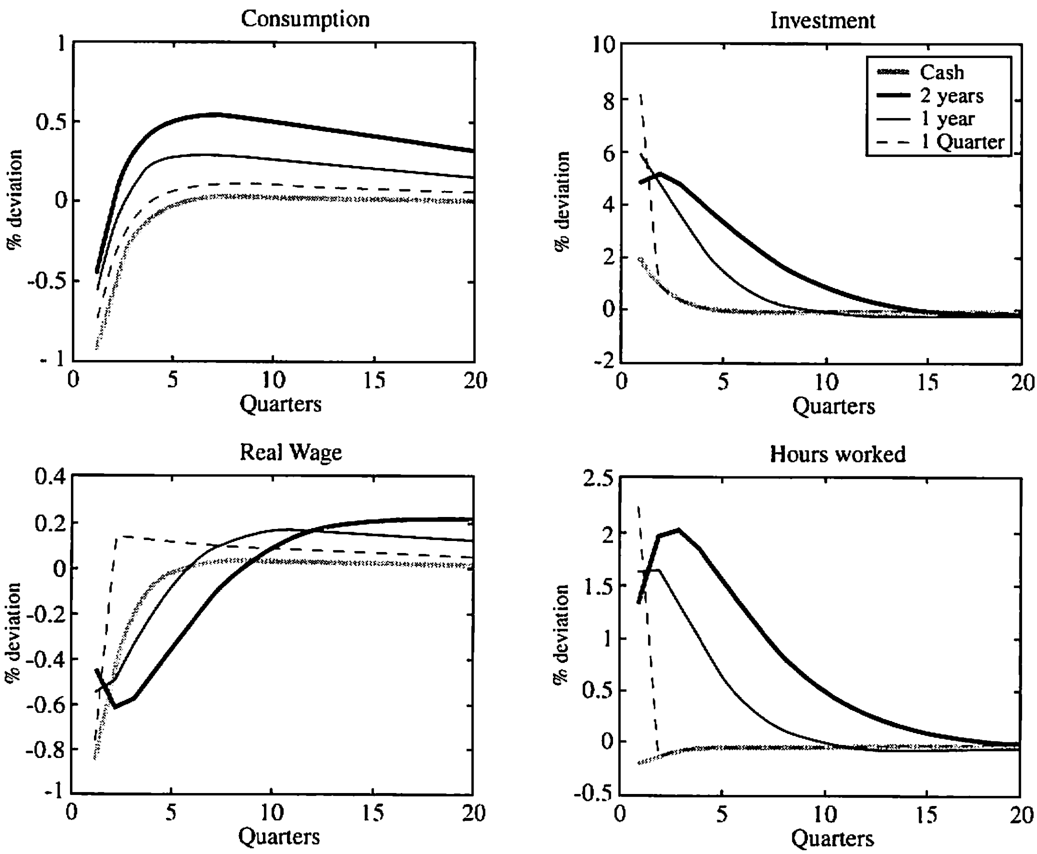
The model with a stochastic contract length can generate a persistent hump-shape response of output to a monetary shock as suggested by empirical studies. As π increases, the mean duration of a contract diminishes[15]. Thus, the length of the period during which firms can benefit from wage stickiness is shorter. So they will respond more strongly to a monetary shock, and will increase more sharply their demand for hours. This explains the higher magnitude and lower persistence of the response of output.

To understand the hump-shaped form of the response of output to the monetary shock, it is useful to think about the household's problem. With a longer contract length, the distortion period is longer and this induces more consumption smoothing. Instantaneously, consumption falls less, investment increases less and more money balances are transferred compared to the case of shorter contracts. In the second period, consumption and investment increases more for a longer contract length. For a contract length higher than one year, this effect induces an even higher response of output in the second period (hump-shaped). In the case of an average length of 2 years,

14 Just recall that the firm has the right to manage employment.

15 The mean duration of a contract is given by $1/\pi$.

Figure 2 : *Impulse response functions to a monetary shock*



even investment an inflation have a hump-shaped response to the monetary shock.

2.3 Quantitative Validation

In this section, we simulate 2 types of models, to assess for their ability to mimic the German business cycle :

- The monetary Business Cycle model without wage contracts;
- The monetary Business Cycle model with wage contracts for different values of mean duration of the contracts.

The data generated by the theoretical model economies are logged and detrended using the Hodrick-Prescott filter. We compute a set of second order moments characterizing the business cycle. These statistics are reported in table 4.

Table 4 : *Cyclical properties of the model*

	Data	Cash	2 years	1 year	2 quarters	1 quarter
σ_y	1.62	1.70	3.05	2.71	2.45	2.55
σ_c/σ_y	0.92	0.72	0.42	0.42	0.46	0.46
σ_i/σ_y	2.82	2.93	3.50	3.87	4.13	4.24
σ_h/σ_y	0.81	0.38	1.13	1.06	1.03	1.08
σ_f/σ_y	0.23	0.67	0.29	0.35	0.42	0.42
$\sigma_{m/p}/\sigma_y$	1.39	0.46	0.40	0.37	0.36	0.32
ρ_f	0.24	0.38	0.32	0.36	0.38	0.37
$\rho(c, y)$	0.70	0.69	0.59	0.33	0.12	0.04
$\rho(i, y)$	0.83	0.85	0.95	0.94	0.94	0.94
$\rho(h, y)$	0.69	0.91	0.91	0.89	0.87	0.87
$\rho(f, y)$	0.03	−0.32	0.07	0.20	0.20	0.15
$\rho(m/p, y)$	0.30	0.62	0.68	0.55	0.38	0.24
$\rho(\Delta y)$	−0.06	−0.02	0.12	−0.02	−0.21	−0.42

Note : For example $\pi = 0.125$ correspond to a average length of wage contract of two years. $c, i, y, m/p$ and f denote respectively consumption, investment, output, real balances and the inflation rate.

The main features of the German data are alike those of the U.S. economy. Investment is more volatile than output while consumption exhibits less variability. Hours are less volatile than output. Inflation is far less volatile than output, while the volatility of the real balances is higher than that of output. All aggregates are procyclical. But, as observed in the data for Germany, the inflation rate is almost acyclical, meaning that demand and supply shocks both matter. Finally, conversely to US data, German output growth does not display that much persistence in the short–run, and 3 lags are needed to get significant persistence in the data (corr(Δy_t, Δy_{t-3})=0.12) meaning that shocks need time to propagate.

For the calibration we have used, the simple cash–in–advance model mimics in a quite good way the volatility of output. As soon as we introduce nominal rigidities, the volatility of output increases. This is due to the abandonment of the intertemporal substitution in labor supply behavior, that traditionally leads to a smoothing in hours worked. Nominal wage contracts reinforce volatility of hours, and thus that of output.

Whatever the model we consider, the general pattern of relative volatilities is reproduced. But, the different models underestimate the relative volatility of consumption. This is essentially due to the permanent income hypothesis which implies a too high consumption smoothing. Furthermore, investment's relative volatility is too high in the model. It appears that as π increases — *i.e.* as the mean duration of contracts diminishes — the volatility of investment increases. This is so because hours react more strongly. Since nominal wages are fixed, the wealth of an individual increases instantaneously. This additional wealth will not be consumed because of the inflation tax, and will be invested.

The relative volatility of inflation is somewhat too high whatever the model we consider. In the simple cash cash-in-advance model the relative volatility is 3 times the one observed in the data. As soon as we introduce nominal wage contracts, it decreases considerably. As seen from figure 1, the magnitude of the response of inflation decreases with the length of the contract. So decreasing π allows to lower, slightly, the volatility of inflation. Nevertheless, the volatility of real balances is badly replicated by all models, including those with contracts, since historical real balances are always more volatile than output.

In terms of correlation of aggregates with output, all models predict procyclical aggregates. We focus on correlations between inflation, real balances and output. The simple cash-in advance model fails to mimic $\rho(f, y)$ and $\rho(m/p, y)$. The correlation between inflation and output is negative. Indeed, as the variance decomposition exercise shows, productivity shocks dominate. Further, after a monetary shock inflation goes up while output falls, as the IRF analysis shows. The introduction of contracts breaks this negative link. After a monetary shock, output and inflation raise. So we obtain a weak positive correlation between output and inflation, which is consistent with German data. Finally, output growth persistence can be easily matched using 1 or 2 years contracts. It should be however stressed that non monetary RBC models should be able to mimic that stylized fact for the German economy as the standard RBC model generates no persistence in the short run (See Cogley and Nason (1995)). Further the model fails to account for the third order autocorrelation as it generates a weak negative number (-0.08).

Concerning the real balances, the cash-in-advance model strongly overestimates the correlation with output. Introducing nominal wage contracts allows to reduce sharply this correlation, so that the model is able to mimic the data but only for high values of π — *i.e.* for short contracts. For a longer average length of contracts, the persistence of inflation is much better replicated by the nominal wage contract model than in the CIA model.

We now turn to the analysis of the variance decomposition for output and inflation rate. This allows us to precise the weight of monetary shock in the dynamics of these aggregates. As can be seen from table 5, monetary shocks do not explain the dynamics of output in the simple cash-in-advance

Table 5 : *Variance Decomposition*

	Cash		2 years		1 year		2 quarters		1 quarter	
	Y	f	Y	f	Y	f	Y	f	Y	f
1	0.67	77.61	22.79	39.56	30.36	51.21	37.25	61.57	43.08	70.07
4	0.24	82.27	42.70	61.62	32.93	69.45	22.68	75.56	23.48	79.20
8	0.14	82.15	42.24	61.82	26.32	69.69	15.40	75.65	15.54	79.22
20	0.08	82.11	33.85	61.99	17.63	69.73	9.38	75.64	9.29	79.21
40	0.07	81.86	29.21	61.89	14.39	69.57	7.43	75.48	7.31	79.05

Note : Each column gives the percentage of variance explained by the monetary shock. Y stands for output and *f* for the inflation rate

model. Since prices freely adjust instantaneously after an increase in money supply, monetary shocks only affect marginally the dynamics of output.

In the nominal wage contracts economy, monetary shocks explain between 22% and 40% of output. As seen from the IRF analysis, the raise in money supply leads to an increase in hours worked, and thus output. The magnitude of this real effect is higher than the one obtained in the cash-in-advance model as the elasticity of labor supply is infinite in the nominal wage contract case. Concerning the inflation rate, the introduction of contracts raises the weight of technological shocks. Since households cannot adjust hours worked, consumption becomes more reactive to a technological shock. So, the response of inflation is reinforced *via* the cash-in-advance constraint.

An even more striking feature, appearing through this variance decomposition, is that monetary shocks exert a long lasting effect on output dynamics. After four quarters, monetary shocks still explain between 22% and 40% of output volatility in the nominal wage contracts model, whereas they only explain less than 1% in the cash-in-advance model. After 20 quarters, the share of output volatility explained by monetary shocks still lies between 9% and 30% in the nominal wage contracts model, for the different lengths of contracts.

3 Concluding remarks

We have built a simple cash-in-advance model, in which we introduced stochastic nominal wage contracts. At each period, a given contract has a positive probability to end. We showed that the introduction of these contracts allows to weaken the negative effect of the inflation tax such that monetary shocks exert a positive and persistent effect on output dynamics.

Contrary to the basic cash-in-advance model, this model is able to mimic the correlation of inflation and real balances with output and still matches most of the moments of real variables. It also lowers not only the standard deviation of inflation relative to that of output but also the persistence of inflation. Note that the relative standard deviation of real balances remains far too low. Further, the variance decomposition analysis indicates that, in this setting, monetary shocks explain between 22 and 40 % of the variance of output, compared to 1% for the cash-in-advance model. Moreover, the model generates a persistent hump-shaped response of output to a monetary shock (9% to 30% after 5 years), as suggested by several studies (See for example Bec and Hairault (1993) or Sims (1992).)

However, due to the presence of nominal wage contracts, the model generates too much volatility, especially for hours worked. One way to circumvent this shortcoming of the model would be to introduce, as in Ambler et al. (1997), labor adjustment costs. This is left for further research.

A Data

If not indicated otherwise, data are from the Deutsche Institut für Wirtschaftsforschung, Berlin (DIW). As in Brandner and Neusser (1990) annual values for total population (Source : Austrian Institute of Economic Research (WIFO) database) have been interpolated to get quarterly series for per capita calculations. All series have been seasonally adjusted (Census X-11).

Output : Real gross national product (GNP), at constant prices 1980, per capita.

Consumption : Real private consumption, at constant prices 1980, per capita.

Fixed investment : Real gross investment, at constant prices 1980, per capita.

Hours : Total hours worked, per capita.

Real wage : Compensation of employees, divided by the deflator of private consumption, per employee.

Price level : Consumer price index (CPI), IMF statistics.

Inflation : Δ LN (CPI).

Money supply : M1, Seasonally adjusted (Census X-11) business statistics.

B Recursive Form of Contracts

$$\log(\chi_t) = \sum_{i=0}^{\infty} \Xi(i) E_t \log\left(W^{\star}_{t+i}\right)$$
$$\Xi(i) = \pi\left(1 - \sum_{j=1}^{i} \xi(j)\right)$$
$$\xi(j) = \pi(1-\pi)^j,\ j \geqslant 0$$

$$\begin{aligned}\Xi(i) &= \pi\left(1 - \frac{\pi}{(1-\pi)}\sum_{j=1}^{i}(1-\pi)^j\right)\\ &= \pi\left[1 - \frac{\pi}{(1-\pi)}\left(\frac{(1-\pi)-(1-\pi)^{i+1}}{\pi}\right)\right]\\ &= \pi(1-1+(1-\pi)^i)\\ &= \pi(1-\pi)^i\end{aligned}$$

$$\begin{aligned}\log(\chi_t) &= \sum_{i=0}^{\infty} \Xi(i) E_t\left[\log(W^{\star}_{t+i})\right]\\ &= \pi\sum_{i=0}^{\infty}(1-\pi)^i E_t\left[\log(W^{\star}_{t+i})\right]\\ &= \pi\sum_{i=0}^{\infty}(1-\pi)^i E_t\left[(F^i \log W^{\star}_t)\right]\\ &= \frac{\pi}{1-(1-\pi)F} E_t \log W^{\star}_t\end{aligned}$$

$$\begin{aligned}E_t\left(1-(1-\pi)F\right)\log(\chi_t) &= \pi \log W^{\star}_t\\ \log(\chi_t) &= \pi \log W^{\star}_t + (1-\pi) E_t \log(\chi_{t+1})\end{aligned}$$

Further, we had :

$$\log(W^c_t) = \sum_{i=0}^{\infty} \Xi(i) \log(\chi_{t-i})$$

Given that $\Xi(i) = \pi(1-\pi)^i$

$$\begin{aligned}\log(W_t^c) &= \sum_{i=0}^{\infty} \pi(1-\pi)^i \log(\chi_{t-i}) \\ &= \sum_{i=0}^{\infty} \pi(1-\pi)^i L^i \log(\chi_t) \\ &= \frac{\pi}{1-(1-\pi)L} \log(\chi_t)\end{aligned}$$

Thus,

$$\log(W_t^c) = \pi \log(\chi_t) + (1-\pi) \log W_{t-1}^c$$

C Optimality conditions and Equilibrium

The Household

$$U_C(t) = \Lambda_t + \xi_t \tag{14}$$

$$U_\ell(t) = \Lambda_t \frac{W_t}{P_t} \tag{15}$$

$$\beta E_t \Lambda_{t+1}(1-\delta+r_{t+1}) = \Lambda_t \tag{16}$$

$$\frac{\Lambda_t}{P_t} = \beta E_t \left(\frac{\Lambda_{t+1}+\xi_{t+1}}{P_{t+1}} \right) \tag{17}$$

$$\lim_{i\to\infty} E_t \left\{ \beta^{t+i} \Lambda_{t+i} K_{t+1+i} \right\} = 0 \tag{18}$$

$$\lim_{i\to\infty} E_t \left\{ \beta^{t+i} \xi_{t+i} M_{t+1+i} \right\} = 0 \tag{19}$$

Λ_t and ξ_t are the lagrangian multipliers associated respectively to the intertemporal budget constraint and the cash–in–advance constraint. Equation (14) and (15) define Frishian demand functions for consumption and leisure. Equation (17) defines the intertemporal behavior of demand for money. Finally, (18) and (19) provide terminal conditions to the evolution of assets and money.

The Firm

$$W_t = \alpha \frac{Y_t}{H_t} \tag{20}$$

$$r_t = (1-\alpha) \frac{Y_t}{K_t} \tag{21}$$

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