Doping in Sport and Competition Design

Nicolas Eber*
IEP, Université Robert Schuman†

Jacques Thépot*
LARGE, Université Louis Pasteur‡

Introduction

Comments on the Festina affair during the *Tour de France 98* put forward the idea that a main reason for athletes to use performance-enhancing drugs (doping) is the economic stakes in modern competitive sports. Thus, doping may be primarily an economic issue. However, very few effort has been devoted to this approach. Bird and Wagner (1997) use arguments directly taken from non-cooperative game theory to analyze the use of doping as an institutional problem of finding social norms likely to deter athletes from using drugs[1].

The present paper contributes to the economic analysis of the doping problem by proposing a formal model. The use of doping is analyzed in a game-theoretic framework so as to capture explicitly the economic ingredients of competitive sports. We develop a two-player game based on the following assumptions : (i) a health cost is incurred by athletes using doping; (ii) using doping allows any athlete to improve his results during the season; (iii) but if both athletes dope, the order of finish remains unchanged; (iv) a doped athlete has a positive probability to be caught by a drug test and, hence, to be punished.

† 47, avenue de la Forêt-Noire, 67082 Strasbourg Cedex, France. e-mail : nicolas.eber@iep.u-strasbg.fr.
‡ 61, avenue de la Forêt Noire, 67085 Strasbourg Cedex, France. e-mail : thepot@cournot.u-strasbg.fr.
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1 Another reference is Breivik (1987) who also uses a game-theoretic reasoning to establish the basic character of prisonners's dilemma in sport doping.

Our contribution deals with identifying four basic factors on which depend the athletes' incentives to use doping: (i) the efficiency of the test system, (ii) the number of events during a season, (iii) the range of prizes from sports events, (iv) prevention measures. We can then discuss the possible directions to deter from doping. What is needed is a global reform of the competitive sports combining an improvement of the test system, fewer events during the season, lower spreads in the prizes from sports events and more prevention. We also study the optimal competition design. We highlight the necessary *complementarity* between high prizes and strong tests: either tests are strong and prizes can then be large, or tests are inefficient and prizes must be low. We also show that the optimal competition design crucially depends on the athletes' "reservation wage". Several (anti-doping) competition designs are possible: very important competitions should optimally feature large prizes and strong tests, while competitions of little importance should optimally feature low prizes and few tests.

The paper is organized as follows. Section 2 lays out the "doping game" and presents the Nash solution. Implications for anti-doping policy are derived in section 3. A discussion on the optimal competition design is provided in section 4. Section 5 concludes.

1 The model

Let us consider two athletes, A and B, involved in a season containing n sports events. They are assumed to be identical in terms of physical abilities and preferences.

1.1 Notations and definitions

The extensive form game is in four stages. At stage 1, each athlete chooses to dope (D) or not to dope (ND). At stage 2, the season is played. At stage 3, each athlete is controlled for doping use. Any doped athlete has a probability p to be caught. Any undoped athlete has no risk to be falsely recognized as doped. At stage 4, the prizes for all the season (i.e. for the n events) are delivered. We assume that each athlete will be controlled for sure and that the tests are run after all the events of the season; this assumption may be viewed as an extreme case of a reality-closed system where athletes have a very high probability to be controlled at least once during the season, the results of the tests being published (sometimes a long time) after the events.

Let w_1 and w_2 be the prizes respectively for the first and the second place gained in any event, and w_0 the global income in case of positive test, with $nw_1 > nw_2 > w_0$. Doping involves a cost c which is the (perceived) health cost (evaluated in pecuniary terms). If an athlete is caught by a doping test, he is banned and not paid for all the events. In the case where only one athlete is caught as doped, he is disqualified and his competitor then recovers all the prizes devoted to the winner (i.e. as if he won all the events). Furthermore, we suppose that both athletes have a utility function u increasing in revenue. If placed in equal conditions (i.e. if they are both doped or both undoped), the athletes are assumed to share the market, winning each half of the events. However, if one athlete dopes while his competitor does not, he is sure to win all the events.

In this context, the game can easily be put under normal form and the strategies of the athletes reduce to choosing between D and ND, in the first stage. Then four cases have to be considered (where U_A and U_B denote the payoffs for athlete A and B) :

- (D, D) : Both athletes dope, thus incurring cost c. Each athlete wins $\frac{n}{2}$ events and gets $\frac{n}{2}$ second places. With probability p^2, both are caught by the doping tests and thus get w_0. With probability $(1-p)^2$, both are not caught and each thus earns $n\frac{w_1+w_2}{2}$. With probability $p(1-p)$, only one is caught : the athlete recognized as doped gets w_0 while the other one (falsely recognized as undoped) earns all the prizes (as if he won all the competitions), namely nw_1. Finally, the expected utility of any athlete, in the (D, D) case, is

$$\begin{aligned} U_A(D,D) &= U_B(D,D) \\ &= (1-p)^2 u\left(\frac{n(w_1+w_2)}{2} - c\right) + p(1-p)\left(u(nw_1 - c) + u(w_0 - c)\right) \\ &\quad + p^2 u(w_0 - c) \\ &= (1-p)^2 u\left(\frac{n(w_1+w_2)}{2} - c\right) + p(1-p)\,u(nw_1 - c) + pu(w_0 - c) \end{aligned}$$

- (ND, ND) : Both athletes do not dope. Each wins half of the competitions, thus earning at the end of the season

$$U_A(ND, ND) = U_B(ND, ND) = u\left(\frac{n(w_1+w_2)}{2}\right)$$

- (D, ND) : Athlete A dopes while athlete B does not. Athlete A wins all the events. However, with probability p, he is caught by the doping test, thus getting w_0 while athlete B then recovers the prizes for the victories, i.e. nw_1. With probability $1-p$, athlete A is not caught by the test and thus earns nw_1, while athlete B then earns nw_2. Finally, the expected utilities are

$$\begin{aligned} U_A(D, ND) &= (1-p)\,u(nw_1 - c) + pu(w_0 - c), \\ U_B(D, ND) &= (1-p)\,u(nw_2) + pu(nw_1) \end{aligned}$$

• (ND, D) : This case is symmetric to the previous one :

$$U_A(ND, D) = (1-p)\,u(nw_2) + pu(nw_1),$$
$$U_B(ND, D) = (1-p)\,u(nw_1 - c) + pu(w_0 - c)$$

1.2 The Nash equilibrium

Solving the game can easily be made. Pair (ND, ND) is a Nash equilibrium of the game if and only if $U_A(D, ND) = U_B(ND, D) \leqslant U_A(ND, ND) = U_B(ND, ND)$, i.e. iff

$$u\left(\frac{n(w_1+w_2)}{2}\right) \geqslant (1-p)\,u(nw_1 - c) + pu(w_0 - c) \tag{1}$$

Let $f(w_1) = \dfrac{u(nw_1 - c) - u\left(\frac{n(w_1+w_2)}{2}\right)}{u(nw_1 - c) - u(w_0 - c)}$. Condition (1) is equivalent to $p \geqslant f(w_1)$.

Pair (D, D) is *not* a Nash equilibrium iff $U_A(ND, D) = U_B(D, ND) > U_A(D, D) = U_B(D, D)$, namely iff

$$(1-p)^2\,u\left(\frac{n(w_1+w_2)}{2} - c\right) + p(1-p)\,u(nw_1 - c) + pu(w_0 - c) \tag{2}$$
$$< (1-p)\,u(nw_2) + pu(nw_1)$$

Proposition 2 *If the athletes are risk-averse, pair (ND, ND) is the (unique) Nash equilibrium of the game if and only if $p \geqslant f(w_1)$.*

Proof. We have to prove that (1) implies (2). (1) implies

$$(1-p)^2\,u\left(\tfrac{n(w_1+w_2)}{2} - c\right) + p(1-p)\,u(nw_1 - c) + pu(w_0 - c)$$
$$\leqslant (1-p)^2\,u\left(\tfrac{n(w_1+w_2)}{2} - c\right) + p(1-p)\,u(nw_1 - c)$$
$$+u\left(\tfrac{n(w_1+w_2)}{2}\right) - (1-p)\,u(nw_1 - c)$$

$$= (1-p)^2\,u\left(\frac{n(w_1+w_2)}{2} - c\right) + u\left(\frac{n(w_1+w_2)}{2}\right) - (1-p)^2\,u(nw_1 - c)$$
$$= (1-p)^2\left(u\left(\frac{n(w_1+w_2)}{2}\right) - \theta_{12}\right) + u\left(\frac{n(w_1+w_2)}{2}\right)$$
$$- (1-p)^2\,(u(nw_1) - \theta_1)$$
$$= \left((1-p)^2 + 1\right)u\left(\frac{n(w_1+w_2)}{2}\right) - (1-p)^2\,(u(nw_1) + \theta_{12} - \theta_1),$$

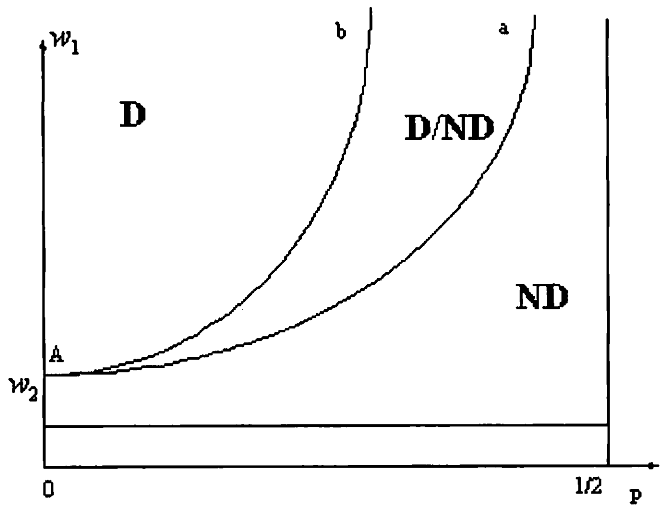

Figure 1 : *The Nash equilibria of the doping game*

with $\theta_{12} = u\left(\frac{n(w_1+w_2)}{2}\right) - u\left(\frac{n(w_1+w_2)}{2} - c\right)$ and $\theta_1 = u(nw_1) - u(nw_1 - c)$. Since function u is concave, then $\theta_{12} \geqslant \theta_1$. For the same reason, we have:

$$\left((1-p)^2+1\right) u\left(\frac{n(w_1+w_2)}{2}\right) - (1-p)^2 \left(u(nw_1) + \theta_{12} - \theta_1\right)$$

$$\leqslant \left((1-p)^2+1\right) \frac{(u(nw_1)+u(nw_2))}{2} - (1-p)^2 \left(u(nw_1) + \theta_{12} - \theta_1\right)$$

which is strictly lower than $(1-p)\,u(nw_2) + pu(nw_1)$ since

$$\left((1-p)^2+1\right) \frac{(u(nw_1)+u(nw_2))}{2} - (1-p)^2 \left(u(nw_1) + \theta_{12} - \theta_1\right) - \left((1-p)\,u(nw_2) + pu(nw_1)\right)$$

$$= -\frac{1}{2}p^2 \left(u(nw_1) - u(nw_2)\right) - (1-p)^2 (\theta_{12} - \theta_1) < 0$$

(because function u is increasing and $w_1 > w_2$). □

The possible equilibria of the game are represented in the parameter space $\{p, w_1\}$ as indicated in Figure 1. Ordinate of point A is $w_2 + 2c/n$, which does not depend on function u. Curve Aa, on which relation (1) is an equality, departs the situations where (ND, ND) is an equilibrium; curve Ab, on which relation (2) is an equality, departs the situations where (D, D) is an equilibrium. Proposition 1 states that zones **D** and **ND** do not overlap when the athletes are risk-averse.

In the risk-neutral case, curves Aa and Ab are respectively of equation $w_1 = \frac{2c/n+w_2-2pw_0}{1-2p}$ and $w_1 = \frac{(1-p^2)w_2+2c/n-2pw_0}{1-2p-p^2}$.

- In zone **D**, the only Nash equilibrium is (D, D); the potential gain from using doping unilaterally outweighs the costs. It is clearly a case of Prisoner's Dilemma. Although each athlete would be better off in a completely undoped world, each one finds optimal to use doping given that his competitor does not. Doping is here a dominating strategy, which results in a preferred outcome regardless of the strategy used by the competitor. Doping use clearly stems from the competition between athletes and the economic stakes of that competition.
- In zone **ND**, the only Nash equilibrium is (ND, ND); the cost of doping outweighs the gain from using drugs unilaterally. Not doping is a dominating strategy; athletes are not involved in a Prisoner's Dilemma anymore.
- In zone **D/ND** both (ND, D) and (D, ND) are Nash equilibria of the game; combined equilibria arise where one athlete does dope while his competitor does not. This case corresponds to a world where both doped and undoped athletes coexist : if one athlete uses doping, it may not be in the interest of his competitor to also dope since the latter will only win half of the events while incurring the doping cost.

Clearly, $f(w_1)$ is increasing with w_0; hence, from proposition 1, the no-doping equilibrium is less likely to occur as w_0 rises. To some extent, w_0 could be interpreted as the non-sporting income since it is the income earned by a disqualified athlete, i.e. by an individual excluded from competitive sport. Thus, as the economy grows and standards of living rise relative to returns to sport, doping should rise. This fits well with the historical record as doping emerged and grew after World War II[2].

Let us define $\theta = \lim_{w_1 \to \infty} f(w_1)$.

Proposition 3

- *If the athletes are risk-averse,* $\theta < 1/2$.
- *If the athletes are risk-neutral,* $\theta = 1/2$.
- *If the athletes are risk-lovers,* $\theta > 1/2$.

Proof. If function u is concave, $f(w_1) < \dfrac{u(nw_1 - c) - \frac{u(nw_1)+u(nw_2)}{2}}{u(nw_1 - c) - u(w_0 - c)}$.
Hence the result. □

As a result, when the athletes are risk-averse, it is sufficient to have a doping test of efficiency $\theta < 1/2$ to deter them from doping for any value of the rewards. *Ceteris paribus*, it is obviously more difficult to deter from doping when athletes are risk-lovers than when they are risk-averse : the required test system efficiency is higher in the former case.

[2] We thank a referee for pointing out this argument to us.

2 Elements to deter from doping

From Figure 1, we can derive some implications for anti-doping policy. Graphically, the problem is how to reach the zone **ND**, starting from zone **D**. The model identifies 4 factors which have to be controlled in order to deter from doping.

1. *The drug test efficiency, p*: Doping tests have not to be perfect to provide sufficiently good incentives. Proposition 2 shows that when athletes are risk-neutral or risk-averse, a doping test system with 50 % efficiency (i.e. one half of the doped athletes are caught by the tests) is sufficient to deter from doping. Moreover, an increase in p clearly lowers the incentives to dope. Graphically, we move away from zone **D** and get closer to zone **ND**. An efficient policy in terms of p is unambiguously positive for both athletes since their payoffs in the undoped world do not depend on this parameter. Hence, measures towards more efficient doping tests should be unambiguously supported by athletes. However, such a policy is costly (efficient tests are very expensive to implement) and its results are very uncertain since it may create counter-productive effects, such as incentives to always produce new and unknown drugs with the creation of an industry of drug developers, testers, lawyers, advisers, enforcers and administrators (Bird and Wagner, 1997).
2. *The number of sports events, n*: It can be proved that a decline in n leads to a shift of curves Aa and Ab in Figure 1 to the left, thus reducing the size of zone **D**. *Ceteris paribus*, a decrease in the number of events reduces the incentives of athletes to use doping: returns from competition are increasing with the number of events while the cost of doping is constant. To some extent, this can be interpreted as increasing returns to scale from doping. However reducing the numbers of events may be negative for athletes (in terms of final payoffs) since, in every zone, their payoffs depend positively on n. If the decrease in n needed to deter from doping is too large, athletes could be finally worse off in the undoped world than they were at the doped equilibrium. In that case, athletes bear the cost of the anti-doping policy and may thus be reluctant to policies aimed at reducing the number of events during the season. Obviously, such policies have a direct economic cost by reducing the size of the competitive sports industries.
3. *The spread of prizes, $w_1 - w_2$*: The larger is the difference, the higher are the incentives to use doping. Thus, a means to deter from doping is also (*ceteris paribus*) to decrease the spread of the prizes from events. (Graphically, we move closer to zone **ND** when w_1 decreases, for given values of w_2 and p.) Athletes should be favorable to this kind of measures since they would earn the same expected payoffs but in a completely undoped world. However, the implementation of such a policy is much more difficult than expected by the model. Prizes include not only financial rewards but also non pecuniary ones such as prestige and the pressure

from sponsors and media tends to exacerbate the prestige of the winner, so that competitive sports are now embodied in a world where "first is first and second is nobody".

4. *The perceived health cost,* c : Prevention has a crucial role. Informing athletes on the risks of doping induces an increase of the perceived health cost. It can be checked that an increase of c implies a shift to the left of curves Aa and Ab on Figure 1, thus shrinking zone **D**. *Ceteris paribus*, prevention measures which increase the perceived health cost c reduce the athletes' incentives to use doping. Notice that such measures are unambiguously positive for both athletes since their payoffs in the undoped world do not depend on c.

Finally, the main insight of this section is to provide a rigorous game-theoretic analysis which confirms the informal intuition frequently put forward. Deterring athletes from doping requires a better drug test system, fewer events, lower spread of prizes and more prevention. Of course, manipulating these factors is very hard (and expensive) to do.

3 Antidoping optimal package

All these factors may be jointly determined : an efficient policy to deter from doping should be considered by the regulators as a "package" $\{p, n, (w_1, w_2), c\}$, encompassing actions on the efficiency of the drug test system, measures on the number of events, on the spread of prizes and the perceived health cost.

Of course, all these actions and measures are costly : improving the efficiency of the test requires high R&D investments, decreasing the number of the sports events and/or the spread of the prizes may reduce the public appeal for such events, medical costs are incurred for prevention. In addition, they resort to different decision makers : pharmaceutical companies, competition organizers and sponsors, leagues and sports federations, public health services, etc.

3.1 A simplified model of competition design

Let us consider, for simplicity, a situation where the package is restricted to $\{p, w_1\}$, with w_2 fixed and $w_0 = c = 0$, $n = 1$, and with risk-neutral athletes. The social planner who can be merely assimilated to the competition organizer is then in charge of designing the package $\{p, w_1\}$ which ensures that the athletes will not dope, namely, that

$$p \geqslant \frac{1}{2}\frac{w_1 - w_2}{w_1} \tag{3}$$

The regulator has to trade off between two components:

- The test cost, which is an increasing and convex function of the efficiency of the test, $\Gamma(p)$, defined on $[0,s] \subset [0,1]$, of the form $\Gamma(p) = \frac{p}{s-p}$, with $\Gamma(0) = 0$ and $\Gamma(s) = +\infty$. Threshold s measures the maximal test efficiency which can be reached; then, we assume that

$$s \in [0, 1/2] \tag{4}$$

(Otherwise, there would exist a test with finite cost ensuring that the athletes do not dope, for any value of w_1.)

- The profit gained by the competition organizer is a concave function $\Pi(w_1)$, which is derived from a simple model for competition participation. We assume that the revenue, R, of the organizer is an increasing and concave function, of the effort e made by the participants, i.e. $R = R(e)$, and that the effort is an increasing and concave function of the winning prize, $e = e(w_1)$. Accordingly, the organizer revenue is an increasing and concave function of $w_1 : R = R(w_1)$, with $R' > 0$ and $R'' \leqslant 0$. As a result, the organizer profit is of the form $\Pi(w_1) = R(w_1) - w_1$.

The optimal social value V of the package $\{p, w_1\}$ is solution to the program:

$$\begin{cases} \max\limits_{p,w_1} V = \left[R(w_1) - w_1 - \dfrac{p}{s-p}\right] \\ \dfrac{1}{2}\dfrac{w_1 - w_2}{w_1} \leqslant p, \\ p \leqslant s \end{cases} \tag{5}$$

Proposition 4 *At the optimum, constraint (3) is binding; therefore, there will always be the following relationship between the optimal winning prize* w_1^* *and the optimal test efficiency* $p^* : p^* = \frac{1}{2}\left(1 - \frac{w_2}{w_1^*}\right)$.

Proof. Immediate from (5). □

Proposition 3 states the necessary *complementarity* between strong tests and large spread of prizes. Indeed, a large spread (i.e. a large value of w_1^*/w_2) is only feasible if the tests are very efficient (i.e. if p is large enough): *large prizes and strong tests are necessarily complementary.*

Condition (3) being necessarily satisfied as an equality, the test cost can be seen as a function $\Gamma(w_1) = \frac{w_1 - w_2}{w_2 - (1-2s)w_1}$, which takes positive values for:

$$w_2 \leqslant w_1 \leqslant w_2/(1-2s) \tag{6}$$

Under conditions (4) and (6), it is easy to prove that function Γ is increasing and convex on $[w_2, w_2/(1-2s)]$.

3.2 The linear profit case

Let us solve program (5) in the case where function R is linear. Then $V = (R-1)\,w_1 - \frac{w_1 - w_2}{(2s-1)w_1 + w_2}$. With $(R-1) > 0$ and $\beta = 1/(R-1) > 0$, program (5) becomes:

$$\begin{cases} \max\limits_{w_1} V = w_1 - \beta \dfrac{w_1 - w_2}{w_2 - (1-2s)\,w_1} \\ w_2 \leqslant w_1 \leqslant w_2/(1-2s) \end{cases} \tag{7}$$

Maximizing function V with respect to w_1 yields to the optimal value:

$$w_1^* = \frac{w_2 - \sqrt{2\beta w_2 s}}{1-2s},$$

which satisfies condition (6) for:

$$\beta \leqslant 2sw_2 \tag{8}$$

The optimal test efficiency is then:

$$p^* = \frac{1}{2}\frac{2w_2 s - \sqrt{2\beta w_2 s}}{w_2 - \sqrt{2\beta w_2 s}}$$

Proposition 5 *Under condition (8),*

1. the optimal winning prize w_1^* is an increasing and convex function of w_2;

2. the optimal test efficiency p^* is increasing in w_2.

Proof. Immediate. □

Notice that w_2 is a kind of "reservation wage" for athletes (taken as given in our analysis); w_2 may come from a bargaining process between athletes (or their unions) and sports events organizers. Proposition 5 clearly states that the value of this "reservation wage" has a crucial influence on the optimal competition design: an increase in the minimum wage w_2 implies an increase both in the optimal test efficiency and in the spread of prizes (w_1^*/w_2 increases with w_2 since w_1^* is an increasing and convex function of w_2). Proposition 5 thus implies that there may be very different efficient (anti-doping) competition designs:

- some with a low minimum wage w_2 and, hence, with low test efficiency and low spreads of prizes, but also
- some with a high minimum wage and, hence, with strong tests and high spreads of prizes.

This result has straightforward implications for the implementation of competition designs. It means that, for competitions of little importance (e.g. for amateur competitions), featured by a low "reservation wage" (i.e. a low w_2), the optimal competition design features small spreads (and, more generally, small prizes) with few tests. For such competitions, the economic stakes are not very important so that low prizes with few tests is the best solution. On the contrary, high-level professional athletes have a high reservation wage, hence requiring a high w_2; organizers of very important events should then optimally offer high spreads (and, more generally, high prizes) with strong tests. For professional sports, the economic stakes are very important; it is then optimal for organizers to offer high prizes in order to generate great participation and effort by athletes and, hence, high revenues. In compensation, of course, they have to pay for efficient tests to fulfill the non-doping constraint.

The point that high w_2 creates tougher problems for administrators may deserve further "real-world" comments. In the Olympics, athletes seem to value just competing even if they do not win. Yet, our model suggests that the most cherished events, the ones with high non-pecuniary rewards to losers, are likely to be highly doped unless strict enforcement of drug control is applied. The International Olympic Committee should therefore be at the forefront of the fighting against doping in sport...

Conclusion

The main conclusion of our economic approach of doping in sports is that what is needed to deter from doping is a global design of sports competitions, including not only investments to improve the test system, but also measures on the number and the prizes of events and prevention. With respect to our analysis, tennis is a good illustrative example : the ATP has designed the worst package, with a small p (few tests), a large n (never-ending season) and a huge spread of prizes (in Grand Slam tournaments, the winner earns twice more than the finalist). One may suspect that (as rumored) incentives to dope are strong in this sport.

This paper is a contribution to the regulation problem of doping in sport. Of course, we have not incorporated here punishments, reputation effects, information-gathering aspects which are crucial in the doping world. However, such phenomena are now well understood by game theorists so that extensions of the model could possibly take them into account. Morever, focusing on information and reputation could stimulate research on institutional aspects of doping (especially mutual enforcement mechanisms), initialized by Bird and Wagner (1997).

References

Bird, E.J., Wagner, G.G. (1997), Sports as a common property resource: A solution to the dilemmas of doping. *Journal of Conflict Resolution*, 41, 749-766.

Thépot, J. (1998), Parrainage sportif et dopage, *Le Monde*, 15 août 1998.

Breivik, G., (1987), The doping dilemma - Some game theoretical and philosophical considerations, *Sportwissenschaft*, 17, 83-94.