On a shareholder constrained efficient criterion for strategic firms

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Introduction

As it has since long (Debreu (1959)) been known, the objective of profit maximization makes sense when shareholders are utility maximizers, markets are competitive and complete (in the case of an economy with uncertainty). That profit maximization may still be a plausible concept in imperfectly competitive environments seems to have been rather accepted in the literature. Tirole (1988), for instance, considers profit maximization as a good proxy for utility maximization, even in a context of general equilibrium.

It seems, however, quite difficult to accept such a viewpoint, for many reasons. First of all, it has been known since the seminal paper by Gabzsewicz and Vial (1972) that the outcome of profit maximization may depend on the normalization chosen by the manager. The reason for that is rather intuitive: maximizing profits with respect to different goods may lead to quite different choices, due to substitutability and complementariness effects between goods. In Cornwall (1977) it was also made clear that not only equilibria might depend on the normalization, but the very existence of equilibria might depend on that.

Utility maximization seems to be a more natural criterion to be adopted by firms, on the simple grounds that agents do not normally desire profits per se, but in view of the utility derived from them.

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As a first observation, which will be useful in what follows, it is worthwhile recalling that production decisions are no longer dependent on price normalizations when they are the outcome of such a process. This has been widely recognized in the case of strategic behaviour in quantity, as it was also illustrated in Dierker and Grodal ((1986), (1993)), and less accepted in the case of strategic behaviour in prices (as the counterexample proposed in Volker Bohm (1994) would seem to imply).

Once we have accepted the idea that utility maximization is a more appropriate criterion than profit maximization also in a set-up of imperfect competition, the indeterminacy linked to the choice of a normalization rule is replaced by another potential source of indeterminacy: when a firm has many owners, whose utility is a manager going to maximize? As will become clearer in the sequel, a unanimity problem haunts the strategic firm’s decision process, as is also the case in other general equilibrium contexts like market incompleteness.

The purpose of this note is to extend to the case of imperfect competition the well known criterion of “shareholders’ constrained efficiency”, originally developed by Drèze (1974) for a set-up of market incompleteness and illustrate its functioning by means of a thoroughly worked out example. In passing, this will also provide the opportunity for drawing a comparison between the two forms of market imperfections, which might bear some independent interest.

It is worth noticing that some of the conclusions of this work have also been reached by Dierker and Grodal ((1995) and (1996)), but from a substantially different standpoint. Relating the profits of a firm to the aggregate demand of its shareholders, the authors introduce a definition of the objective of the firm which they call “maximization of shareholders’ real wealth”; this objective function, which does not depend on the particular normalization chosen, turns out to provide very similar results to those obtained by using the idea of “shareholder constrained efficiency” analyzed in the present paper.

The rest of the paper is organized as follows: the model we use is rapidly sketched in section 2. In the same section we will argue about the importance, also on positive grounds, of firms’ adopting utility maximization rather than profit maximization as a decision criterion, and the resulting unanimity problem will be illustrated. In section 3 the “shareholder constrained efficient criterion” will be introduced, and used to deal with the unanimity issue. In section 4, a thoroughly worked out example will be illustrated, in which shareholders have to decide over two strategic variables. Section 5 concludes, with some remarks on existence of equilibria and on the relationship between imperfect competition and market incompleteness.
1 The model

To fix ideas, we are going to make use of an extremely simplified model, particularly simple with regard to the production sector of the economy. The economy extends over one period, which can be ideally divided into two stages: in the first one, the only firm present in the economy decides over the input it is to use, and therefore over the output level. In the second stage all agents act competitively as to their consumption decisions, and markets clear at walrasian equilibrium prices. The outcome of the second stage of the game is anticipated by the strategic firm in deciding over the output level. For simplicity, we do not consider any kind of uncertainty in what follows.

More formally, we consider a set \( L \equiv \{1, \ldots, L\} \) of goods, and a set \( I \equiv \{1, \ldots, I\} \) of individuals, characterized by:

- \( U^i(x^i_1, \ldots, x^i_L) \): agent \( i \)'s utility function, satisfying standard textbook assumptions, plus homotheticity;
- \( w^i \equiv (w^i_1, \ldots, w^i_L) \) a vector of initial endowments in goods;
- \( \theta^i \), an ownership share in the only firm active in the economy, \( \theta^i \geq 0 \).

With respect to this last feature, we can define the set of shareholders as the set \( S = \{i \in I \mid \theta^i \neq 0\} \). We will further assume that \( |S| > 1 \), i.e. the firm is not a single ownership.

All agents behave competitively with regard to consumption decisions.

The production sector is made up of just one firm, producing a single output \( y_1 \) out of one input, \( y_2 \), according to the production function:

\[
y_1 = f(y_2)
\]

with \( f \) satisfying standard textbook assumptions such as:

(i) \( f'(y_2) > 0 \)
(ii) \( f''(y_2) < 0 \)
(iii) \( \lim_{y_2 \to 0} f'(y_2) = \infty \)

The assumptions made on utility functions guarantee the existence of a differentiable vector of objective demand functions (general equilibrium price functions):

\[
p(y_2) \equiv (p_1(y_2), \ldots, p_L(y_2))
\]

with \( \frac{\partial p_1(y_2)}{\partial y_2} < \infty \).

It is worth noticing that under more general assumptions than the ones made here the existence of such functions cannot be taken for granted. Given that the focus of the paper is rather on the decision criteria that strategic corporative firms might adopt, we deliberately chose not to deal more at depths with the existence problem, which is still open in the literature on imperfect competition in general equilibrium.
Our only firm is strategic, in the sense that it chooses an optimal amount of input, and consequently of output, on the basis of its (perfect) knowledge of the objective inverse demand functions. On the other hand its owners behave as perfect competitors in consuming. This apparently schizophrenic behaviour can easily be rationalized on two different grounds: on the one hand the impact of production on equilibrium market prices can be much larger than the impact of single consumption decisions (and thus be ignored by shareholders). Moreover, while production decisions are, so to speak, concentrated (and maybe carried out by an "omniscient" manager), consumption decisions have a genuinely private nature, and are spread across shareholders.

As mentioned before, there are two main possible behaviours that the firm can adopt to select a production plan, which are introduced in the remainder of the section.

1.1 Profit maximization

Under this hypothesis, the objective of the firm is that of

\[
\max_{y_2} \pi(y_2) = p_1(y_2)f(y_2) - p_2(y_2)y_2
\]

As it has since long (Debreu (1959)) been known, this objective is not questionable when shareholders are utility maximizers, markets are competitive and complete. That profit maximization may still be a plausible concept in imperfectly competitive environments seems to have been rather accepted in the literature. As it was mentioned in the introduction Tirole (1988), for instance, considers profit maximization as a good proxy for utility maximization, even in a context of general equilibrium. The idea underlying this conjecture is that revenue accruing to individuals via profits is what shareholders take mostly into account, since negative price effects, possibly caused by firms' strategies, should normally be negligible (because, for instance, shareholders' consumption of the firms' goods are marginal).

Gabzsewicz and Vial (1972) demonstrated that the outcome of profit maximization may depend on the normalization chosen by the manager. In fact, they clearly show that the functional form of the profit function depends on the particular normalization rule which is adopted, and that, therefore, maximizing profits with respect to different goods may lead to quite different choices, due to substitutability and complementariness effects between goods. On the other hand they also showed that profit maximization, for a given normalization rule, might not be in the best interest of shareholders (put in another way, and under the light of what will be shown in the sequel of the paper, we could say that shareholders might strictly prefer one normalization rule to all the others). As a matter of fact Cornwall (1977) showed that not only equilibria might depend on the normalization, but even the existence of equilibria.
1.2 Utility maximization

Utility maximization seems to be a more natural criterion to be adopted by firms, on the simple ground that agents do not normally desire profits per se, but in view of the utility derived from them.

As a first observation, which will be useful in what follows, it is worthwhile recalling that production decisions are no longer dependent on price normalizations when they are the outcome of such a process. This has been widely recognized in the case of strategic behaviour in quantity, as it was also illustrated in Dierker and Grodal ((1986), (1993)), and less accepted in the case of strategic behaviour in prices (as the counterexample proposed in Volker Bohm (1994) would seem to imply).

In what follows we will show that utility maximization, though perfectly sensible, creates a problem when there is not a single owner. Intuitively enough, the production plan which is selected by means of utility maximization very much depends on whose utility is being maximized.

Let us consider a generic shareholder $i$, and suppose this shareholder can strategically (i.e. exploiting the inverse demand functions) choose the production plan. The problem he solves can therefore be formulated as:

\[
\begin{align*}
\max_{z^i, y^i} & \quad u^i(x^i) \\
\text{s.t.} \quad & \quad p(y^i) z^i(p(y^i)) = \theta^i \pi(y^i) = \theta^i (p_1(y^i) f(y^i) - p_2(y^i) y^2) 
\end{align*}
\]  

where $z^i$ is the vector of net demands of individual $i$ at prices $p(y^i)$. (Of course, problem (2) will also be the optimization problem of a simple consumer, for whom $\theta = 0$).

The first order, necessary conditions for problem (2) are:

\[
\nabla u^i(x^i) = \lambda^i p
\]

and

\[
\theta^i (p_1(y^i) f'(y^i) - p_2(y^i)) = \frac{\partial p_1}{\partial y_2} (x_{1i} - f(y^i) - \omega_{1i}) + \frac{\partial p_2}{\partial y_2} (x_{2i} + y^2 - \omega_{2i})
\]

\[
+ \sum_{i=3}^{L} \frac{\partial p_i}{\partial y_2} (x_{ii} - \omega_{ii})
\]

plus the budget constraint and the usual non negativity constraints on consumption and input.

A simple inspection of equation (4) clearly shows that, despite the equality of $\frac{\partial p_1}{\partial y_2}, ..., \frac{\partial p_L}{\partial y_2}$ across shareholders, the outcome of the production decision will depend on the “dominant” shareholder’s characteristics, as they get reflected in equilibrium net demands, and in the ownership quotas. We can also observe that the value of a marginal change in the production decision is made up of two components: the marginal profit coming out of production,
and the change in income brought about by the revaluation, or devaluation, of individual net demands, as is clear from the right hand side of (4).

One last remark is maybe in order, as for the relationship between profit and utility maximization: in some recent contributions (Groth (1984)) in the case of a Cournot-Walras oligopoly and Böhm (1990) in that of a monopoly) it was shown that under different price normalizations virtually any feasible allocation can be reached as an equilibrium of an imperfectly competitive economy. Intuitively, the same result should hold true in the case of utility maximization, under different choices of the utility function. In fact, that maximizing profits under a suitably chosen normalization rule should correspond, in terms of outcome, to maximizing a particular utility function, is the content of a kind of folk theorem circulating in this branch of literature.

It is quite clear from (4) that any “equivalent” (in the aforementioned sense) normalization rule should crucially incorporate individual i’s (whose utility is being maximized) net demands. In fact, we can easily state and prove the following

**Proposition 1** For the case of \( \pi \neq 0 \) (non-zero profits, which is always true in our simple economy, given the assumptions), the following two problems admit the same solutions, if any:

\[
\max_{y_2} \pi = \hat{p}_1(y_2)f(y_2) - \hat{p}_2(y_2)y_2
\]

(5)

where

\[
\hat{p} = \left( \frac{p_1}{\sum_{i=1}^{L} p_i(y_2)z^*_i}, \ldots, \frac{p_L}{\sum_{i=1}^{L} p_i(y_2)z^*_i} \right)
\]

and \( z^*_i \)'s are agent i’s net demands calculated at his own (consumption) equilibrium, and:

\[
\text{find a } y_2 \text{ satisfying (4)}
\]

(6)

**Proof**: see appendix.

Proposition 1 permits to conclude that a firm which maximizes its owner’s satisfaction chooses the same strategy it would select if it maximized profits computed in terms of the optimal expenditure of its owner. This kind of normalization is very close in nature to “true cost of living” index in the literature on price indices.

2 Shareholder constrained efficiency

We noticed in the previous section that, unless shareholders are identical, (in preferences and endowments), unanimity is extremely unlikely to
fall upon a particular choice of production. It would therefore be useful if we could find a rule such that “stable” decision could be taken.

Many stability concepts might be put forward. One such criterion is that of “shareholder constrained efficiency”, proposed by Geanakoplos et al. (1990) for an economy with production and incomplete markets.

**Definition 1** (see Geanakoplos et al. (1990)) : a production plan $y_2^*$ is said to be “shareholder constrained efficient” if there does not exist another plan $\bar{y}_2 = y_2^* + dy_2$, technologically feasible, and a vector of income transfers $\tau \equiv (\tau_1, \ldots, \tau_S)$, with $\sum_{i \in S} \tau_i = 0$, such that:

$$V^i(\bar{y}_2, \tau^i) \geq V^i(y_2^*, 0), \forall i \in S$$

(7)

with at least one strict inequality ($V^i(\cdot)$ being the indirect utility associated to the plan $y_2$ and the transfer $\tau^i$, in the second stage of the game).

From Definition 1 it should be clear that the proposed criterion of efficiency is local in nature, because shareholders can only evaluate infinitesimal changes from any given plan. In other words an investment plan might qualify as shareholder constrained efficient according to our first order conditions, even though there existed another plan, maybe very different, which would make all shareholders better off, after implementing a suitable compensation scheme. Our plan would therefore be shareholder constrained efficient only locally, and not “globally”. We should add, however, that on the one hand, “global” shareholder constrained efficiency will a fortiori be “local”; on the other hand, we might think that shareholders have “limited” computational abilities, which enable them to assess only the consequences of small departures from a given plan.

The right hand side of (7) represents the level of satisfaction reached by shareholder $i$ at the “status quo”. The left hand side represents the level of satisfaction he would get if plan $\bar{y}_2$ were implemented, and if he were in addition given the transfer $\tau^i$. These transfers can be effected using any good (or basket of goods), provided the amount of good(s) transferred be equivalent, in utility terms, to the loss or benefit received by the agent following the change in production (this makes the compensation scheme, it should be noticed, always implementable, regardless of shareholders’ initial endowments). However, we should add that these transfers are meant not to influence equilibrium prices, which is normally true if, for example, agents cannot re-trade after receiving or paying the compensations (say that these transfers are in “utils”, which cannot be traded).

If we allowed shareholders to trade after the compensations have been implemented and the production plan decided upon\(^1\), and if some trade actually occurred, then shareholders would reach a level of satisfaction at least as high as the one they get without re-trading (by a straightforward

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\(^1\) Otherwise, a possible change in equilibrium prices and net demands might bring about a change in the selected production plan.
argument of revealed preferences). Thus, if agents were allowed to retrade, smaller compensations should actually be paid to those suffering from the proposed change in production, and this would be likely to make the set of "shareholder constrained efficient" plans smaller.

The choice of our simpler, myopic "shareholder constrained efficiency" concept, could be justified on two grounds; on the one hand we might say that compensations are paid in the form of "fringe benefits" or other non tradable goods, which shareholders have to consume and cannot exchange. On the other hand the full-fledged concept of "shareholder constrained efficiency" (including retrading of compensation bonuses) would impose onto managers a heavier information burden; in fact, they should not only know equilibrium price functions and aggregate net demands, but they should also solve the utility maximization problems of all shareholders.

However, retrading would not constitute a problem if prices and net demands remained constant. The latter would approximately hold if the set of shareholders and the compensations were both small. In this sense the "shareholder constrained efficient" criterion might be used as an approximation even if retrading were allowed.

It should be quite apparent that this definition involves a notion of veto power (but one could maybe conceive of a "k-percent" shareholders' constrained efficiency definition), and that it is particularly suitable for a small group of shareholders (maybe a control group).

This notion of stability, which has received much attention in the incomplete markets literature, can also be profitably applied in our model. This is, in short, the content of the following proposition.

**Proposition 2** Under standard differentiability assumptions, a production plan $y^*_2$ is shareholder constrained efficient in the economy described above if and only if it satisfies the following equation:

$$f'(y^*_2) - p_2(y^*_2) = \frac{\partial p_2}{\partial y_2} |_{y_2=y^*_2} (z^{*S}_2 + y^*_2) + \sum_{l=3}^{L} \frac{\partial p_l}{\partial y_2} |_{y_2=y^*_2} z^{lS}_2$$

(with $z^{*S}_l = \sum_{i \in S} z^{i*}_{l}$, $z^{i*}_l$ the net demand of agent $i$ for good $l$) or, in a more compact form,

$$\nabla_{y_2} |_{y_2=y^*_2} p(y_2) z^{S*} = \nabla_{y_2} |_{y_2=y^*_2} \pi(y_2)$$

**Proof**: let us consider the decision problem from the perspective of shareholder $i$, as it was already discussed in the previous paragraph.

We may recall that, from (4), the marginal (indirect) utility of an increase in $y_2$, evaluated at the plan $y^*_2$, is:

$$\lambda_i \left[ \frac{\partial p_2}{\partial y_2} |_{y_2=y^*_2} (\omega_{2i} - \theta_{i} y^*_2 - x^{i*}_2) - \sum_{l=3}^{L} \frac{\partial p_l}{\partial y_2} |_{y_2=y^*_2} (\omega^{i}_{l} - x^{i*}_l) \right]$$

$$+ \theta^i (f'(y^*_2) - p_2(y^*_2))$$
where $\lambda^i$ is the marginal utility of income of shareholder $i$, corresponding to optimal net consumption $(x^* - w^i)$. The derivation of (9) from expression (4) is quite straightforward, once we recall that the price of good 1 has been normalized to unity.

As it was suggested above, (9) will in general yield a value different from 0. Therefore, agent $i$ is willing to increase (if (9) > 0) or decrease (if (9) < 0) the production level with respect to $y^*_2$. Let for the moment assume that (9) is positive.

For any other shareholder $k$ in S, an expression similar to (9) is obtained. Dividing it by $\lambda^k$, agent $k$'s marginal utility of income, we obtain the amount of income transfer that agent $k$ should receive (if (9) is negative) or pay (if (9) is positive) to stay on the same indifference surface (utility equivalent income adjustment).

Summing all these minimal income transfers to the utility equivalent income gain of agent $i$, we obtain expression (8), which is zero at $y^*_2$. In other words, all the gains brought about by an infinitesimal change in $y^*_2$ are spent in compensating shareholders suffering a loss. This makes it impossible, therefore, to find a bona fide transfer scheme which satisfies Definition 1.

If the sum of all income transfers were negative, then one could consider an opposite motion (i.e. one with the opposite sign) and get a dominating plan. This explains the strict equality sign in expression (8), for an interior solution.

Remark 1: criterion (8) is a combination of shareholders' objectives in that it weighs derivatives of the equilibrium price functions with the sum of shareholders' net demands.

Remark 2: it is evident from (8) that if all agents in the economy own shares in the firm (which is, for example, the case of a public monopoly) and the economy is closed, then the shareholder constrained efficient solution coincides with the competitive solution (this can be seen by considering (3) and market clearing conditions).

Remark 3: if the owners of a firm have to decide over many strategic variables (an input vector, for example), as will be the case in the example of section 4, then a condition like (8) has to hold for each of them, separately.

Remark 4: by the same arguments used in proposition 1, one can show that maximization of profits:

$$\pi(y_2) = \hat{p}_1(y_2)f(y_2) - p_2(y_2)y_2,$$

where

$$\hat{p} = \left( \frac{p_1}{\sum_{i=1}^L p_i(y_2)z^*_i}, \ldots, \frac{p_L}{\sum_{i=1}^L p_i(y_2)z^*_i} \right), \quad z^*_i = \sum_i z^*_it$$

leads to a shareholder constrained efficient solution.

As an additional characterization of the “shareholder constrained efficient” rule we can introduce the following:
Proposition 3: The "shareholder constrained efficient" solution is a necessary condition of the following shareholder welfare maximization program:

$$\max \sum_{i \in S} u^i(w^i + z^i)$$

subject to:

$$p(y_2)z^i = \theta^i(p_1(y_2)f(y_2) - p_2(y_2)y_2) + \tau^i, \quad \forall i \in S$$

$$\sum_{i \in S} \tau^i = 0$$

($\tau$ being a vector of transfers of numeraire, which can only be consumed and not transferred thereafter).

Proof: See Appendix.

Proposition 3 shows that applying the shareholder constrained efficient rule is tantamount to maximizing the sum of shareholders' utilities, provided nominal transfers (or numeraire transfers) are allowed. Transfers of revenue will equalize marginal utilities of income across shareholders. In practice, this means that the s.c.e. criterion can be considered as a criterion of "Pareto efficiency with ex post transfers", limitedly to shareholders.

A limit case of proposition 3, in which all $\tau^i$ would be set to zero, is when all shareholders have preferences representable by a quasi-linear utility function with a common numeraire. In this case, of course, marginal utilities of income across shareholders would coincide.

A similar point has also been made by Dierker and Grodal (1994), who show how their "shareholders' real wealth maximization" criterion can be recovered via maximization of all shareholders' social surplus, including shareholders' surplus as consumers (consumer surplus), and as capitalists (producer surplus or profit). This is shown for the special case in which shareholders' preferences are represented by quasi linear utility functions.

A shareholder constrained efficient equilibrium can be defined as follows.

Definition 2: A shareholder constrained efficient equilibrium of the economy outlined above will be a triple $(y_2^*, p^*, z^*)$, such that:

i) $z^{i*}$ solves problem (2) at prices $p^*$ for all $i$;

ii) $y_2^*$ is a solution of (4) at $z^*$;

iii) markets clear.

Conditions (i) states that the $z^{i*}$'s are the agents' best choice at $y_2^*$ and at prices $p^* = p(y_2^*)$, both for shareholders and consumers; condition (ii) means that $y_2^*$ is the input choice that maximizes the owner's utility, taking the price function $p(y_2)$ into account.
3 An example

In this section we are going to apply the shareholder constrained efficient criterion to the example of an economy involving a competitive sector and an oligopolistic sector, represented by two identical strategic firms deciding both on their optimal production and on the quota of their production to take to the market. This example is closely inspired by the work of Gabszewicz-Michel (1994), whose notation and set-up we will follow.

The economy features two goods and five agents. The first good is owned by the first agent, who behaves competitively. The second good is produced out of the first one, by two firms owning the same linear technology. Each firm is owned, in equal amounts, by two shareholders, with different preferences. The behaviour of the firms is not competitive, in that they strategically decide about their production level and about the share of their output to take to the market. In the second stage of the game all agents behave competitively taking prices as parameters, and markets clear at walrasian prices. Firms anticipate competitive prices to set their strategic variables. Moreover, shareholders are inactive on the market for the good they produce in the second stage.

Formally, preferences of the five agents are represented by the following utility functions:

\[ u^1(x_1, x_2) = 14 + \log x_1 + \log x_2 \]
\[ u^2(x_1, x_2) = u^4(x_1, x_2) = 14 + \log x_1 + 2 \log x_2 \]
\[ u^3(x_1, x_2) = u^5(x_1, x_2) = 14 + 2 \log x_1 + \log x_2 \]

and the endowment of the first, competitive agent is \( u^1 = (1, 0) \), whereas the other four agents are only endowed with shares of the two firms\(^2\).

The two firms produce good 2 out of good 1 according to the technology:

\[ y_j = \frac{1}{\alpha_j} z_j, \quad \alpha_j > 0 \]

\( y_j \) being the output of firm \( j \) and \( z_j \) the input level. Agents 2 and 4 each own half of firm 1, and agents 3 and 5 own each half of firm 2. In other words, \( \theta^2 = \theta^4 = \theta^3 = \theta^5 = 0.5 \).

Out of total output \( y_j \) firm \( j \) decides to take to the market a quantity \( q_j \). The quantity left, \( y_j - q_j \), is used for self-consumption. We assume that this quantity is shared between shareholders in proportion to their respective ownership quotas (i.e. equally, in our example).

The ownership quota, \( \theta \), entitles every shareholder to receive the corresponding share of total profits, computed at market prices \( p_1, p_2 \):

\[ \pi(q_j, y_j) = p_2 q_j - p_1 z_j = p_2 q_j - p_1 \alpha_j y_j \]

\(^2\) To avoid additional difficulties, we assume that shareholders own shares of one firm only.
and of total self-consumption \( y_j - q_j \).

An equilibrium for this economy will be a vector \((y_j^*, q_j^*, x_i^*, x_h^*, x_{1*}, p^*)\) such that:

1. \((y_j^*, q_j^*)\) maximize shareholders' utility (in a sense to be made more precise below) given the choice of the shareholders of the other firm and given the equilibrium price \( p^* = p(q_j^*, q_{-j}^*) \), which represents the competitive market clearing price in the second stage of the game;

2. \(x_i^*, x_h^*, x_{1*}\) denote, respectively, the competitive demands of shareholders \(i\) and \(h\) and of the competitive agent \(1\), at prices \( p^* = p(q_j^*, q_{-j}^*) \), in the second stage of the game.

As individuals decide on self-consumption of good 2 in the first stage of the game, \(x_2 = 0\) in shareholders' budget constraints for the competitive stage of the game. Therefore the budget constraints of the shareholders are:

\[
p_1 x_i^* = \theta(p_2 q_j - p_1 \alpha_j y_j) \implies x_i^* = \theta \left( \frac{p_2}{p_1} q_j - \alpha_j y_j \right), \quad \forall i = 2, 4
\]

\[
p_1 x_h^* = (1 - \theta)(p_2 q_j - p_1 \alpha_j y_j) \implies x_h^* = \theta^h \left( \frac{p_2}{p_1} q_j - \alpha_j y_j \right), \quad \forall h = 3, 5
\]

Solving the optimization problem for the competitive individual, we obtain as optimal demands of good 1 and 2:

\[
x^1(p_1, p_2) = \left( \frac{1}{2}, \frac{p_1}{2p_2} \right)
\]

Since the competitive individual is the only one who expresses a competitive demand of good 2, the equilibrium condition on the market for this good turns out to be:

\[
\frac{p_2}{p_1} = \frac{1}{2(q_1 + q_2)}
\]

which we can use to compute the indirect utilities:

\[
v^i = 3 + \log \left[ \theta \left( \frac{q_j}{2(q_j + q_{-j})} - \alpha_j y_j \right) \right] + 2 \log[\theta(y_j - q_j)], \quad i = 2, 4; j = 1, 2
\]

\[
v^h = 3 + 2 \log \left[ (1 - \theta) \left( \frac{q_j}{2(q_j + q_{-j})} - \alpha_j y_j \right) \right] + \log[(1 - \theta)(y_j - q_j)],
\]

\[
h = 3, 5; j = 1, 2
\]

If we suppose that firms are run by only one of the two agents, firms' decisions will depend on which agent we choose. If it is the first shareholder who decides we obtain, at the symmetric equilibrium (setting \( \alpha_j = \alpha_{-j} = 2 \)):

\[
y_1 = y_2 = \frac{5}{48}; \quad q_1 = q_2 = \frac{1}{16}; \quad y_1 - q_1 = y_2 - q_2 = \frac{1}{24}
\]
On the other hand, if we considered the second shareholder as the one who decides we would get:

\[ y_1 = y_2 = \frac{1}{12}; \quad q_1 = q_2 = \frac{1}{16}; \quad y_1 - q_1 = y_2 - q_2 = \frac{1}{48} \]

As we may immediately check from equilibrium configurations, the strategic behaviour proposed by the first shareholder differs from the one chosen by the second shareholder, with respect to the share of product to take to the market.

One possibility to come out of the decisional impasse, then, is to use the criterion of "shareholder constrained efficiency" illustrated in the previous sections. Here oligopolists decide strategically on both the quantity to produce and the share of product to take to the market, leaving the rest for self consumption. In such a context, therefore, strategic choices directly influence shareholders' utilities, via self-consumption.

Following the argument we developed in section 3, a couple \((y^*_j, q^*_j)\) has to satisfy the following necessary condition, to qualify as an interior shareholder constrained efficient strategy:

\[
\left( \frac{\partial v^i}{\partial y^*_j} / \lambda^i + \frac{\partial v^h}{\partial y^*_j} / \lambda^h, \frac{\partial v^i}{\partial q^*_j} / \lambda^i + \frac{\partial v^h}{\partial q^*_j} / \lambda^h \right) dt = 0, \text{ for } i = 2, 4; \quad h = 3, 5
\]

(10)

where

\[
\lambda^i = \frac{1}{\theta \left( \frac{p_2}{p_1} q_j - \alpha_j y_j \right)}
\]

\[
\lambda^h = \frac{2}{(1 - \theta) \left( \frac{p_2}{p_1} q_j - \alpha_j y_j \right)}
\]

represent the marginal utilities of good 1 (or marginal utilities of income, if good 1 is taken to be a numeraire good) for the two types of agents. Dividing marginal increases in indirect utility by these coefficients permits to express them in a common unit of measure, i.e. utility equivalent quantities of good 1. Then, if transfers had to be implemented, they would be in terms of good 1. Of course, the same arguments would apply if the other good was chosen as a benchmark.

In expression (10) \(dt\) is a feasible direction of movement ("motion") in the strategy space, starting from the proposed allocation \((y^*_j, q^*_j)\).

Since, at equilibrium, \(0 < q_j < y_j\), let us consider the feasible "motions"

\[ dt = (0, \varepsilon), \varepsilon \in V(0) \text{ and } dt = (\varepsilon, 0) ; \]

In fact, for a plan to qualify as shareholder constrained efficient it is necessary and sufficient that (10) be verified with respect to \(dt = (0, \varepsilon)\) and
\[ dt = (\varepsilon, 0), \text{ which implies that } (10) \text{ can also be stated as:} \]

\[ \frac{\partial v^i_j}{\partial y_j} \lambda^i + \frac{\partial v^h_j}{\partial y_j} \lambda^h = \frac{\partial v^i_j}{\partial q_j} \lambda^i + \frac{\partial v^h_j}{\partial q_j} \lambda^h = 0 \] \quad (11)

Computing expressions (11) for our example (with \( \alpha = 2 \)), and looking for a symmetric equilibrium, we obtain:

\[ y_j = \frac{2 + 3\theta}{24(1 + \theta)^2}; \quad q_j = \frac{1}{16}; \quad y_j - q_j = \frac{1 + 6\theta}{48(1 + \theta)} \]

In the following table we summarize the utility levels attained by agents in this economy in the three kinds of equilibria, when \( \theta^i = \theta^h = 0.5 \):

<table>
<thead>
<tr>
<th></th>
<th>1st equil.</th>
<th>2nd equil.</th>
<th>s.c.e. equil.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_j )</td>
<td>0.10</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>( q_j )</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>( v^i )</td>
<td>11.22</td>
<td>11.22</td>
<td>11.22</td>
</tr>
<tr>
<td>( v^h )</td>
<td>2.38</td>
<td>1.69</td>
<td>2.30</td>
</tr>
<tr>
<td>( v^i^h )</td>
<td>2.38</td>
<td>3.07</td>
<td>2.77</td>
</tr>
</tbody>
</table>

(12)

We should immediately notice two things: 1) the shareholder constrained efficient plan lies in between the investment plans that would be chosen, independently, by the two shareholders, and this is also reflected in the respective welfare levels; 2) the utility of the competitive agent is identical in the two cases; this last finding, however, is specific to the example, and depends on the fact that the quantity taken to the market is the same in all cases.

4 Concluding remarks

The main purpose of this work has been that of illustrating some aspects of corporative firms' decision problem in an imperfectly competitive set-up. The stress was put on utility maximization as a more natural criterion to be pursued by managers of imperfectly competitive firms, rather than profit maximization.

It is also interesting to notice a similarity between the frameworks of imperfect competition and market incompleteness, which we can sum up as follows: a marginal change with respect to a proposed production plan, \( y^* \), is evaluated by any shareholder \( i \) as indicated in the following two expressions, respectively for the case of market incompleteness and imperfect
\[ v_{i.m.} = (-\theta^i, \theta^i \frac{\lambda_1}{\lambda_0}, \ldots, \theta^i \frac{\lambda_S}{\lambda_0}) dy \]  

(13)

\[ v_{i.c.} = (\theta^i, -z^i_1, \ldots, -z^i_L) \frac{\partial}{\partial y} (\pi, p) \]  

(14)

where for simplicity we omitted the asterisks on all the relevant variables.

As we may notice, in both cases the evaluation is made up of two components: a subjective one, consisting of ownership quotas and state prices in the case of market incompleteness, and of ownership quotas and (optimal) net demands in that of imperfect competition. The second component, that we might define an "activity" vector, is represented by the marginal change from the proposed plan in the case of incomplete markets, and by the gradient of profit and equilibrium price functions in the case of imperfect competition.

In both cases (market incompleteness and imperfect competition) marginal changes in production decisions are evaluated differently by the shareholders, which brings about a problem of unanimity.

When markets are incomplete, this is so because individuals' sensitivity vis à vis income at the various date-event pairs, as measured by the marginal utility of income (the Lagrangian multipliers associated to the various budget constraints) generally differ across shareholders.

When markets are not competitive, on the other hand, that happens because net demands differ across shareholders, which causes production induced income changes to be generally different.

For the case of market incompleteness, Drèze (1974) proposed to aggregate the "subjective" vectors and use the resulting convex combination to evaluate \( dy \). We propose to do the same for the case of strategic firms.

We could exploit the structural similarity between the two set-ups, however, to transpose to the set-up of imperfect competition some other concepts of equilibrium, such as that of "Majority Rule Production Equilibrium" or that of "Designated Manager Production Equilibrium", illustrated by DeMarzo (1993) for the case of market incompleteness. Obviously, the possibility of extending these concepts to the case of imperfect competition does not entail anything about the existence of an equilibrium in those economies; existence would still crucially depend on the characteristics of equilibrium price correspondences (or aggregate demand correspondences, as the case may be). Moreover, as clearly mentioned by Dierker-Godal ((1995) and (1996)), an additional source of non-convexity appears in our case: dependence of prices on strategic variables will normally cause the relevant budget set(s) to be non convex; even in the case of a single owner, therefore, the optimal response mapping could be multi-valued, and equilibrium might fail to exist.
5 Appendix

Proof of proposition 1: Let us consider the following problem of profit maximization:

\[
\max_{y_2} \pi = \hat{p}_1(y_2) f(y_2) - \hat{p}_2(y_2) y_2
\]

with

\[
\hat{p}_1 = \frac{p_1}{\sum_{l=1}^{L} p_l(y_2) z_{l}^{i*}}; \hat{p}_2 = \frac{p_2}{\sum_{l=1}^{L} p_l(y_2) z_{l}^{i*}}
\]

where agent i is the only shareholder of the firm. The first order, necessary condition of this problem with respect to the strategic variable \(y_2\) is:

\[
\left( \frac{\partial p_1}{\partial y_2} f(y_2) + f'(y_2) p_1(y_2) \right) \sum_{l=1}^{L} p_l(y_2) z_{l}^{i*} - \sum_{l=1}^{L} \frac{\partial p_1}{\partial y_2} z_{l}^{i*} p_1 f(y_2) - \left( \sum_{l=1}^{L} p_l(y_2) z_{l}^{i*} \right)^2 \left( \sum_{l=1}^{L} p_l(y_2) z_{l}^{i*} \right)^2
\]

\[
- \left( \frac{\partial p_2}{\partial y_2} y_2 + p_2 \right) \left( \sum_{l=1}^{L} p_l(y_2) z_{l}^{i*} - \sum_{l=1}^{L} \frac{\partial p_1}{\partial y_2} z_{l}^{i*} (p_2 y_2) \right)
\]

\[
= 0
\]

By simplifying expression (15) we obtain:

\[
p_1(y_2) f'(y_2) - p_2(y_2) = \frac{\partial p_1}{\partial y_2} (z_{i}^{i*} - f(y_2)) + \frac{\partial p_2}{\partial y_2} (z_{i}^{i*} + y_2) + \sum_{l=1}^{L} \frac{\partial p_l}{\partial y_2} z_{l}^{i*}
\]

(16)

where we have used the fact that \(\sum_{l=1}^{L} p_l(y_2) z_{l}^{i*} \neq 0\) or, which is equivalent, given agent i’s budget constraint, that profits are non zero.

The conclusion follows from the fact that (16) is equal to condition (4).

Proof of proposition 3: let us consider the program:

\[
\max \sum \limits_{i} u^i (w^i + z^i)
\]

s.t. \( p(y_2) z^i = \theta^i (p_1(y_2) f(y_2) - p_2(y_2) y_2) + \tau^i \)

\[
\sum \limits_{i} \tau^i = 0
\]

whose first order conditions with respect to \(z^i, \tau^i\) and \(y_2\) are respectively:

\[
\nabla_{z^i} u^i (w^i + z^i) = \lambda^i p
\]

(17)
\[ \lambda^i = \lambda^k, \forall i \neq k \]  
(18)

\[ \sum_i \left( \lambda^i \left( \frac{\partial p}{\partial y_2} z^i - \theta^i (f'(y_2) - p_2(y_2) + y_2) \right) \right) = 0 \]  
(19)

To derive condition (18) we have substituted from the condition

\[ \sum_i \tau^i = 0 \]

in the budget constraint of shareholder k, and taken derivatives with respect to \( \tau^i, \forall i \neq k \).

Therefore, we can substitute \( \lambda^i = \lambda^k = \lambda \) in expression (19) and factor it out, which yields, summing across shareholders:

\[ f'(y_2^*) - p_2(y_2^*) = \frac{\partial p_2}{\partial y_2} \bigg|_{y_2=y_2^*} (z_{2i}^* + y_2^*) + \sum_{i=3}^{L} \frac{\partial p_i}{\partial y_2} \bigg|_{y_2=y_2^*} z_i^* \]  
(20)

In view of strict positivity of \( \lambda \) and of the comparison of (20) with (4) the conclusion follows.
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