Multi-product cost functions: An application to the production characteristics of secondary education in Flanders

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1 Introduction

The theory on economies of scale was originally developed for and applied to single-product outputs in industry. Later on the concepts of ray average cost, multi-product scale economies and economies of scope were introduced to relax the restrictive assumption of producing a single output with a single input. The quadratic cost function and the translog cost function are among the most commonly used functional forms in this multi-product context. Originating from the theory of firms producing multiple-product outputs, this method can also be used to derive returns to scale and scope for non-profit organisations such as schools. In research on education, this multi-product framework is merely applied to analyse scale and scope effects in institutions of higher education, especially (research) universities.

Most of the research on the production characteristics of secondary education however, use single-output production functions (e.g. Watt [1980], Bee and Dolton [1985], Dougherty [1990]). The drawback of this approach is that it can give insight in the degree of (overall) economies of scale only. Until now only a few studies apply the more advanced multi-product framework to analyse whether the production by secondary schools is characterised by economies of scale and scope or by diseconomies. Jimenez [1986] uses 1975 data on 43 primary and secondary Bolivian schools and 41 primary Paraguayan schools to estimate

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a generalised translog multi-product cost function. Callan and Santerre [1990] use a sample containing pooled data over a four school year period (1980-1984) including all of the 165 school districts in Connecticut to estimate a translog cost function. The findings of both studies are inconclusive and do not support each other.

The purpose of this study is to quantify the degree of (ray and product specific) (dis)economies of scale and scope in secondary education in Flanders. Therefore multi-product cost functions need to be estimated. In this paper we use data on the entire population of Flemish secondary schools.

The paper is structured as follows. Section 2 covers the theoretical concepts needed to measure the extent of economies of scale and scope while section 3 describes the data used in this study. In section 4 the econometric model is specified and its consequences for the calculation of the degree of economies of scale and scope are discussed. Empirical results are discussed in section 5. Some concluding comments are presented in section 6.

2 Theoretical framework

Following Baumol, Panzar and Willig [1988], the degree of ray economies of scale at output vector \( y \) in a multi-product environment is defined as:

\[
S_R(y) = \frac{\sum_{i=1}^{n} y_i \frac{\partial C(y)}{\partial y_i}}{\sum_{i=1}^{n} y_i} = \frac{1}{\sum_{i=1}^{n} \varepsilon C_{y_i}}
\]

where \( C(y) \) is the (total) cost, \( y_i \) (\( i = 1, \ldots, n \)) are the individual components of vector \( y \) and \( \varepsilon C_{y_i} \) are the individual cost elasticities of each output. Ray economies of scale measures overall economies of scale and assumes that the composition of the output bundle remains fixed while the size of the composite output bundle can vary, i.e. every output has to be expanded with the same proportion. The production is characterised by constant returns to scale if \( S_R(y) \) equals unity. Ray economies or diseconomies of scale are said to exist if \( S_R(y) \) is greater than or less than one, respectively. \( S_R(y) \) can be interpreted as "the elasticity of the output of the relevant composite commodity with respect to the cost needed to produce it" (Baumol, Panzar and Willig [1988], p. 51).

To determine whether the production is characterised by product specific scale economies or not, the incremental cost for each output has to be calculated. The incremental cost of product \( i \) is equal to:

\[
IC_i(y) = C(y) - C(y_{N-i})
\]
with $y = y_i + y_{N-i}$. $IC_i(y)$ is thus the increase in total costs if the amount $y_i$ of product $i$ is added to output bundle $y_{N-i}$. The average incremental cost of product $i$ is then defined as $AIC_i(y) = IC_i(y)/y_i$.

The degree of scale economies to product $i$ at vector $y$ can be measured by:

$$S_i(y) = \frac{IC_i(y)}{y_i \partial C_i(y) / \partial y_i} = \frac{AIC_i(y)}{MC_i(y)} \quad (3)$$

Product specific returns to scale are said to be constant, increasing or decreasing if $S_i(y)$ equals, is greater than or smaller than one. The concept of product specific economies of scale allows the composition of the output bundle to change. It measures how costs react if the quantity of one product varies, while holding the quantity of all other outputs constant.

Finally, the degree of economies of scope between distinct product sets can be determined. Suppose $S(2 \leq S \leq n)$ is the number of product sets and let $Y_p$ be any combination of elements from vector $y$ (except vector $y$ itself). For all $p \neq q$, no component $y_i$ of $Y_p$ is an element of $Y_q$, otherwise not only scope effects would be taken into account but also product specific economies of scale.

The degree of economies of scope between two or more product sets can be calculated as follows:

$$SCS(y) = \sum_{p=1}^{S} \frac{C(Y_p) - C(Y_1, \ldots, Y_S)}{C(Y_1, \ldots, Y_S)}$$

(4)

where $C(Y_1, \ldots, Y_S)$ is the cost of producing the $S$ sets jointly.

Economies or diseconomies of scope between the $S$ product sets are said to exist if $SCS(y)$ is greater or smaller than zero, respectively. The degree of economies of scope measures the difference in costs in terms of percentage between a system where the different products (or product sets) are produced separately and a system where all products (or product sets) are produced jointly by one firm. If the production is characterised by economies of scope, it is cheaper to produce the different product sets jointly rather than to split up the production.

In case one wants to examine whether there exist economies of scope between two outputs (not product sets), it can be shown (Baumol, Panzar and Willig [1988], p. 89) that weak cost complementarities between two products are a sufficient condition for economies of scope. These inter-product cost complementarities are measured by the change in marginal cost of one product as a result of a change in the output quantity of another, jointly produced, product.
Weak cost complementarities are said to exist if:

$$\frac{\partial^2 C(\tilde{y})}{\partial y_i \partial y_j} \leq 0$$

with $i \neq j$ and for all $\tilde{y}(0 \leq \tilde{y} \leq y)$.

3 Data

The data used in this study are obtained from the Ministry of the Flemish Community, more specific the Department of Education. The cross-sectional data (school year 1994-1995) include the entire population of secondary schools in Flanders ($n = 1011$). From an organisational point of view, three major types of schools can be identified: (Flemish) government organised schools (ARGO) ($n = 276$), privately (mainly catholic) organised schools (VGO) ($n = 633$), and schools organised by local authorities or provinces (OGO) ($n = 102$). The so-called School pact from 1959 contains the legal obligation for the government to ensure that every student has the "free choice" between a confessional (VGO or OGO) and a non-confessional (ARGO) school within a "reasonable distance" (the average reasonable distance for secondary education was determined at 16 kilometres) from his or her home. This legal obligation implies that a parallel network of schools has to be maintained and has severe implications on the current mechanism to finance the schools. The number of teachers that schools are allowed to employ (and therefore the subvention they receive) depends heavily on the number of students. This system is strongly regressive however. The non-confessional (ARGO) network which has to ensure the free choice is obliged to supply a minimum range of study fields in its schools, even if there are (almost) no students interested. To compensate for this obligation, ARGO schools can benefit from a more generous subvention mechanism than VGO and OGO schools.

Only schools supplying a full six-year study program ($n=773$) were included in the analysis, this means that 238 schools (mainly schools organising only the first and the second year) were excluded. In the data bank 43 study fields can be distinguished. For each school the total cost (in Belgian Francs) as well as the number of students in each of these fields is reported. These 43 fields can be aggregated to three main clusters: general education (GEN), technical education (TEC), and vocational education (VOC).

The number of students in each of these three clusters will be used as output variable in the estimation of the cost function. Although the student load is not a perfect measure of output (it fails for example to capture the quality of the teaching output), it is especially in empirical
estimations of educational cost functions often the only proxy commonly available for every single school. Other possible proxies for output such as success rates, achievement test scores, moral values, pedagogical efficiency, etc. are often not (widely) available or difficult to measure. Moreover, using the number of students as output variable is very useful regarding the aim of this paper: examining the relationship between cost and scale (and scope) of activity.

Table 1: Descriptive statistics

<table>
<thead>
<tr>
<th>Main study field</th>
<th>Type of school(*)</th>
<th>ARGO</th>
<th>VGO</th>
<th>OGO</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>General education (GEN)</td>
<td>( n )</td>
<td>127</td>
<td>329</td>
<td>21</td>
<td>477</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>167</td>
<td>445</td>
<td>180</td>
<td>360</td>
</tr>
<tr>
<td></td>
<td>std.</td>
<td>131</td>
<td>276</td>
<td>145</td>
<td>272</td>
</tr>
<tr>
<td>Technical education (TEC)</td>
<td>( n )</td>
<td>127</td>
<td>328</td>
<td>70</td>
<td>525</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>94</td>
<td>296</td>
<td>197</td>
<td>234</td>
</tr>
<tr>
<td></td>
<td>std.</td>
<td>69</td>
<td>205</td>
<td>132</td>
<td>192</td>
</tr>
<tr>
<td>Vocational education (VOC)</td>
<td>( n )</td>
<td>133</td>
<td>326</td>
<td>70</td>
<td>529</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>151</td>
<td>188</td>
<td>195</td>
<td>180</td>
</tr>
<tr>
<td></td>
<td>std.</td>
<td>112</td>
<td>143</td>
<td>120</td>
<td>134</td>
</tr>
<tr>
<td>TOTAL</td>
<td>( n )</td>
<td>170</td>
<td>530</td>
<td>73</td>
<td>773</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>311</td>
<td>567</td>
<td>424</td>
<td>497</td>
</tr>
<tr>
<td></td>
<td>std.</td>
<td>172</td>
<td>286</td>
<td>221</td>
<td>281</td>
</tr>
</tbody>
</table>

(*) ARGO: (Flemish) government organised; VGO: privately organised; OGO: organised by local authorities or provinces.

Table 1 reports, for the three different types of schools (ARGO, VGO and OGO) as well as for entire population of schools, a number of descriptive statistics. For each aggregated study field, \( n \) indicates the number of schools that organises that type of education. The mean and the standard deviation of the number of students are calculated on these schools only. So only study fields with non-zero student loads are included to obtain the mean and the standard deviation. The last three lines of this table report the total number of schools for each organisational type as well as the average number of students and the standard deviation(1).

(1) These averages and standard deviations are based on the total number of students in each school, so they cannot be compared directly with the means and standard deviations of the different study fields which did not take into account zero output levels.
Because the wages of teaching staff are centrally (i.e. by law) determined, there is little variation across schools. These wages account for 88% of the total secondary education expenditures. Furthermore, Flanders is a relatively small region with limited price variation for other inputs. Consequently it does not make any sense to include input prices in the cost equation. Therefore, by not including input prices, the function we estimate, is in fact a “pseudo-cost function”. This is not only a problem for schools, it is indeed a common problem for non-profit organisations and public sector institutions. Is assumed that this cost function, written in terms of output, results from the implicit minimisation of the total costs, subject to the production function constraint.

Another assumption we make is the absence of rigidities in costs at the level of individual schools (i.e. we suppose that costs can adjust to any changes in outputs). The current mechanism to finance the schools allows to adjust costs relatively easily to changes in outputs. If the number of students is reduced, the surplus of teachers receive an equivalent of their wage from a special (central) fund. This system implies that these wages are removed from the subvention for that particular school.

4 Model specification

The specific aim of this paper is to determine the extent of ray and product specific (dis)economies of scale and (dis)economies of scope in the Flemish secondary school system. In this section two of the most commonly used multi-product cost functions were estimated. From the estimated parameters of a quadratic and a generalised (or hybrid) translog (Box-Cox) cost function the degree of economies of scale and scope as well as the marginal cost functions and cost elasticities of the study fields can be derived. The estimation procedure for the quadratic cost function was OLS, the generalised translog was estimated using the iterative Marquardt technique.

4.1 Quadratic cost function

A multi-product quadratic cost function of the following form is used\(^{(2)}\):

\[ C(y) = \alpha_0 \frac{1}{y} + \sum_{i=1}^{n} \beta_i \frac{y_i}{y} + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \frac{y_i y_j}{y} \]

with \( y = \sum_{i=1}^{n} y_i \) the total number of students.

\(^{(2)}\) To avoid heteroscedasticity this function is actually estimated using the average cost function:
\[ C(y) = \alpha_0 + \sum_{i=1}^{n} \beta_i y_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} y_i y_j \]  

(6)

where \( y \) is the vector of the composite output bundle (i.e. the study fields supplied by each school) consisting of \( y_i \) (\( i = 1, \ldots, n \)) students and with the symmetry constraint \( \beta_{ij} = \beta_{ji} \) applying.

In the case of the quadratic cost function specified above, the marginal cost equation for product \( i \) is obtained by:

\[ MC_i(y) = \frac{\partial C(y)}{\partial y_i} = \beta_i + \sum_{j=1}^{n} \beta_{ij} y_j . \]  

(7)

Individual cost elasticities can be derived using the following formula:

\[ \varepsilon_{Cyi} = \frac{MC_i y_i}{C(y)} . \]  

(8)

The degree of ray economies of scale \( S_R(y) \) can be calculated as:

\[ S_R(y) = \frac{C(y)}{\sum_{i=1}^{n} y_i MC_i(y)} = \frac{C(y)}{\sum_{i=1}^{n} \beta_i y_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} y_i y_j} = \frac{1}{\sum_{i=1}^{n} \varepsilon_{Cyi}} . \]  

(9)

The degree of product specific economies can be computed as follows:

\[ S_i(y) = \frac{IC_i(y)}{y_i MC_i(y)} = \frac{IC_i(y)}{\beta_i y_i + \sum_{j=1}^{n} \beta_{ij} y_i y_j} . \]  

(10)

The derivation of the degree of economies of scope is straightforward as it follows from definition (4). The alternative condition for the existence of weak cost complementarities between two products can be written as:

\[ \frac{\partial^2 C(y)}{\partial y_i \partial y_j} = \frac{\partial}{\partial y_i} [MC_j(y)] = \beta_{ij} \leq 0 . \]  

(11)
4.2 Generalised translog cost function

A multi-product cost function often used in empirical research is the translog cost function. However, because quite a lot of schools have zero output levels for some of the study fields (see Table 1), one should be careful to avoid errors when taking logarithms during the estimation of an ordinary translog function. A solution for this problem is to make the translog cost function more flexible by modifying the exogenous variables using a Box-Cox transformation (see for example Caves, Christensen and Trethewey [1980]):

$$y_i^{(\lambda)} = \frac{y_i^\lambda - 1}{\lambda}$$

and by L'Hôpital's rule:

$$\lim_{\lambda \to 0} \frac{y_i^\lambda - 1}{\lambda} = \ln y_i .$$

The so-called hybrid or generalised translog function can then be written as:

$$\ln C(y) = \alpha_0 + \sum_{i=1}^{n} \beta_i \left( \frac{y_i^\lambda - 1}{\lambda} \right) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \left( \frac{y_i^\lambda - 1}{\lambda} \right) \left( \frac{y_j^\lambda - 1}{\lambda} \right)$$

where $y$ is the vector of the composite output bundle (i.e. the study fields supplied by each school) consisting of $y_i$ ($i = 1, \ldots, n$) and with the symmetry constraint $\beta_{ij} = \beta_{ji}$ applying. For sufficiently small values of $\lambda$ ($\lambda$ close to zero) the hybrid translog function approximates the ordinary translog function. If, on the other hand, $\lambda$ is close to one, the function tends to be a quadratic cost function.

Deriving $\ln C(y)$ with respect to $y_i$ yields the following expression:

$$\frac{\partial \ln C(y)}{\partial y_i} = \beta_i y_i^{\lambda - 1} + \sum_{j=1}^{n} \beta_{ij} y_i^{\lambda - 1} \left( \frac{y_j^\lambda - 1}{\lambda} \right) .$$

To calculate individual cost elasticities the following formula is used:

$$\frac{\partial \ln C(y)}{\partial y_i} = \frac{\partial \ln C(y)}{\partial \ln y_i} \times \frac{\partial \ln y_i}{\partial y_i} = \frac{\varepsilon_{C y_i}}{y_i} .$$

Consequently the cost elasticity of the $i$th output can be written as:

$$\varepsilon_{C y_i} = y_i \frac{\partial \ln C(y)}{\partial y_i} = \beta_i y_i^{\lambda} + \sum_{j=1}^{n} \beta_{ij} y_i^{\lambda} \left( \frac{y_j^\lambda - 1}{\lambda} \right) .$$
The marginal cost equations can be found using:

\[ MC_i(y) = C(y) \frac{\partial \ln C(y)}{\partial y_i} . \]  \hspace{1cm} (18)

The degree of ray economies of scale can be calculated as follows:

\[ S_R(y) = \frac{C(y)}{\sum_{i=1}^{n} y_i MC_i(y)} = \frac{1}{\sum_{i=1}^{n} \epsilon C y_i} \]

\[ = \frac{1}{\sum_{i=1}^{n} \beta_i y_i^\lambda + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} y_i^\lambda \left( \frac{y_j^\lambda - 1}{\lambda} \right)} . \]  \hspace{1cm} (19)

The degree of product specific economies of scale can be written as:

\[ S_i(y) = \frac{IC_i(y)}{y_i MC_i(y)} = \frac{IC_i(y)}{\beta_i y_i^\lambda + \sum_{j=1}^{n} \beta_{ij} y_i^\lambda \left( \frac{y_j^\lambda - 1}{\lambda} \right)} . \]  \hspace{1cm} (20)

A sufficient condition for economies of scope between output \( i \) and output \( j \) is that:

\[ \frac{\partial^2 C(y)}{\partial y_i \partial y_j} = \frac{\partial}{\partial y_i} \left[ \frac{C(y)}{y_j} \frac{\partial \ln C(y)}{\partial y_j} \right] \]

\[ = \frac{C(y)}{y_i} \frac{\partial \ln C(y)}{\partial y_i} \times \frac{\partial \ln C(y)}{\partial y_j} + C(y) \frac{\partial^2 \ln C(y)}{\partial y_i \partial y_j} \]

\[ = C(y) \left[ \frac{\partial \ln C(y)}{\partial y_i} \times \frac{\partial \ln C(y)}{\partial y_j} + \frac{\partial^2 \ln C(y)}{\partial y_i \partial y_j} \right] \]

\[ = \frac{C(y)}{y_i y_j} \left[ \epsilon C_{y_i} \epsilon C_{y_j} + y_i y_j \frac{\partial^2 \ln C(y)}{\partial y_i \partial y_j} \right] \leq 0 . \]  \hspace{1cm} (21)

Dropping \( C(y)/y_i y_j \) will preserve the sign of this expression, so weak cost complementarities between \( i \) and \( j \) are said to exist if:

\[ \left[ \beta_i y_i^\lambda + \sum_{j=1}^{n} \beta_{ij} y_i^\lambda \left( \frac{y_j^\lambda - 1}{\lambda} \right) \right] \left[ \beta_j y_j^\lambda + \sum_{i=1}^{n} \beta_{ij} y_j^\lambda \left( \frac{y_i^\lambda - 1}{\lambda} \right) \right] \]

\[ + \beta_{ij} y_i^\lambda y_j^\lambda \leq 0 . \]  \hspace{1cm} (22)
5 Empirical results

Since ARGO schools can benefit from a more generous subvention mechanism than VGO and OGO schools (see Section 3), a dummy variable (on the intercept) for ARGO schools was included to control for these differences. The results of the estimation procedure show that this dummy is highly significant\(^\text{(3)}\).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Quadratic cost function</th>
<th>Generalised translog cost function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Std. Error</td>
</tr>
<tr>
<td>Box-Cox parameter ((\lambda))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>19438882**</td>
<td>1672985</td>
</tr>
<tr>
<td>Dummy ARGO</td>
<td>21457504***</td>
<td>1334579</td>
</tr>
<tr>
<td>Dummy OGO</td>
<td>2504941</td>
<td>1829016</td>
</tr>
<tr>
<td>Linear terms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN</td>
<td>112420***</td>
<td>10043</td>
</tr>
<tr>
<td>TEC</td>
<td>188283***</td>
<td>15452</td>
</tr>
<tr>
<td>VOC</td>
<td>162920***</td>
<td>19315</td>
</tr>
<tr>
<td>Quadratic terms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN \times GEN</td>
<td>23.052*</td>
<td>13.379</td>
</tr>
<tr>
<td>TEC \times TEC</td>
<td>-82.356**</td>
<td>37.002</td>
</tr>
<tr>
<td>VOC \times VOC</td>
<td>178.508***</td>
<td>58.168</td>
</tr>
<tr>
<td>Cross product terms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN \times TEC</td>
<td>-8.154</td>
<td>69.265</td>
</tr>
<tr>
<td>GEN \times VOC</td>
<td>363.397***</td>
<td>103.695</td>
</tr>
<tr>
<td>TEC \times VOC</td>
<td>72.743</td>
<td>69.050</td>
</tr>
</tbody>
</table>

\(^\text{3)}\) A dummy for OGO schools was also added. Its sign is also positive but less pronounced (compared with the ARGO dummy) however.

Notes: *** denotes significant at 1% level, ** denotes significant at 5% level, * denotes significant at 10% level.
cross product terms that is significantly different from zero. The generalised translog function has an R-squared of 90% and all the estimated parameters are significant at the 1% level or better. The resulting value of the Box-Cox parameter \( \lambda \) is 0.27 and is also estimated very precisely. The coefficients obtained in these estimations (together with the output levels for the different study fields) are now used to derive the degree of economies of scale and scope. First cost elasticities, marginal costs and incremental costs need to be calculated however.

Table 3 shows (for the different types of schools) the cost elasticity as well as the marginal cost at the mean output level of all three study fields. The individual cost elasticities reveal the increase in total costs in terms of percentage if one output (i.e. the number of students) is expanded by 1%. The cost elasticity of technical education is, in both cost functions and in the different types of schools, smaller than the elasticity of general and vocational education.

<table>
<thead>
<tr>
<th>Main study field</th>
<th>Quadratic cost function</th>
<th>Generalised translog cost function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ARGQ</td>
<td>VGO</td>
</tr>
<tr>
<td>Cost elasticities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN</td>
<td>0.251</td>
<td>0.459</td>
</tr>
<tr>
<td>TEC</td>
<td>0.148</td>
<td>0.230</td>
</tr>
<tr>
<td>VOC</td>
<td>0.370</td>
<td>0.403</td>
</tr>
<tr>
<td>( \sum_{i=1}^{n} \varepsilon_{Cy_i} )</td>
<td>0.769</td>
<td>1.091</td>
</tr>
</tbody>
</table>

Marginal costs

|                  |       |       |       |       |       |       |       |       |
| GEN              | 174226 | 198841 | 190436 | 192521 | 231334 | 184949 | 187358 | 191576 |
| TEC              | 182423 | 149576 | 168471 | 159899 | 194972 | 110348 | 123932 | 122629 |
| VOC              | 284355 | 413282 | 315914 | 375028 | 284691 | 324303 | 240796 | 311838 |

The sum of the individual elasticities indicates the percentage increase in total costs if all outputs are equally expanded by 1%. This total cost elasticity shows that (ray) economies of scale could be realised in ARGQ schools and to a smaller extent in OGO schools too. VGO schools are operating with constant returns to scale. Comparing marginal costs, Table 3 shows that an additional student in VOC costs more than double than an additional student in TEC.

Table 4 reports the average incremental cost of all outputs at their mean. These incremental costs are necessary to calculate the degree of
product specific economies of scale. Vocational education has the highest average incremental cost.

Table 4: Average incremental costs at output means

<table>
<thead>
<tr>
<th>Study field</th>
<th>Quadratic cost function</th>
<th>Generalised translog cost function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ARGO</td>
<td>VGO</td>
</tr>
<tr>
<td>GEN</td>
<td>170376</td>
<td>188583</td>
</tr>
<tr>
<td>TEC</td>
<td>190164</td>
<td>173953</td>
</tr>
<tr>
<td>VOC</td>
<td>257400</td>
<td>379723</td>
</tr>
</tbody>
</table>

The degree of ray economies of scale is defined as the inverse of the sum of the cost elasticities of the n outputs and is, for various output levels, presented in Table 5. Using the parameters obtained from the estimation of the quadratic cost function, only ARGO schools could realise considerable economies of scale by increasing their actual size. Even at an output level of 150% the actual size, economies of scale would not yet be completely exhausted. VGO schools would operate at constant returns to scale at an output level of 75% their actual size, while OGO schools are currently operating at constant returns to scale.

Table 5: Ray economies of scale

<table>
<thead>
<tr>
<th>Scale</th>
<th>Quadratic cost function</th>
<th>Generalised translog cost function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ARGO</td>
<td>VGO</td>
</tr>
<tr>
<td>50%</td>
<td>2.00</td>
<td>1.12</td>
</tr>
<tr>
<td>75%</td>
<td>1.54</td>
<td>0.99</td>
</tr>
<tr>
<td>100%</td>
<td>1.30</td>
<td>0.92</td>
</tr>
<tr>
<td>125%</td>
<td>1.16</td>
<td>0.87</td>
</tr>
<tr>
<td>150%</td>
<td>1.06</td>
<td>0.83</td>
</tr>
<tr>
<td>200%</td>
<td>0.93</td>
<td>0.77</td>
</tr>
<tr>
<td>300%</td>
<td>0.80</td>
<td>0.71</td>
</tr>
</tbody>
</table>

The generalised translog function suggests that both ARGO and OGO could benefit from increasing their size. At the current output level, VGO schools are characterised by (nearly) constant returns to scale. These results indicate that, from a cost effectiveness point of view, especially ARGO schools are too small and that increasing the average size of these schools would result in considerable cost savings.

Global economies of scope compare the costs of splitting up the production of n outputs in n firms, each producing a single output, with
the costs of producing these n outputs jointly in one unit\(^{(4)}\). More precisely global economies of scope indicate the difference in costs in terms of percentage between a system where each school specialises and offers only one study field and a system where each school supports the whole range of study fields. The degree of global economies of scope is, for various output levels, reported in Table 6. The values for the global economies of scope (quadratic cost function) show that creating specialised schools would, at the current output level, lead to a substantial increase in total costs for all types of schools. Based on the generalised translog function, the increase in costs ranges from 40\% (VGO) to more than 100\% (ARGO).

**Table 6: Global economies of scope**

<table>
<thead>
<tr>
<th>Scale</th>
<th>Quadratic cost function</th>
<th>Generalised translog cost function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ARGO</td>
<td>VGO</td>
</tr>
<tr>
<td>50%</td>
<td>3.24</td>
<td>1.12</td>
</tr>
<tr>
<td>75%</td>
<td>2.53</td>
<td>0.69</td>
</tr>
<tr>
<td>100%</td>
<td>2.03</td>
<td>0.43</td>
</tr>
<tr>
<td>125%</td>
<td>1.65</td>
<td>0.26</td>
</tr>
<tr>
<td>150%</td>
<td>1.36</td>
<td>1.14</td>
</tr>
<tr>
<td>200%</td>
<td>0.94</td>
<td>-0.04</td>
</tr>
<tr>
<td>300%</td>
<td>0.44</td>
<td>-0.24</td>
</tr>
</tbody>
</table>

Product specific economies of scale are calculated using the cost elasticities and the incremental costs reported in Table 3 and Table 4. Table 7 shows the product specific economies of scale at the mean output level. Based on the quadratic cost function, economies of scale could be realised for technical education only. All other study fields are characterised by diseconomies. Product specific economies of scale obtained from the generalised translog cost function indicate a large degree of economies of scale for technical education. General and vocational education show product specific economies too, however less pronounced.

Pairwise scope effects at the output means are reported in Table 8. All possible combinations yield economies of scope (except for general

\[^{(4)}\] The exact formula to calculate the degree of global economies of scope is:

\[
SC_G(y) = \frac{\sum_{i=1}^{n} C(y_i) - C(y)}{C(y)}.
\]
combined with vocational education in VGO schools [quadratic cost function]. Cost savings by supplying two study fields jointly range from five percent to more than fifty percent, depending on the cost function.

### Table 8: Pairwise economies of scope at output means

<table>
<thead>
<tr>
<th>Main study fields</th>
<th>Quadratic cost function</th>
<th>Generalised translog cost function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ARGO</td>
<td>VGO</td>
</tr>
<tr>
<td>GEN × TEC</td>
<td>0.53</td>
<td>0.17</td>
</tr>
<tr>
<td>GEN × VOC</td>
<td>0.32</td>
<td>-0.08</td>
</tr>
<tr>
<td>TEC × VOC</td>
<td>0.46</td>
<td>0.14</td>
</tr>
</tbody>
</table>

### 6 Concluding comments

In this paper we used two distinct multi-product cost functions to estimate the degree of ray and product specific economies of scale as well as the extent of economies of scope in Flemish secondary schools. Using a quadratic cost function, only ARGO schools could realise considerable ray economies of scale. VGO schools are characterised by diseconomies of scale at the current output level, while OGO schools operate at constant returns to scale. The degree of global economies of scope reveals that splitting up the production of the schools would raise costs dramatically. At the current output level, only technical education is characterised by product specific economies of scale. Based on the generalised translog cost function the production of Flemish ARGO and OGO schools is characterised by a considerable degree of ray economies of scale, indicating that cost savings could be made by increasing the average size of these schools. The results for VGO schools on the other hand reveal constant returns to scale. Calculation of global economies of scope shows that a system with specialised schools, each producing a single output, would cost 40 (VGO) to more than 100 percent (ARGO) more than a system where every school supplies all three study fields.
Having a closer look at the individual study fields, evidence is found for
the existence of product specific economies of scale for all three study
fields. It is more pronounced in technical education however. Pairwise
scope effects range from about fifteen to twenty percent.

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