Quality choice and specialization in North-South trade

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1 Introduction

The role of product differentiation has received wide attention in the literature on trade. The interplay between preferences, technology, and the pattern of production and trade under imperfect competition has been focused upon by the New Trade Theory stemming from the seminal contributions of Krugman [1979] and Helpman [1981], mainly concerned with horizontal product differentiation. Likely differences in quality across the set of traded goods were not considered as the producers' response to differences in per capita income and thus in the demand expressed by each country.

Vertical differentiation has been frequently associated to trade in manufactured goods, since the pioneering book of Linder [1961], where trade specialization is affected by per capita income and consumer preferences. Linder's book provided a theoretical basis for the effect of both vertical differentiation and macroeconomic growth upon trade specialization. According to the endogenous growth literature, trade has an enhancing repercussion on development, which in turn modifies specialization, as recent contributions have shown (see, mainly and more generally, Grossman and Helpman [1991]).

Both casual observation and statistical evidence suggest that vertical differentiation can be found also in trade among countries with different per capita income and is often associated to Intra-Industry Trade (IIT). This is the case of the EU and other rich areas vis à vis NIC's (newly industrializing countries) and NEC's (newly exporting countries).

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The econometric explanation provided in the literature appears dichotomous. On the one hand we have Vertical IIT (VIIT), i.e. IIT in vertically differentiated goods, which seems due to differences in factor endowments, leading poor countries to specialize in low-quality goods (Greenaway, Hine and Milner [1994, 1995], Torstensson [1996]). On the other hand, some industry studies reveal a more complex picture, where Horizontal IIT (HIIT), i.e. IIT in horizontally differentiated goods, takes place also between rich and poor countries (Tharakan, Kerstens and Gleijser [1994], Tharakan and Kerstens [1995]).

On the theoretical side, VIIT has been modelled according to two alternative views: a supply-side approach, where vertical differentiation can be traced back to divergences either in relative factor endowments or technological efficiency across countries (Falvey [1981], Falvey and Kierzkowski [1987], Flam and Helpman [1987]); and a demand-side approach, where vertical differentiation is due to heterogeneous consumer preferences arising from different per capita income across the population of consumers.

In the latter vein, Krishna [1987, 1990] has tried to endogenize the choice of product quality by a monopolist on the trace of Spence [1975] model. Within a framework of oligopolistic strategic interaction Shaked and Sutton [1982, 1983], have provided new insights into the choice of product quality under Bertrand competition in a closed economy. Shaked and Sutton [1984], and Motta [1992] extended this analysis to international trade, in order to investigate the effect of quality choice on firm profitability and the spectrum of goods being offered. Yet not many instances exist of analysis of trade in vertically differentiated goods between rich and poor countries.

Following these contributions we attempt to analyse the twin problem of price competition and the short run impact on welfare, by analysing the surplus distribution of the opening of trade between two countries which differ only for the level of per capita income. In this sense, our paper is an attempt to interpret some aspects of North-South relations. We consider two single product monopolists operating in their domestic markets and choosing quality in autarky as a long run commitment. Subsequently, the separation between the two markets vanishes, giving rise to an international duopoly and firms compete in prices. The distinction between autarky and free trade reflects a feature of firms' life cycle. At the outset, firms are mostly affected by their respective domestic market, since a slight degree of natural protection can always be thought to exist, making the domestic market handler to serve for a newly established firm. As a result, after trade liberalization, firms adjust only the market variable.
Trade associated with this vertically differentiated duopoly is one-way with no IIT. While in the framework presented here there is no overlap between the income distribution of the two partner countries, elsewhere a similar model with overlapping income distributions gives rise to IIT (Lambertini, 1997). In this paper, close in spirit to the traditional Heckscher-Ohlin-Samuelson approach, we show that a poor country can largely benefit from trade with a rich country, if there is vertical differentiation. Most of the gain is enjoyed by the firm of the poor country and by the consumers of the rich country. However, there are circumstances (some configurations of the relative per capita income of the two countries) in which also the consumers of the poor country may be better off. A net looser is the firm of the rich country.

Real wages in both countries increase owing to liberalization. Even though our results are cast in an environment in which there is only one factor of production, we find that the opening of trade brings about an increase in the real wage which is higher in the rich country than in the poor. This provides a partial reply to some of the worries of rich countries trading with poorer partners (see Wood [1994], Krugman [1995]).

Finally, we explore the effect of an import reducing tariff, set by the government of the rich country facing a trade deficit. Although such a tariff doesn’t generally increase welfare, there are some circumstances in which it can benefit both countries. Although we derive our results from a duopoly setting, we may expect many of our conclusions to hold also in a more general oligopoly framework with a larger number of firms, as long as firms enjoy a non-negligible amount of market power.

2 The autarky model

We consider two monopolies operating in two separated countries, each producing a vertically differentiated good. Each monopolist operates in its domestic market and faces consumers heterogeneous in terms of their incomes. Preferences and technology are described following Mussa and Rosen's [1978] formulation. To model income distribution, we assume consumers uniformly distributed according to \( f(\theta) \) defined over \([\underline{\theta}, \bar{\theta}]\), \( \bar{\theta} \geq 0, \bar{\theta} - \underline{\theta} \geq \bar{\theta}/3 \). Parameter \( \theta \) indicates the marginal willingness to pay for quality, which grows as income increases. It can be interpreted as the reciprocal of the marginal utility of income. The population of consumers in each country is normalised, for the sake of simplicity, to \( \bar{\theta} - \underline{\theta} = 1 \). Every consumer has unit demand, i.e., either he buys one unit of the differentiated good or nothing.\(^{(1)}\)

\(^{(1)}\)The condition \( \bar{\theta} - \underline{\theta} \geq \bar{\theta}/3 \) comes from the optimization program of the monopolist in autarky. The consequence of the optimal monopolist behaviour is
A consumer's surplus function determines the choice between buying and not buying. He buys if the following condition is met:

\[ U = \theta q - p \geq 0, \quad (1) \]

where \( p \) is the price of the good, while \( q \) is the quality.

For each price-quality pair set by the producer, we are able to derive the demand function\(^{(2)}\):

\[ x^M = \frac{\theta - p}{q}. \quad (2) \]

On the production side we assume that costs are convex in quality and linear in quantity:

\[ C = wL = q^2 x. \quad (3) \]

The above cost function implies that \( w = q^2 x / L \), which means that the nominal wage depends on the productivity of labour w.r.t. both quantity (linearly) and quality (non-linearly). If we assume that the average productivity of labour w.r.t. quantity is constant, and, for the sake of simplicity, equal to one, we obtain \( w = \theta^2 \). Therefore, firm's quality choice has an immediate feedback on the structure of wages, while technology does not play any role.

The monopolist maximizes profit with respect to quality and price. Quality is deemed a long run commitment and each firm chooses it at the

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that part of the population is not served, due to the restricted supply of goods. In this case, the monopoly's inefficiency applies only to quantity and not to quality (Spence [1975], pp. 419-21). Since every consumer has unit demand, the normalised population has to be greater than or equal to \( \theta / 3 \).

\( ^{(2)} \)Since consumers are uniformly distributed \([ \theta, \bar{\theta} ]\) the value of \( \theta \) associated with the last consumer who buys is \( p/q \) (from condition (1) met as an equality). Therefore, demand is given by the difference between the \( \theta \) of the richest consumer \( (\bar{\theta}) \) and the price-quality ratio offered by the monopolist \( (p/q) \), multiplied by the cumulative uniform distribution

\[ \int_{\theta}^{\bar{\theta}} f(\theta) d\theta = 1. \]

This implies that the maximum value \( \bar{\theta} \) can reach is 3, since the difference \( \bar{\theta} - \theta \) cannot be lower than \( \theta / 3 \) and population has been normalised to 1. On this last assumption, see Mussa and Rosen [1978] and Cremer and Thisse [1994], inter alia.

\( ^{(3)} \)The assumption of convex costs is the only one giving rise to meaningful solutions when dealing with an oligopolistic market, that we shall consider in the next section. For the analysis of the relationship between product quality and the curvature of the cost function, see Lambertini [1993].
beginning on the domestic market, without any possibility of reversing it after the opening of trade. The objective function is:

$$\pi^M = \left(\bar{\sigma} - \frac{p}{q}\right)(p - q^2)$$  \hspace{1cm} (4)$$

where superscript $M$ stands for monopoly. We can therefore derive two first order conditions:

$$\frac{\partial \pi^M}{\partial p} = \bar{\sigma} - \frac{2p}{q} + q = 0$$  \hspace{1cm} (5)$$

$$\frac{\partial \pi^m}{\partial q} = p + \frac{p^2}{q^2} - 2\bar{\sigma}q = 0.$$  \hspace{1cm} (6)$$

from which we get the following solutions:

$$p^{M^*} = \frac{2}{9} \bar{\sigma}^2$$  \hspace{1cm} (7)$$

$$q^{M^*} = \frac{\bar{\sigma}}{3}.$$  \hspace{1cm} (8)$$

As a result, the money wage amounts to $w^{M^*} = \bar{\sigma}^2/9$, and consequently the real wage is $w^{M^*}/p^{M^*} = 1/2$. This is the quantity of a good contingent on a particular choice of quality. If we compare real wages across countries, we must weigh such quantities by the respective quality levels.

It can be shown that the monopolist behaviour, as far as the quality choice is concerned, is efficient, since market demand is linear, as shown by Spence ([1975], pp. 419-21). Then, maximum profit and optimal quantity will be:

$$\pi^{M^*} = \frac{\bar{\sigma}^3}{27}$$  \hspace{1cm} (9)$$

$$x^{M^*} = \frac{\bar{\sigma}}{3}.$$  \hspace{1cm} (10)$$

\footnote{This amounts to assuming that there is a sunk cost independent of quality and linked to the acquisition of technology. Since this does not affect first order conditions it can be assumed to be nil.}
We now calculate, using (1) and (3), the social welfare of the monopoly setting, as the sum of consumer's and producer's surplus\(^{(5)}\):

\[
SW^M = \int_{(2/3)\overline{\theta}}^{\overline{\theta}} (\theta q - q^2) \, d\theta
\]  

(11)

that yields, in equilibrium:

\[
SW^M = \frac{3}{54} \overline{\theta}^3
\]  

(12)

while consumer's surplus amounts to

\[
CS^M = \frac{1}{54} \overline{\theta}^3.
\]  

(13)

3 The duopoly model

The monopolist choice considered in the previous section is the result of a decision taken in conditions of autarky\(^{(6)}\), since we assume that quality choice is an \textit{infant industry} decision. Each firm is considered to be confined only to the domestic market at the very beginning of her life cycle and produces according to the tastes of domestic consumers. When she becomes older, she competes with the foreign firm, but she is constrained not to change her quality strategically. Since this would require bearing a further sunk cost, she resorts to price competition. This amounts to analysing the impact effects of trade in the short-run \(^{(7)}\).

Now we investigate what happens when trade opens, considering two twin countries, whose only difference is the level of per capita income. Neither natural nor administrative barriers separate them. A

\(^{(5)}\) The lower limit of the integral is given by

\[
\overline{\theta} - x^M = \frac{2}{3} \overline{\theta}.
\]

The value of \(SW^M\) over \([\overline{\theta}, (2/3) \overline{\theta}]\) is nil, since no consumer buys in this interval, because below \((2/3) \overline{\theta}\) the price-quality ratio is higher than consumer's marginal willingness to pay.

\(^{(6)}\) The same assumption is made by Motta [1994] and appears to be plausible in a long run perspective, because changing quality usually requires the adoption of a different technique, with the associated sunk cost.

\(^{(7)}\) For a discussion of short-run vs long-run effects of trade liberalization, see Shaked and Sutton [1984] and Motta [1992].
new market setting emerges: a Bertrand duopoly made up by the two former monopolists.

We assume that country \( D \) (domestic) is richer than country \( F \) (foreign). As a consequence, country \( D \) specializes in the production of a good of higher quality than the one produced in country \( F \). Consumers have heterogeneous marginal willingness to pay for quality and the richest consumer of the poorer country has the same marginal willingness to pay of the poorest consumer of the richer country.\(^8\) This assumption makes this model suitable to unravel trade between rich and poor countries, i.e., North-South. By normalising each population to one, we exclude any dimension effect and concentrate on relationships driven only by per capita income differences.

Then we can define both the locus of indifference between the two goods and the threshold beyond which the consumers in the two countries buy the low quality good, and below which they do not buy the good, since net surplus is negative.

In figure 1 we describe the distribution of consumers after the opening of trade.

\[
\begin{array}{cccc}
\tilde{\Theta}_F & k & \tilde{\Theta}_F = \Theta_D & h & \tilde{\Theta}_D \\
\end{array}
\]

\textbf{Figure 1:} Distribution of consumers after the opening of trade

Parameters \( h \) and \( k \) define the location in the price-quality space \( (\theta) \) of two indifferent consumers: in \( k \), we have a consumer who is indifferent between either buying the low quality good or nothing, while \( h \) locates the consumer indifferent between the two goods. The value of \( k \) is derived according to the following indifference condition:

\[
\theta_F q_F - p_F = 0. \tag{14}
\]

Solving (14), we get:

\[
\theta_F = \frac{p_F}{q_F} = k. \tag{15}
\]

The value of \( h \) is given by the solution of another indifference condition \(\(^9\)\):

\[
\theta q_F - p_F = \theta q_D - p_D \tag{16}
\]

\(^8\) This assumption is needed to avoid discontinuous demand functions, and implies that the supports of \( f_D(\theta) \) and \( f_F(\theta) \) are contiguous, i.e., \( \tilde{\Theta}_D = \tilde{\Theta}_F \).

\(^9\) Unlike the case of \( k \), we haven't any prior knowledge of the country where the
leading to:
\[ \theta = \frac{3(p_D - p_F)}{\bar{\theta}_D - \bar{\theta}_F} = 3(p_D - p_F) = h, \] (17)

since \( \bar{\theta}_D - \bar{\theta}_F = 1 \) We are now able to derive the demand function for each duopolist:
\[ x_D = \bar{\theta}_D - 3(p_D - p_F), \] (18)
\[ x_F = 3(p_D - p_F) - 3\frac{p_F}{\bar{\theta}_F}. \] (19)

Each duopolist maximizes profit, using his price as a control variable. The solution concept is a Bertrand-Nash equilibrium. The profit functions are, respectively:
\[ \pi_D = (\bar{\theta}_D - 3(p_D - p_F))(p_D - q_D^2) \] (20)
\[ \pi_F = \left(3(p_D - p_F) - 3\frac{p_F}{\bar{\theta}_F}\right)(p_F - q_F^2). \] (21)

The first order conditions are:
\[ \frac{\partial \pi_D}{\partial p_D} = \frac{18\bar{\theta}_D p_D - 9\bar{\theta}_F p_D - \bar{\theta}_D \bar{\theta}_F^2}{3\bar{\theta}_F} = 0 \] (22)
\[ \frac{\partial \pi_F}{\partial p_F} = \frac{4\bar{\theta}_D^2 + 9p_F - 18p_D - 3\bar{\theta}_D \bar{\theta}_F}{3} = 0. \] (23)

Solving the system (22-23) with respect to \( p_D \) and \( p_F \) we obtain the following equilibrium prices:
\[ p_D^* = \frac{\bar{\theta}_D (2\bar{\theta}_D - \bar{\theta}_F)}{9} \] (24)
\[ p_F^* = \frac{\bar{\theta}_D \bar{\theta}_F}{9}. \] (25)

Observe that \( p_i^{M*}/p_i^* > 1, i = D, F \) for \( \bar{\theta}_D < 2\bar{\theta}_F \). In order to calculate the real wage in the rich country, we have to deflate the money wage by the weighted sum of \( p_D^* \) and \( p_F^* \), where the weights are given, respectively, by \( x_D \) and \( h - \bar{\theta}_F \), i.e. country \( D \)'s imports from country \( F \). The same weights are needed to construct the average quality level purchased by a representative consumer living in country \( D \), i.e.:
\[ q_{D}^{avg} = q_D x_D + q_F (h - \bar{\theta}_F). \]

consumer identified by \( h \) is located. Therefore, we do not use any subscript for \( \theta \) in (16).
The real wage of the representative individual in country $D$ under free trade is larger than the real wage observed in autarky for all admissible values of $\overline{\theta}_D$. The difference between real wages in the two countries after liberalization is positive in the relevant range.

An immediate implication is that the real wage increases in both countries after the opening of trade, but in different proportion. Real wages in the two countries, after trade liberalization, are $\overline{\theta}_D/\overline{\theta}_D$ in country $F$ and $3\overline{\theta}_D/(5\overline{\theta}_D - 3)$ in country $D$. As a result, we can state the following proposition.

**Proposition 1** Since money wages are unchanged w.r.t. autarky, the opening of trade implies that real wage increases in both countries, but proportionally more in the rich country.

Then, substituting the equilibrium prices (24-25) into the original profit functions (20-21), we obtain the equilibrium profits:

\[
\pi^*_D = \frac{\overline{\theta}_D^2}{27} \tag{26}
\]

\[
\pi^*_F = \frac{\overline{\theta}_D \overline{\theta}_F}{27} \tag{27}
\]

It follows that:

\[
\frac{\pi^*_D}{\pi^*_F} = \frac{\overline{\theta}_D}{\overline{\theta}_F} > 1 \tag{28}
\]

while in the former autarky equilibrium, the ratio between profits of the two monopolists was:

\[
\frac{\pi^{M^*_D}}{\pi^{M^*_F}} = \left(\frac{\overline{\theta}_D}{\overline{\theta}_F}\right)^3 \tag{29}
\]

In the duopoly setting, we have a new ratio between profits of the two firms operating, respectively, in the rich and the poor market. The opening of trade has an immediate effect on the two firms that can be summarized in the following proposition.

**Proposition 2** The opening of trade under vertical differentiation (i) always reduces the profit of the firm operating in the rich country, while (ii) it increases the profit of the firm in the poor country if per capita income in the latter is sufficiently low, i.e.,

\[
\overline{\theta}_F < \frac{1 + \sqrt{5}}{2}.
\]
Proof: A straightforward comparison between (26-27) and (9) is sufficient to establish this result. If we wish to push forward the analysis to the case of overlapping income distributions between the two countries, we can reach some conclusions which are partially at odds with the content of the above Proposition. As for point (i), we can expect this result to hold as long as the overlapping is not so wide to give rise to IIT. In the latter circumstance, the high-quality firm serves rich consumers in the foreign country offsetting the negative effect on profits due to duopolistic competition. Part (ii) of Proposition 2 will not hold within the same interval of parameters. It may be expected to hold in the case of overlapping per capita income distributions with one-way trade. In that case the low-quality firm enters the richer market and is better off as compared to autarky.

Opening of trade between countries with far apart per capita income may have a beneficial effect on the firm in the poor country, even though it behaves as a Bertrand duopolist. If the per capita income of the poor country is not far lower than that of the rich, also the firm operating in the former may loose. The closer countries are in terms of per capita income, the less the firm in the poor country benefits from trade. The threshold for $\bar{\theta}_F$ and the consequent dual effect of the opening of trade, is due to two opposite effects: a negative effect on the profit of the firm in the poor country due to increased competition, and a positive one due to a higher consumer surplus coming from the rich country.

The firm in the rich market faces both an absolute and a relative decrease of its profit vis-à-vis the firm in the poor market. With overlapping income distributions the high-quality firm may find not profitable to set such a low price. Then IIT appears, unless the two countries are very similar. Therefore producers of the poor country may favour trade liberalization, while the opposite happens for producers in the rich country.

To have a complete view we have to evaluate the effects of the opening of trade on consumer surplus and then on social welfare. To this purpose, we first compute the consumer surplus in the poor country:

$$\begin{align*}
CS_F &= \int_{\underline{\theta}_F}^{k} (\theta q_F - p_F) d\theta + \int_{k}^{\bar{\theta}_F} (\theta q_F - p_F) d\theta.
\end{align*}$$

The first part of the right-hand side of (30) is equal to zero, because it covers the non-buying area of the poor country. Taking into account (15), equation (30) becomes:

$$\begin{align*}
CS_F &= \int_{\bar{\theta}_F/3}^{\bar{\theta}_F} (\theta q_F - p_F) d\theta,
\end{align*}$$

(31)
which yields
\[ CS_F = \frac{4}{9} \bar{\theta}_F^2 - \frac{2}{3} \bar{\theta}_F p_F. \] (32)

Substituting into (32) the equilibrium values of \( q_F \) and \( p_F \) we get
\[ CS_F = \frac{2}{27} \bar{\theta}_F^2 (2\bar{\theta}_F - \bar{\theta}_D), \] (33)

which is the consumer surplus of the poor country.\(^{(10)}\) Adding (33) to (27), we get the social welfare of the poor country:
\[
SW_F = \frac{2}{27} \bar{\theta}_F^2 (2\bar{\theta}_F - \bar{\theta}_D) + \frac{\bar{\theta}_D}{27} \bar{\theta}_F (\bar{\theta}_D - \bar{\theta}_F) \\
= \frac{\bar{\theta}_D}{27} \left( 4\bar{\theta}_F^2 + \bar{\theta}_D^2 - 3\bar{\theta}_F \bar{\theta}_D \right). \] (34)

While the consumer surplus of the rich country is:
\[
CS_D = \int_{\bar{\theta}_D}^{\theta} (\theta q_F - p_F) d\theta + \int_{\theta}^{\bar{\theta}_D} (q\theta_D - p_D) d\theta, \] (35)

which yields
\[ CS_D = \frac{\bar{\theta}_D^3 - 9\bar{\theta}_F^3 + 2\bar{\theta}_D^2 \bar{\theta}_F + 6\bar{\theta}_D^2 \bar{\theta}_D}{54}. \] (36)

Adding together (26) and (36), we get the social welfare of the rich country:
\[ SW_D = \frac{3\bar{\theta}_D^3 + 6\bar{\theta}_D \bar{\theta}_F^2 - 9\bar{\theta}_F^3}{54}. \] (37)

We have to establish a sort of viability condition, defining the lower bound of the ratio between \( \bar{\theta}_D \) and \( \bar{\theta}_F \), below which a disequilibrium in the rich country (excess supply) appears. This condition is the extension of the parallel condition stated in section 2 and explained in footnote 1:
\[ \frac{\bar{\theta}_D}{\bar{\theta}_F} \geq \frac{3}{2}. \] (38)

Then, if we compare the social welfare levels of country \( D \) and \( F \) there is no value of \( \bar{\theta}_D/\bar{\theta}_F \) belonging to the viable region, such that the welfare of the rich country is lower than the welfare of the poor country. However, in order to address the question of any commercial policy, we

\(^{(10)}\) Since \( CS_F \) has to be non-negative, \( \bar{\theta}_D \leq 2\bar{\theta}_F \).
have to evaluate the change in welfare induced by the opening of trade in both countries. To this purpose, we analyse trade flows.

4 The structure and direction of trade flows

We calculate the equilibrium values of \( k \) and \( h \), i.e., the two discriminant points in the price-quality spectrum of the duopoly market demand. As we already know, \( k \) defines the price-quality ratio corresponding to the borderline consumer who buys the low quality good. We see in which country this consumer is located:

\[
k = \frac{\bar{\theta}_D}{3}
\]

while the consumer who is indifferent between the two goods lies in

\[
h = 2 \frac{\bar{\theta}_D}{3}.
\]

Therefore, we can derive the distribution of demand between the two countries and the two goods. Together with condition (38), condition (40) states that the high quality good is not traded, since it is demanded only by the rich country, where also the low quality good is consumed. This happens if the indifferent consumer lies in the rich country, i.e.:

\[
h > \bar{\theta}_F.
\]

The intuition behind (41) is that, if the consumer indexed by \( h \) lies within \([\bar{\theta}_F, \bar{\theta}_D]\)\(^{(11)}\), the consumers who cannot afford the high quality good buy the imported low quality good. From (40) we notice that the demand for the high quality good is the same observed under autarky. However, under condition (41), opening of trade allows the less wealthy consumers in the rich country to buy the imported good, giving rise to a trade flow. Hence, the poor country exports to the rich country, while the latter does not export anything to the poor country. The kind of trade we obtain is a one way trade in vertically differentiated goods. This result might appear quite odd. It comes from the assumption that the spectrum of income distribution of one country does not overlap to any extent the spectrum of income distribution of the other. However, one-way trade is observed also with a limited overlapping of income distributions [Lambertini, 1997]. Under condition (41), the total demand of the rich

\(^{(11)}\text{We consider an interval open on the left, since we cannot have simultaneously trade and the coincidence of } h \text{ with } \bar{\theta}_F. \text{ Notice that (41) requires that (38) be satisfied as a strict inequality.}\)
country is:

\[ X_D = \frac{\bar{\theta}_D}{3} + h - \bar{\theta}_F = \bar{\theta}_D - \bar{\theta}_F = 1, \tag{42} \]

which means that under free trade every consumer of country \( D \) buys either the domestic or the imported good. Since the production of the high quality good \( y_D = \bar{\theta}_D/3 \) remains constant, (42) can be rewritten as:

\[ X_D = y_D + h - \bar{\theta}_F. \tag{43} \]

Hence, \( h - \bar{\theta}_F \) defines the demand for the imported good.

Let us turn to the poor country and define its total demand. If condition (41) is met, the latter is:

\[ X_F = \bar{\theta}_F - k = \bar{\theta}_F - \frac{\bar{\theta}_D}{3}. \tag{44} \]

In this case, exports amount to:

\[ y_F - X_F = \frac{2}{3} \bar{\theta}_D - \bar{\theta}_F, \tag{45} \]

which, according to condition (38), must be non-negative. If (45) is nil, trade disappears and the demand of the poor country becomes:

\[ X_F = h - k = \frac{\bar{\theta}_D}{3}. \tag{46} \]

After having analysed trade flows, we can derive some additional insights as to the benefits of the opening of trade. A textbook “pro-competitive” effect arises owing simply to opening of trade even in the absence of trade flows. Then, if we compare equilibrium demand under autarkic monopoly in the poor country, i.e. equation (10), with the corresponding demand under free trade, i.e. equation (44), we realize that the total number of consumers being served in the poor country increases if \( \bar{\theta}_D < 2 \bar{\theta}_F \), i.e., the marginal willingnesses to pay of the two countries are not too far apart.\(^{(12)}\) This effect is accompanied by a lower price which enables some consumers of the poor country to buy the low quality good, which was not affordable in autarky. At the same time the production of firm \( F \) rises \( \bar{\theta}_D/3 \).

\(^{(12)}\) Using (10) and (44), we obtain the following inequality:

\[ \bar{\theta}_F - \frac{\bar{\theta}_D}{3} - \frac{\bar{\theta}_F}{3} > 0, \]

which is satisfied for \( \bar{\theta}_F > \bar{\theta}_D/2 \).
Under condition (46), the richer market is being completely served only with the high quality good since the opening of trade does not give rise to any trade flow. Consumers in the rich country enjoy a decrease of the price of the domestic good as compared to autarky.\(^{(13)}\) We may then compare the terms of trade between the two countries with the relative price of goods produced in autarky, using (7), (24) and (25). The terms of trade between the rich and the poor country are lower than the relative price in autarky, implying that \(p_D\) has decreased more than \(p_F\).

We can summarize the findings concerning trade flows in the following proposition.

**Proposition 3** Trade liberalization increases demand in the rich country via a decrease in the price of the domestic good; in the poor country, the same effect occurs if \(\bar{\theta}_D < 2\bar{\theta}_F\).

What is stated in Proposition 3 can be extended *a fortiori* to the case of overlapping income distributions, since in such a setting price competition would be more intense, due to the higher substitutability between the two varieties.

5 The analysis of welfare distribution in countries D and F

We now compare the distribution of welfare in the two countries before and after the opening of trade by seeking for which values of the relevant parameters trade is welfare-improving. As for country \(D\) it appears that

\[ SW_D^T > SW_D^M \quad \text{iff} \quad \bar{\theta}_D > \frac{3}{2} \bar{\theta}_F, \tag{47} \]

where superscript \(T\) stands for trade. This condition holds since it corresponds to the viability condition already established in (38). For country \(F\):

\[ SW_F^T > SW_F^M \quad \text{iff} \quad \frac{\bar{\theta}_F}{27} \left( 4\bar{\theta}_F^2 + \bar{\theta}_D^2 - 3\bar{\theta}_D \bar{\theta}_F \right) - \frac{3}{54} \bar{\theta}_F^3 > 0, \tag{48} \]

which simplifies to:

\[ \bar{\theta}_F \left( 5\bar{\theta}_F^2 + 2\bar{\theta}_D^2 - 6\bar{\theta}_F \bar{\theta}_D \right) > 0. \tag{49} \]

This condition is met for all admissible values of \(\bar{\theta}_D, \bar{\theta}_F\). We can therefore state the following proposition.

\(^{(13)}\) To see this, compare (24) with (7).
Proposition 4 The opening of trade increases the social welfare in both countries in the admissible range of the relevant parameters.

We now turn to the analysis of the welfare distribution between consumers and producers, as a consequence of trade, by splitting Proposition 4 into three subpropositions.

As far as the consumer surplus in the rich country is concerned, we have that:

\[ CS_D^T > CS_D^M \iff 2 \bar{\theta}_D^2 \bar{\theta}_F + 6 \bar{\theta}_D \bar{\theta}_F^2 - 9 \bar{\theta}_F^3 > 0, \]  

(50)

which can be rewritten as:

\[ \bar{\theta}_F \left( 2 \bar{\theta}_D^2 + 6 \bar{\theta}_D \bar{\theta}_F - 9 \bar{\theta}_F^2 \right) > 0. \]  

(51)

This holds by virtue of the viability condition (38).

In the poor country,

\[ CS_F^T > CS_F^M \iff \frac{2}{27} \bar{\theta}_F^2 \left( 2 \bar{\theta}_F - \bar{\theta}_D \right) - \frac{\bar{\theta}_F^3}{54} > 0, \]  

(52)

which is met for

\[ \frac{2}{3} \bar{\theta}_D \geq \bar{\theta}_F > \frac{4}{7} \bar{\theta}_D. \]  

(53)

This implies that, if

\[ \bar{\theta}_F \in \left[ \frac{\bar{\theta}_D}{2}, \frac{4}{7} \bar{\theta}_D \right], \]

subset of the viable region, the consumer surplus of country \( F \) is lower after the opening of trade, because of a decrease in the number of consumers who buy the good. These results can be summarized in the following proposition.

Proposition 4.1 The opening of trade increases the consumer surplus in both countries if

\[ \bar{\theta}_F \in \left[ \frac{4}{7} \bar{\theta}_D, \frac{2}{3} \bar{\theta}_D \right]. \]

However, if

\[ \bar{\theta}_F \in \left[ \frac{\bar{\theta}_D}{2}, \frac{4}{7} \bar{\theta}_D \right], \]

the opening of trade increases the consumer surplus of the rich country while decreasing the consumer surplus of the poor country.
The above Proposition can be interpreted in terms of the marginal willingnesses to pay (proxies of per capita income) of the two countries: the higher is the marginal willingness to pay in the rich country vis-à-vis that of the poor, the lower is the benefit accruing to consumers in the poor country. If the income distributions overlap, Proposition 4(1) can be expected to hold over a wider range of parameters, since the degree of differentiation is lower and consequently price competition is fiercer. Moreover, the proportion of consumers in the poor country that are not served is going to shrink.

We now turn to producer surpluses. For the rich country we get:

\[ \pi^M_D > \pi^T_D \iff \bar{\theta}_D^3 - \bar{\theta}_D^2 > 0, \]  

which is always true, since the monopolist of the rich country looses when becoming a duopolist.

In the poor country, we notice that:

\[ \pi^M_F > \pi^T_F \iff \bar{\theta}_F^3 - \bar{\theta}_D \bar{\theta}_F > 0, \]  

or

\[ \bar{\theta}_F > \frac{\sqrt{5} - 1}{2} \bar{\theta}_D = 0.618 \bar{\theta}_D. \]

If \( \bar{\theta}_F \in [\bar{\theta}_D/2, 0.618 \bar{\theta}_D] \), the firm of country \( F \) is better off after trade liberalization, since it exports to the rich country. In the interval \( (4/7) \bar{\theta}_D, 0.618 \bar{\theta}_D \), the firm and the consumers of country \( F \) are both better off. For \( \bar{\theta}_F < (94/7) \bar{\theta}_D \), the effect of trade is positive for the firm but not for consumers of the poor country, because exports are high, and, as a consequence, a great chunk of consumers of country \( F \) are not served. In the interval \( [0.618 \bar{\theta}_D, (2/3) \bar{\theta}_D] \), consumers are better off while the firm of the poor country is worse off because it exports less. This highlights a conflict of interests between the firm and the consumers of the poor country: the firm is better off when per capita incomes between the two countries are far apart, while consumers in the poor country gain as per capita incomes get closer. This may make quite difficult the implementation of any commercial policy.\(^{(14)}\)

We can finally state the following propositions.

**Proposition 4.2 Trade decreases the profit of both firms if**

\[ \bar{\theta}_F > 0.618 \bar{\theta}_D. \]

\(^{(14)}\)This results holds in a two-country framework. A more general model considering more than two countries may not confirm it.
Otherwise, it decreases the profit accruing to the firm of the rich country, while increasing the profit of the firm in the poor country.

**Proposition 4.3** There is a region of the parameters, identified by

\[ (4/7) \bar{\sigma}_D, 0.618 \bar{\sigma}_D, \]

in which both consumers and the producer of the poor country gain from trade.

The intuition behind the above Propositions is that the firm of the poor country gains from trade if it is sufficiently poor in terms of per capita income. In case of overlapping income distributions the range of parameters wherein the firm in the poor country gains from trade shrinks, because price competition becomes more intense, while the market share that the low-quality firm may acquire in the richer country becomes smaller (see Lambertini, [1997]).

6 An import-reducing tariff

The trade flows we have just examined give rise to a range of commercial deficits by the rich country. The maximum deficit is equal to one half of total production of country D (\( \bar{\sigma}_D/6 \) when \( \bar{\sigma}_F = \bar{\sigma}_D/2 \) while it disappears when \( \bar{\sigma}_F = 2\bar{\sigma}_D/3 \)). The deficit of the rich country grows directly with the difference in per capita income of the two countries: the incentive to export by the low quality firm increases as the rich country becomes relatively richer vis à vis the poor country.

When it faces a trade deficit \( (\lfloor \bar{\sigma}_D/2 \rfloor < \bar{\sigma}_F < \lfloor 2/3 \bar{\sigma}_D \rfloor) \), the rich country may decide to set an import reducing tariff, provided that total welfare is not decreased. We assume that a quantity tariff is levied on imports of the rich country, giving rise to the following demand functions:

\[
x_D = \bar{\sigma}_D - h = \bar{\sigma}_D - \frac{(p_D - p_F - t)}{q_P - q_F}
\]

\[
x_F = h - k = \frac{(p_D - p_F - t)}{q_D - q_F} - \frac{p_F}{q_F}
\]

where \( t \) is the unit tariff. Notice that the introduction of a tariff modifies only the location of \( h \).

The two new profit functions are:

\[
\pi_D = (p_D - q_D^2) \left( \bar{\sigma}_D - \frac{(p_D - p_F - t)}{q_D - q_F} \right)
\]
\[ \pi_F = \left( p_F - q_F^2 \right) \left( \bar{\theta}_F - \frac{p_F}{q_F} \right) \\
+ \left( p_F + t - q_F^2 \right) \left( \frac{p_D - p_F - t}{q_D - q_F} - \bar{\theta}_F \right). \] (60)

If we differentiate (59) and (60) with respect to prices we obtain the first order conditions yielding the following equilibrium prices:

\[ p_D^* = \frac{6 \bar{\theta}_F \bar{\theta}_D^2 - \bar{\theta}_D \bar{\theta}_F^2 - 8 \bar{\theta}_D^3 + 18 \bar{\theta}_F t - 18 \bar{\theta}_D t}{9 \left( \bar{\theta}_F - 4 \bar{\theta}_D \right)} \] (61)

\[ p_F^* = \frac{\bar{\theta}_F \left( \bar{\theta}_D \bar{\theta}_F - 4 \bar{\theta}_D^2 + 27t \right)}{9 \left( \bar{\theta}_F - 4 \bar{\theta}_D \right)}. \] (62)

We are now ready to produce some comparative statics on the tariff imposed by country \( D \). We first work out prices:

\[ \frac{\partial p_D}{\partial t} = \frac{2}{4 \bar{\theta}_D - \bar{\theta}_F} > 0 , \ \forall \bar{\theta}_F, \bar{\theta}_D \] (63)

\[ \frac{\partial p_F}{\partial t} = \frac{3 \bar{\theta}_F}{\bar{\theta}_F - 4 \bar{\theta}_D} < 0 , \ \forall \bar{\theta}_F, \bar{\theta}_D . \] (64)

This entails a change in the terms of trade in favour of the rich country which accords to standard protection theory (Corden [1971]). Then we look at the profits after substituting the equilibrium prices (61) and (62) into (59) and (60):

\[ \pi_D^* = \frac{(18t + 4 \bar{\theta}_D^2 - \bar{\theta}_D \bar{\theta}_F)^2}{27 \left( \bar{\theta}_F - 4 \bar{\theta}_D \right)^2} \] (65)

\[ \pi_F^* = \left( 9 \bar{\theta}_F^3 \bar{\theta}_D^2 - \bar{\theta}_F^4 \bar{\theta}_D - 24 \bar{\theta}_F^2 \bar{\theta}_D^3 + 16 \bar{\theta}_F \bar{\theta}_D^4 - 18 \bar{\theta}_F^3 t \\
+ 216 \bar{\theta}_F^2 \bar{\theta}_D t - 648 \bar{\theta}_F \bar{\theta}_D^2 t + 288 \bar{\theta}_D^3 t - 81 \bar{\theta}_F t^2 \\
- 648 \bar{\theta}_D^4 t^2 \right) / \left( 27 \left( \bar{\theta}_F - 4 \bar{\theta}_D \right)^2 \right) \] (66)

and proceeding with comparative statics we obtain:

\[ \frac{\partial \pi_D^*}{\partial t} > 0 , \ \forall \bar{\theta}_D, \bar{\theta}_F . \] (67)

Since \( \pi_D \) is strictly monotone in \( t \), there is no stationary value of it associated with any finite level of \( t \). As for the firm operating in the
poor country, we have:

$$\frac{\partial \pi_F^*}{\partial t} < 0 \quad \text{iff} \quad t > \frac{16 \bar{\theta}_D^3 - \bar{\theta}_F^3 + 12 \bar{\theta}_F^2 \bar{\theta}_D - 36 \bar{\theta}_F \bar{\theta}_D^2}{9 \left( \bar{\theta}_F + 8 \bar{\theta}_D \right)}$$

(68)

which implies that there is a positive $t$ for which $\frac{\partial \pi_F^*}{\partial t} > 0$, as indicated in figure 2 by the area $A$.

![Figure 2: Effect of the tariff on profits](image)

The vertical intercept in figure 2 represents the maximum absolute level of the tariff that benefits both firms. It is easy to verify that the lower bound of $t_{\text{max}}$ is 0.04575, when $\bar{\theta}_D = 2$. The absolute level of $t_{\text{max}}$ is increasing in $\bar{\theta}_D$, while it is constant in percentage terms and is equal to 22.58% of $p_F$.

**Proposition 5** For

$$\frac{\bar{\theta}_D}{2} < \bar{\theta}_F < 0.5359 \bar{\theta}_D,$$

any tariff ranging between 0 and 22.58% of $p_F$ benefits both firms.\(^{(15)}\)

We can calculate the level of $t$ that eliminates imports. This can be obtained by imposing:

$$\frac{p_D^* - p_F^* - t}{q_D - q_F} - \bar{\theta}_F = 0,$$

(69)

\(^{(15)}\) As emphasized by Davidson (1984) and Rotemberg and Saloner (1989), the introduction of a tariff may increase the stability of a cartel in a multiperiod setting.
yielding:

$$t = \frac{(\bar{\theta}_F - 4\bar{\theta}_F)(3\bar{\theta}_F - 2\bar{\theta}_D)}{18}, \quad (70)$$

which is always positive and very large.\(^{(16)}\) In our framework, the rich country has to set an extremely high tariff to stop imports from the poor country. The high tariff needed is due to the large degree of differentiation between the two varieties in the quality spectrum. When varieties get closer as a result of overlapping between the two income distributions, the level of the tariff needed to stop imports decreases and there may not be any longer a positive tariff that benefits also the low-quality firm.

What is the effect of the tariff on consumer surplus and social welfare in both countries? In the poor country, the consumer surplus, after the imposition of the tariff, is:

$$CS_F = \frac{\bar{\theta}_F \left(4\bar{\theta}_D^2 - 13\bar{\theta}_D \bar{\theta}_F + 3\bar{\theta}_F^2 - 27t\right)^2}{54 \left(4\bar{\theta}_D - \bar{\theta}_F\right)^2}. \quad (71)$$

Differentiating (71) with respect to \(t\) and solving we get:

$$t = \frac{(\bar{\theta}_D - 3\bar{\theta}_F)(4\bar{\theta}_D - \bar{\theta}_F)}{27} \quad (72)$$

which is always negative and corresponds to a minimum of the consumer surplus, so that \(CS_F\) is increasing for all positive \(t\). Taking into account (64) the increase in \(CS_F\) is due to the decrease in \(p_F\) as a consequence of the tariff. Total welfare of country \(F\) is the sum of (66) and (71). A stationary value of \(SW_F\) is obtained when

$$t = \frac{(8\bar{\theta}_D - 11\bar{\theta}_F)(4\bar{\theta}_D - \bar{\theta}_F)}{9 \left(16\bar{\theta}_D - \bar{\theta}_F\right)} \quad (73)$$

\(SW_F\) increases to the right of the stationary point. The critical value of \(t\) in (73) is strictly positive in the viable region of parameters.

Let us now consider country \(D\). The consumer surplus after the tariff is:

$$CS_D = \left(16\bar{\theta}_D^5 + 24\bar{\theta}_D^4 \bar{\theta}_F + 81\bar{\theta}_D^3 \bar{\theta}_F^2 - 190\bar{\theta}_D^2 \bar{\theta}_F^3 + 78\bar{\theta}_D \bar{\theta}_F^4 - 9\bar{\theta}_F^5 - 144\bar{\theta}_D^2 t^2 + 612\bar{\theta}_D^2 \bar{\theta}_F t - 792\bar{\theta}_D \bar{\theta}_F^2 t + 162\bar{\theta}_F^3 t - 972\bar{\theta}_D t^2\right) / 54 \left(4\bar{\theta}_D - \bar{\theta}_F\right)^2. \quad (74)$$

\(^{(16)}\)When \(\bar{\theta}_D = 2\) this tariff amounts to 700% of \(p_F\).
A maximum of $CS_D$ is obtained when

$$t = \frac{-8\bar{\theta}_D^3 + 34\bar{\theta}_D^2 \bar{\theta}_F - 44\bar{\theta}_D \bar{\theta}_F^2 + 90\bar{\theta}_F^3}{108\bar{\theta}_D} \quad (75)$$

which is always negative in the viable region of parameters. Total welfare of country $D$ is the sum of the profits (65) plus the consumer surplus (74) plus the revenue of the tariff, i.e., the imports (58) time the tariff. $SW_D$ is maximized when:

$$t = \frac{(4\bar{\theta}_D - \bar{\theta}_F) (3\bar{\theta}_D^2 + 2\bar{\theta}_D - 3\bar{\theta}_D \bar{\theta}_F - 3\bar{\theta}_F^3)}{18(7\bar{\theta}_D - \bar{\theta}_F - 2)} \quad (76)$$

Hence a positive tariff maximizes $SW_D$ for all acceptable values of $\bar{\theta}_D$ and $\bar{\theta}_F$. Then, there exists a positive tariff which is welfare improving for both countries, inducing a positive effect on the poor country because the domestic producer ends up by serving a larger share of consumers.

If we compare the welfare maximizing tariff in the rich country (76) with the tariff eliminating imports (70), we observe that in the admissible range of parameters:

$$\frac{\bar{\theta}_D}{2} < \bar{\theta}_F < \frac{2}{3} \bar{\theta}_D,$$

the tariff that maximizes social welfare in country $D$ is always lower than the tariff that eliminates imports. The rich country can maximize welfare without stopping trade.

7 Conclusions

We analysed the short-run effects of trade in vertically differentiated goods between a rich and a poor country, i.e., in a North-South framework. We consider a Bertrand duopoly setting. Firms compete after having chosen quality irreversibly in an autarkic environment in which they have a monopoly power. The result is a one way trade, from the poor to the rich country, when no barrier exists. No IIT appears. Trade benefits consumers of the rich country in all circumstances, while it increases the surplus of consumers of the poor country only if per capita income of the poor country is sufficiently high vis à vis that of the rich. This effect is due to the increase in the intensity of price competition which is observed when per capita incomes of the two countries get closer. In such a case a large number of consumers in the poor country are able to buy the low-quality good. The firm of the rich country always looses from trade, while the producer of the poor country may gain
from trade if per capita incomes in the poor country are sufficiently low. This points to a possible conflict of interests between consumers and the producer of the poor country.

Trade has also an impact on the real price of the labour input. In both countries, real wages increase with respect to autarky. In the rich country, the increase in real wage is proportionally larger than in the poor country. In this sense, trade between rich and poor countries may exert a mutually beneficial effect on real factor incomes, without the negative consequences envisaged in some recent literature (Wood [1994]).

Because of a trade deficit, the rich country may decide to set an import reducing tariff. In a subset of the viable region of parameters a positive tariff may increase both countries' welfare. If the poor country is sufficiently poor vis à vis the rich country, a welfare maximizing tariff is a viable trade policy, leading to an increase of domestic surplus of both consumers and the producer.

The above results apply also in the case of partially overlapping income distributions and fixed costs of quality improvements, provided that one-way trade obtains (Lambertini [1997], Motta [1992]). A limited degree of overlapping is consistent with the North-South trade setting and with empirical observations of scanty IIT between rich and poor countries (Tharakan [1984], Havrylyshyn and Civan [1985]). Long-run quality adjustments are considered only in models where quality entails a fixed cost. Shaked and Sutton [1984] show that the increase in global market size favours firms' R&D efforts, leading to an increase in the quality level. Under Bertrand competition, the welfare effect is ambiguous. Motta [1992] extends the analysis to Cournot competition between firms located in countries of different size, proving that, if firms can modify quality after the opening of trade, net gains always arise for both countries. Finally, the issue of the persistence of quality leadership in the international market is addressed by Motta, Thissé and Cabrales [1997], where quality can be modified bearing an adjustment cost. Leapfrogging obtains if and only if the initial quality gap is not too wide.
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