Capital operating time and economic fluctuations

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1 Introduction

Although capital stock is a quasi-fixed factor, most existing stochastic dynamic general equilibrium models neglect variations in capital utilization rates when trying to explain the cyclical fluctuations of output. It is therefore not surprising that they possess so weak propagation mechanisms and display output dynamics which are essentially the same as input dynamics (see for example Cogley ans Nason [1995] or Rotemberg and Woodford [1994]). In contrast, empirical literature investigating output dynamics in both univariate and multivariate frameworks documents strong endogenous propagation mechanisms, witnessed by significantly positive correlations in GNP growth rate and hump-shaped impulse response functions.

Factor demand literature makes it clear that factor utilization rates vary as well as factor stocks do, and explain variations in factor productive services flows (recent contributions including Shapiro [1993], Burnside, Eichenbaum and Rebelo [1995] and Cueva and Heyer [1995]). For example, Shapiro [1986]'s data show that capital operating time is actually more volatile than individual workweek and aggregate hours at business cycle frequencies. To be provocative, assuming a constant capital utilization rate has the same empirical relevancy than assuming constant employment rate and hours and focusing only on variations of the population!

Accounting for variations in factor utilization should therefore appear as a straightforward though useful generalization of the standard stochastic dynamic general equilibrium model when used to explain fluctuations. The model developed in this paper acknowledges that factor

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stocks are costly to instantaneously move and that most of the adjustment is achieved through utilization rates of these factors. This adjustment cost is higher when a single input varies than when inputs adjust altogether simultaneously, because of reorganization costs which can be interpreted as some kind of complementarity between inputs. These realistic features imply the following interesting properties:

(i) both employment and individual hours vary over the cycle;

(ii) utilization rates of both inputs, modelled respectively as labour and capital operating time, endogenously response to shocks (as employment only does in the standard model), inducing shocks to have largely magnified effects;

(iii) adjustment of input stocks take several periods, displaying persistence in responses that closely resembles to observed dynamics.

Focusing on the workweek of capital and labour, we model intensive under-utilization, as in the seminal work of Kydland and Prescott [1988]: for example, there are some idle capacities, likely to be employed during booms, as soon as the capital operating time is less than 24 hours a day. In contrast, Cooley, Hansen and Prescott [1995] or Fagnant, Licandro and Portier [1996] explain idle capacities as the existence of available factors which are not engaged at all in the production process. Intuitively, their point mostly makes sense if the costs of operating factors are large respective to the costs of having them available or to the value of output. Capital vintage models are typical of the first situation, since old vintages are already installed and therefore available at no cost. The second situation requires a great degree of complementarity between capital and labour and an extraordinarily large unexpected decrease in factor productivity\(^{(1)}\). As long as capital is homogenous and only aggregate shocks are considered, factor can only be underutilized along the intensive margin.

This paper also shares Bils and Cho [1994]'s point of view that "firms choose to utilize capital less than 100 percent of the time because it is costly to work labour around the clock". It therefore differs from a collection of papers on the topic including Greenwood, Hercowitz and Huffman [1998], Finn [1995], Licandro and Puch [1995], Burnside and Eichenbaum [1996] and Wen [1997], which rely upon the "depreciation-in-use" hypothesis (Taubman and Wilkinson [1970], Winston [1974]) according to which increased wear-and-tear is what prevent firms from operating their stock of capital during its maximum technological workweek. We do not believe that the largest cost of operating capital longer

\(^{(1)}\)This last point explains that extensive underutilization models use idiosyncratic shocks and not only aggregate shocks.
is a higher depreciation rate\(^{(2)}\). On the contrary, the model presented here emphasizes labour costs associated to an increase in the workweek of capital.

The equilibrium law of motion of capital operating time is not computed in this paper (Burnside [1997] or Dupaigne [1997] providing recent attempts of endogeneization); variations in capital operating time are accounted for by variations in employment and in individual workweek. This assumption is justified as follows: the workweek of capital can be increased through longer individual hours or through an increase in the number of shifts, requiring more workers to be hired\(^{(3)}\). Our approximations of the capital operating time are shown to be equivalent to assumptions on the evolution of the number of jobs in the economy. In comparison, standard RBC models implicitly assume that the number of jobs is equal, up to a multiplicative constant, to total hours worked.

When capital operating time increases with hours or employment, the impact of aggregate shocks are largely magnified: the initial response of labour input generates an increase in capital services yielding, \textit{ceteris paribus}, an increase in the marginal product of labour. Both labour input and its marginal product display “self-sustained” increases which explain the amplification of shocks. Mid-term propagation is affected too, since the initial burst in output enables larger investment, which is desirable since the productivity of capital has raised along with labour input. The capital stock therefore significantly deviates from its steady state path, causing long lasting effects on the level of output. In one of our specification, the successive variations of the different inputs (hours and capital operating time, then employment and capital operating time, then the capital stock) give rise to hump-shaped responses to shocks, similar to those described in the empirical literature.

The paper is organized as follows. Next section describes the model, focusing on the quasi-fixity of inputs and the modelling of factor utilization. Third section presents the results concerning the instantaneous magnification of shocks and its propagation and persistence over time. Last section concludes.

\(^{(2)}\) Among many criticisms, Johnson [1994] concludes from his empirical analysis that if the depreciation rate increases with utilization, the dependance is linear. Explaining fluctuations with the "depreciation-in-use" hypothesis on the contrary requires the relation to be convex and is therefore inconsistent with this finding.

\(^{(3)}\) An increase in the number of shifts with constant employment but less workers per shift is unlikely to occur when labour and capital have a positive degree of complementarity.
2 The model

In this section, we describe a stochastic dynamic general equilibrium model which incorporate quasi-fixity of inputs, weak complementarity between capital and labour, and variations in all utilization margins.

2.1 Quasi-fixity and varying utilization rates

What obviously explains the attention paid to utilization rates is the quasi-fixed nature of inputs. This rigidity prevents inputs' quantities to perfectly adjust instantaneously, and makes adjustment through intensity of use necessary. The quasi-fixity of the capital stock is straightforward since it is precisely a stock. To model the quasi-fixity of employment, we allow for the simple form of labour hoarding described in Burnside, Eichenbaum and Rebelo [1993]: employment is assumed to be chosen at the beginning of a period, whereas shocks occur whose innovations are only revealed to agents during the period. The present decision for employment is therefore predetermined with respect to some state variables, and it is not possible to adjust its value once the shocks are observed, at any cost. The labour market thus remains to be cleared, requiring variations in individual hours, this latter margin being more flexible than employment.

We extend this quasi-fixity of inputs by modelling weak complementarity between capital and labour. As in Bils and Cho [1994], we assume that it is costly to modify capitalistic intensity (the ratio of capital stock to employment) quicker than its deterministic growth rate:

\[ K_{t+1} = (1 - \delta) K_t + Y_t - C_t - G_t - \gamma \left( \frac{K_t/N_t}{K_{t-1}/N_{t-1}} - \gamma \chi_N \right)^2 K_t. \]

Factor stocks are not only impossible to adjust after the shock is revealed, but also costly to modify ex ante. This structure of adjustment costs explains that stocks of both factors will tend to move simultaneously, and adds a clay-clay flavour to our specification.

We now turn to the modelling of utilization rates. Incorporating the workweek of labour is straightforward: longer hours increase production with diminishing returns at the expense of a decrease in leisure time. However, it must be noticed that both employment and individual hours do vary here, while these simultaneous fluctuations are not often reproduced (major exceptions being Kydland and Prescott [1991] and Cho and Cooley [1994]). In the existing literature, either hours can be chosen freely and there is full employment (hence no variations in employment), or workweek is assumed to be fixed (Hansen [1985]).
Fluctuations in the two margins arise here because of labour hoarding, the existence of a fixed cost of working and adjustment costs involving employment. Such a structure of adjustment costs increases the cost of not moving employment along with capital, with respect to the standard case. Employment will thus fluctuate in order to smooth the evolutions of the capitalistic intensity (see Appendix A). But individual hours remain the only margin able to clear the labour market, and will thus also have cyclical evolutions.

Our treatment of capital utilization is symmetrical to labour: capital productive flows can be increased or decreased instantaneously through variations in the capital operating time. Despite the "depreciation-in-use" appears as a widely accepted setup, we believe, as previously stressed, that the costs of operating capital longer are mainly due to the associated increase in labour costs. Obviously, an increase in the utilization rate of capital requires either an increase in employment or individual hours or an increase in the number of shifts, and none of this possibility is costless.

Capital is defined to be operated as long as labour is employed in the productive process\(^{(4)}\). Total hours worked by the two factors are thus equal by definition:

\[
H_{K,t} J_t \equiv H_{L,t} N_t,
\]

where \(H_{L,t}\) and \(H_{K,t}\) respectively denote the workweek of labour and capital, \(N_t\) employment and \(J_t\) the number of jobs. In this framework, the number of shifts is equal to the number of workers operating successively the same job \(N_t / J_t\). We do not however explicitly model here preferences and technology regarding this variable (recent attempts including Burnside [1997] and Dupaigne [1997]). On the contrary, we will restrict ourselves to consider, throughout the paper, three \textit{ad hoc} approximations for capital operating time; in connection with the previous analysis, we will explicit the corresponding assumptions concerning the evolution of the number of jobs in the economy:

- \(H_{K,t} = \mathcal{H}\) (a constant, say 1), referred as model 1. This is of course the standard assumption that only capital stock matters. Formally, it requires that the number of jobs evolves with total hours worked. This model will be used as a benchmark.

\(^{(4)}\) This customary definition is somehow restrictive: there exists some forms of capital that produce output even without any labour input, such as automatic gas stations. However, since the cost of operating this capital is very low (energy for example), it is typically used continuously. Since this paper focuses on fluctuations, this restriction is valid as long as the proportion of unmanned capital does not vary at these frequencies.
• $H_{K,t} = H \cdot H_{L,t}$, referred as model 2.
This second case is the most commonly studied in existing literature (for example, Kydland and Prescott [1988]'s variable workweek polar case, Bils and Cho [1994]'s case B if effort is constant or Burnside and Eichenbaum [1996]'s alternative model): it assumes that the number of shifts is constant over time, and that capital operating time only increases if each worker works longer hours. It also means that there are as many jobs created as workers hired.

• $H_{K,t} = H \cdot H_{L,t} \cdot N_t$, referred as model 3.
Finally, this last case acknowledges that an increase in employment may induce longer hours for capital if newly employed workers work during late shifts. If the number of jobs in the economy is constant, as is assumed here, any new worker is affected to a new shift. Of course, increasing individual hours still affect capital operating time.

It is clear that these specifications can only be considered as rough approximations of the 'true' equilibrium law of motion for capital operating time, which in this framework remains to be modelled. We however believe that models 2 and 3 can be viewed as reasonable lower and upper bounds for this still undefined equilibrium process.

2.2 The program of the benevolent social planner

The economy is populated by a large number of identical, infinitely living, labour supplier agents. The individual incurs a fixed cost $\zeta$ to go to work, best interpreted as a commuting time. An agent working $H_{L,t}$ hours has the following level of utility:

$$\log(C_t^W) + \theta\log(T - \zeta - H_{L,t})$$

where $T$ is the agent's productive time endowment, while the utility of a non-working agent simply writes

$$\log(C_t^U) + \theta\log(T).$$

As we discussed earlier, this economy will display fluctuations in employment. This means that some workers will be unemployed during some periods. It is thus natural to assume, as in Rogerson [1988], that there exists a market for lotteries, and that agents can choose probabilities of employment and state-contingent hours. A well-known outcome of this representation is that the planner equals consumption between employed workers and non-employed workers, so that the benevolent planner program becomes:
\[ V^*(K_t, A_t, G_t) \]
\[
= \max_{\Omega_t^*} \left\{ \sum_{r=t}^{\infty} \beta^r \left[ \log C_r + \theta N_r \log \frac{T - \zeta - H_{L_r}}{T} + \theta \log(T) \right] |_{\Omega_t^*} \right\}
\]
\[
\text{s.t.} \quad \left\{ \begin{array}{l}
K_{t+1} = (1 - \delta) K_t + F(A_t, H_{L,t}, N_t, H_{K,t}, K_t) \\
-C_t - G_t - \gamma \left( \frac{K_t/N_t}{K_{t-1}/N_{t-1}} - \gamma_X \right)^2 K_t.
\end{array} \right.
\]
\[
(1)
\]

and

\[ V(K_t, N_t, A_t, G_t) \]
\[
= \max_{\{G_t, H_{L,t}, H_{K,t}, K_{t+1}\}} \left\{ \sum_{r=t}^{\infty} \beta^r \left[ \log C_r + \theta N_r \log \frac{T - \zeta - H_{L_r}}{T} + \theta \log(T) \right] |_{\Omega_t} \right\}
\]
\[
\text{s.t.} \quad \left\{ \begin{array}{l}
K_{t+1} = (1 - \delta) K_t + F(A_t, H_{L,t}, N_t, H_{K,t}, K_t) \\
-C_t - G_t - \gamma \left( \frac{K_t/N_t}{K_{t-1}/N_{t-1}} - \gamma_X \right)^2 K_t.
\end{array} \right.
\]
\[
(2)
\]

The \textit{ex ante} information set \( \Omega_t^* \) includes all lagged variables and the predetermined capital stock at time \( t \), \( K_t \). The first expression means that agents have to compute employment only on the basis of this information subset and have to rely on their expectations for some current variables whose decisions will be taken later. The \textit{ex post} information set \( \Omega_t \) also includes current values of the shocks, \( A_t \) and \( G_t \) (\( \Omega_t^* \subset \Omega_t \)). What should be noticed concerning the second stage of optimization is that agents have committed themselves regarding employment at the beginning of the period, and that this choice now must be considered as a state variable.

For both factors, stocks and utilization rates are assumed to be perfectly substitutable within productive services. The production function is a Cobb-Douglas on productive flows.

\[ Y_t = A_t [X_{N_t}, H_{L,t}, N_t]^\alpha [H_{K,t}, K_t]^{1-\alpha}, \quad 0 < \alpha < 1. \]

(3)

In this expression, \( X_{N_t} = \gamma_X \) and \( A_t \) respectively denote the exogenous labour augmenting technical progress and the stochastic total factor productivity, whose process is:

\[ \log(A_t) = \rho_A \log(A_{t-1}) + (1 - \rho_A) \log(\bar{A}) + \varepsilon_{A,t} \]

with \( \rho_A \leq 1 \). \( \bar{A} \) denotes the steady state level of TFP and \( \varepsilon_{A,t} \) the innovation of this process, of standard deviation \( \sigma_{\varepsilon_A} \).
The law of motion of capital describes the quadratic adjustment costs on the evolution of the capitalistic intensity. According to this specification, due to Bils and Cho [1994], it is costly to increase the ratio of capital to employment faster (or slower) than the capital stock. Accumulation also involves taxes $G_t = \gamma_X g_t$, which finance a lump sum transfer growing at the same rate than other variables. The stochastic component of government spending follows an autoregressive process, of mean $\bar{g}$:

$$\log(g_t) = \rho_g \log(g_{t-1}) + (1 - \rho_g) \log(\bar{g}) + \varepsilon_{g,t}$$

with $\rho_g \leq 1$ and $\varepsilon_{g,t}$ a serially uncorrelated process, whose standard deviation is $\sigma_{\varepsilon_g}$.

These assumptions guarantee the existence of a balanced growth path. We here extract their common trend from these variable, and define some auxiliary variables:

$$c_t = \frac{C_t}{\gamma_{X,N}}, \quad k_t = \frac{K_t}{\gamma_{X,N}}, \quad i_t = \frac{I_t}{\gamma_{X,N}}, \quad Z_t = \frac{N_t}{N_{t-1}}, \quad X_t = K_{t-1}, \quad x_t = \frac{X_t}{\gamma_{X,N}}.$$  

In the recursive form, the Bellman equations rewrite as:

$$\mathcal{V}^*(k_t, x_t, N_{t-1}, A_t, g_t)$$

$$= \max_{\{Z_t\}} \left[ \kappa + \log c_t + \theta Z_t N_{t-1} \log \frac{T - \zeta - H_{L,t}}{T} \right.$$

$$\left. + E \{ \mathcal{V}^*(k_{t+1}, x_{t+1}, N_t, A_{t+1}, g_{t+1}) | \Omega_t^* \} \right]$$

$$= \left\{ \begin{array}{ll} \gamma_{X,N} k_{t+1} & = (1 - \delta)k_t + A_t [H_{L,t} Z_t N_{t-1}]^\alpha [H_{K,t} k_t]^{1-\alpha} \quad (4) \\
- c_t - g_t - \gamma \left( \gamma_{X,N} \frac{k_t}{x_t Z_t} - \gamma_{X,N} \right)^2 k_t \\
\gamma_{X,N} x_t & = k_{t-1}. \end{array} \right.$$  

$$\mathcal{V}(k_t, x_t, N_t, A_t, g_t)$$

$$= \max_{\{c_t, H_{L,t}, H_{K,t}, k_{t+1}, x_{t+1}\}} \left[ \kappa + \log c_t + \theta Z_t N_{t-1} \log \frac{T - \zeta - H_{L,t}}{T} \right.$$  

$$\left. + E \{ \mathcal{V}(k_{t+1}, x_{t+1}, N_t, A_{t+1}, g_{t+1}) | \Omega_t \} \right]$$

$$= \left\{ \begin{array}{ll} \gamma_{X,N} k_{t+1} & = (1 - \delta)k_t + A_t [H_{L,t} Z_t N_{t-1}]^\alpha [H_{K,t} k_t]^{1-\alpha} \quad (5) \\
- c_t - g_t - \gamma \left( \gamma_{X,N} \frac{k_t}{x_t Z_t} - \gamma_{X,N} \right)^2 k_t \\
\gamma_{X,N} x_t & = k_{t-1}. \end{array} \right.$$
2.3 Calibration and resolution

Values for the structural parameters, presented in tables 1 and 2, are fixed either according to micro evidence or in order to match some long run ratios on US postwar data (see appendix B). The discount factor \( \beta \) is set to .99, implying a discount rate of 1% per quarter. Following Burnside, Eichenbaum and Rebelo [1993], we assume that the individual time endowment and the commuting time respectively equal 1369 and 60 hours for each quarter. The elasticity of labour in the production function, \( \alpha = .58 \)\(^{(5)} \), the annual depreciation rate of 10% (\( \delta = .025 \)) and the technical progress growth rate \( \gamma_{X_N} = 1.004 \) are taken from King, Plosser and Rebelo [1988]. The last parameter is the coefficient of the adjustment cost \( \gamma \). Bils and Cho [1994]'s estimations give the value \( \gamma = .00969 \). Finally, the weight of labour disutility will be chosen as to obtain a steady-state employment ratio of .58\(^{(6)} \).

<table>
<thead>
<tr>
<th>Table 1: Preferences</th>
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<td>( \beta )</td>
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<tr>
<td>.99</td>
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<th>Table 2: Production and accumulation</th>
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<tr>
<td>( \alpha )</td>
</tr>
<tr>
<td>.58</td>
</tr>
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Since the traditional Solow residual is no longer equal to the technological process when utilization rates vary, the calibration of the technological shock process requires attention. As soon as we acknowledge that evolutions in the capital operating time occur along the business cycle and that they matter for output, traditional Solow residual which

\(^{(5)}\) Using electricity consumption as a proxy for the capital operating time, Burnside, Eichenbaum and Rebelo [1995]'s estimate for the elasticity of labour is .54 on economy wide data. Corresponding estimates when the workweek of capital is not taken into account are 1.23 and 1.31 according to the capital stock data used.

\(^{(6)}\) This figure, computed on our data-set, is larger than the employment rates used in many standard models, because we disentangled labour services in employment and individual hours. Our measure is the ratio of employment to population over sixteen, whereas many studies compute the ratio of hours worked to hours available for work.
only corrects for variations of capital stock and hours worked is misspecified\(^{(7)}\). Because our specifications of the capital operating time are only approximations, we cannot compute, from observed workweek data, a Solow residual totally consistent with our theoretical model. We will thus follow Cooley, Hansen and Prescott [1995]: the parameters of the technological shock are set so that the traditional Solow residual match statistical properties of its observed counterpart. For the same reasons, we estimate an order 1 univariate autoregressive process on US government spending, and do not perform a joint estimation of the processes of the two shocks, as is done in Hairault and Portier [1993]. This yields an autoregressive coefficient of \( .97 \) and a standard deviation of innovation of \( .02 \) (table 3).

<table>
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<th>Table 3: Shocks</th>
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<tr>
<td>( \rho_{SR} )</td>
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<tr>
<td>.95</td>
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Due to the labour hoarding hypothesis, the resolution of the log-linearized version of the system of optimality conditions will be twofold. In a first stage, the social planner will determine employment (formally here \( Z_t \) which describes its evolution) conditionally on its expectations based on ex ante information \( \Omega^*_t \), that is excluding current values of shocks. This solution of a first optimization will take the form of a decision rule depending on current states and lagged shocks. Once this decision is taken, the social planner solves the second stage in order to compute optimum consumption, individual hours, capital operating time and investment paths. Of course, this optimization is run with full knowledge of the employment decision rule.

3 Results

In the typical RBC model, the period impact of a change in productivity is only amplified by intertemporal substitution, which shifts labour supply onwards and thus enables an increase of employment. For reasonable individual labour supply elasticity however, this effect has not much chance to be large. Such a model does not either substantially affect output persistence comparatively to the autoregressive nature of shocks. If shocks were \( i.i.d. \), persisting effects would in fact only be due to the initial increase in capital accumulation. Once again this effect is

\(^{(7)}\) Furthermore, as will become clear in next section, other shocks than the technological one ("demand" shocks) may affect the Solow residual as well.
likely to be small, since an increase in the investment flow at date $t$ has only small influence on the path of the capital stock at future dates.

Variable utilization rates have long been claimed to strengthen RBC's propagation mechanisms (see Kydland and Prescott [1988] or Greenwood, Hercowitz and Huffman [1998]). We show in this section that introducing variable factor utilization rates the way we did substantially improves both the instantaneous amplification of shocks and its persistence.

3.1 Magnification of the instantaneous propagation

We focus on the first few periods dynamics to highlight how variable capital utilization do magnify the impacts of shocks to the economy.

In a totally elastic labor supply model, the factor demand effect of a positive technological shock is an increase in the (both average and marginal) product of labour at any point, causing an increase in the demand for labour until the marginal product of labour has reached its initial value. In such cases, the response of hours worked is clearly limited by the decrease of the marginal product of labour. If capital utilization rates increase as well during periods of high productivity, the decrease of the marginal product of labour when hours increase should be smaller, meaning a larger response of hours worked.

This factor demand effect is however not the only one incurring when utilization varies over the cycle. Allowing for variations in the capital utilization rate may modify the labour supply decisions of the household as well as it modifies the labour demand of the firm. This is best understood when one thinks of a positive government spending shock. Such a shock has a positive impact on output if private and government consumption are not perfect substitutes, since households forecast the increase in taxes required to fulfill the intertemporal budget constraint. An increase of government consumption therefore means a decrease of their future wealth and private consumption. In order to smooth their utility path, the household supplies more labour (given the same wage rate) to counter the decrease in its income and consumption. When capital utilization varies with employment or hours, this labour supply response is smaller than in the benchmark model. It is due to the larger effect on output of an increase of labour. From the household point of view, this means that only a smaller increase of labour is enough to overcome the fall in wealth.

These two (intra-temporal substitution and wealth) effects are of opposite signs. The total magnification of the response of hours worked is therefore a priori indeterminate.
A third reason might explain the different responses to shocks according to the patterns of capital utilization: changes in steady state. Increasing the returns to labour as our specifications of capital utilization do may of course induce a higher steady state value of hours worked. As soon as marginal disutility is convex, starting from a higher level necessarily yields larger increases of the disutility of labour when hours worked rise, and therefore smaller hours’ responses. On the basis of relative deviations to the steady state, observed differences between the model with and without varying capital utilization thus underestimate the magnifying effect at a given point. When willing to investigate the pure effects on short term dynamics of variable capital utilization, we should therefore control for the steady state of the model, as is done in figure 1, 2, 3 and 4.

Our preliminary analysis emphasizes the role of the “slope” of the marginal product of labour along an increasing labour path. Marginal productivity of labour writes in the general case:

\[
\frac{dF(A_t, H_{L,t}, N_t, H_{K,t}, k_t)}{dH_{L,t}} = \frac{\partial F(\cdot)}{\partial H_{L,t}} + \frac{\partial F(\cdot)}{\partial H_{K,t}} \cdot \frac{\partial H_{K,t}}{\partial H_{L,t}} = \alpha A_t H_{L,t}^{2-\alpha} N_t^\alpha [H_{K,t} k_t]^{1-\alpha} + (1 - \alpha) A_t [H_{L,t} N_t]^{\alpha} H_{K,t}^{\alpha} k_t^{1-\alpha}.
\]

Differentiating it with respect to hours gives:

\[
\frac{d^2 F(A_t, H_{L,t}, N_t, H_{K,t}, k_t)}{dH_{L,t}^2} = \frac{\partial^2 F(\cdot)}{\partial H_{L,t}^2} + \frac{\partial^2 F(\cdot)}{\partial H_{K,t} \partial H_{L,t}} \cdot \frac{\partial H_{K,t}}{\partial H_{L,t}} = \alpha(\alpha - 1) A_t H_{L,t}^{\alpha - 2} N_t^{\alpha} [H_{K,t} k_t]^{1 - \alpha} + \alpha(1 - \alpha) A_t H_{L,t}^{\alpha - 1} N_t^{\alpha} H_{K,t}^{\alpha} k_t^{1 - \alpha}.
\]

Obviously, \(\partial H_{K,t} / \partial H_{L,t} = 0\) in model 1 since capital operating time is assumed to be constant. The first term of equation (6) then reduces to \(\partial^2 F(\cdot) / \partial H_{L,t}^2\), which is negative as soon as the marginal product of labour is decreasing. After a positive productivity shock, the decrease from a high level of marginal product of labour to its optimal value is hence quick. In the other cases (model 2 and 3), output does depend on capital operating time \((\partial F(\cdot) / \partial H_{K,t}) > 0\) which itself depends on individual hours \((\partial H_{K,t} / \partial H_{L,t}) > 0\). Since \((\partial^2 F(\cdot) / \partial H_{K,t} \partial H_{L,t}) > 0\), it tends to diminish the derivative of marginal product of hours with respect to hours. In other words, marginal product of labour does decrease
slower when hours increase, and thus enables longer adjustment paths when starting from a high level of productivity\(^{(8)}\).

\[\text{Figure 1: Impulse response function to the technological shock (1)}\]

As expected, figure 1 shows that the workweek instantaneously increases after a positive technological shock twice as much when capital utilization is allowed to vary (models 2 and 3) as in model 1 (where it does not). Although employment does not increase in turn (since it is predetermined), the rise in output (on figure 3) is also nearly twice as large as in the benchmark model (due to longer hours and greater returns to hours). Wealth gets up since (present and future) income does, but the deviation from its benchmark path is only moderate, because wealth is a stock. The increase in consumption is therefore limited, and a large part of the supplementary output is invested—the rise of investment is 50% higher in model 2 than in model 1.

Let us notice that an exogenous increase of total factor productivity of 1% induces a larger increase of the traditional Solow residual in

\(^{(8)}\) The same analysis could be undertaken concerning employment (at period 2) for the model 3.
models 2 and 3. Since the capital operating time varies with individual hours, output increases more than what the only response of hours would predict. Neglecting other production factors, traditional Solow residual thus overstates the response of total factor productivity.

The employment response occurs at period 2. The initial response of hours induced a rise of marginal product of employment, which is higher in model 2. We therefore expect a rise in employment in period 2 in the benchmark model, and a bigger one in model 2. This last prediction fails to happen, since adjustment cost increase the cost of boosting employment up.

Changes in the responses to a government spending shock, displayed in figure 2 and 4, are explained by the larger amount of output able to be processed when hours worked increase. Falls in wealth are consequently less severely valued by the social planner. Consumption therefore decreases less than in the benchmark case. In model 2, household substitute hours, whose returns are higher, for employment. Once again, the traditional Solow residual increases since variations in unaccounted factors happen: a demand shock seem to influence technology
as soon as the production function is misspecified, ignoring (cyclical) variations in capital services that do occur trough capital utilization rates.

Impact response of model 3 display basically the same features as model 2: more or less substantial increase (with respect to the benchmark model) of hours worked, output, consumption and investment. On the whole, the forms of variable capital utilization specified here do largely magnify the effect of aggregate shocks on hours worked, output, investment and the Solow residual. The mid term dynamics, however, substantially differ in the three models. Next section will investigate this issue.

Finally, table 4 presents the ratio of (filtered) output and technological shock volatilities of single shock versions of the models. The lack of propagation mechanism of standard RBC model is exemplified by model 1, which in fact smoothes output fluctuations: output varies less than the technological shock. Variable capital utilization as in model 2 and 3 substantially magnifies the effect of a shock to technology; the standard error of cyclical output increases from 1 to 3 after a productivity shock.

<table>
<thead>
<tr>
<th>model 1</th>
<th>model 2</th>
<th>model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_Y/\sigma_A$</td>
<td>.61</td>
<td>1.66</td>
</tr>
</tbody>
</table>

3.2 Persistence: hump-shaped response vs monotonic path

As figures 3 or 4 make it clear, persistence properties of the various models and the mid term dynamics of output differ in many points. The most striking one concerns the monotonicity of the output path: both models 1 and 3 displays hump shaped responses (to both shocks), whereas the output trajectory implied by model 2 decreases monotonically towards its steady state.

Burnside, Eichenbaum and Rebelo [1993] show that their labour hoarding assumption gives rise to this non-monotonic feature. Employment being predetermined, it only responds to shocks with a one period delay. If the shock has a positive effect on employment, the employment response will therefore necessary display an initial statu quo, then a rise and finally a decrease to return to the steady state. This non-monotonic path displays the so-called hump-shaped response.

However, even model 1 has a richer employment and output dynamics than the simple labour hoarding model. Contrarily to the example just described, employment does increase here not only in the first
period after the shock but also during the next one. Adjustment costs on capitalistic intensity cause this employment smoothing. The design of adjustment costs thus appears as a crucial feature for the persistence of employment and output dynamics.

![Figure 3: Impulse response function to the technological shock (2)](image)

In contrast, model 2 only displays a monotonic adjustment path to its steady state (see figure 3), although it also embodies both labour hoarding and adjustment costs on capitalistic intensity. As a consequence, the mid term dynamics in model 2 is very different to that of the two other models. We emphasized in the previous section that its major property with respect to the benchmark model was the much larger response in individual hours explained by the slower decrease of the marginal product of hours. This larger impact has a great influence on the general shape of hours worked and output. In model 1, hours worked are nearly constant from period one to period 2, since the delayed impact response of employment offsets the decrease of individual hours getting down towards their long run value. This cumulative effect on hours worked is notably affected in model 2, as the individual hours response gets larger, since employment response does not vary much due...
Figure 4: Impulse response function to the government spending shock (2)

to adjustment costs. The sharp drop in individual hours between period 1 and 2 now exceeds the impact response of employment. Hours worked therefore decrease from the very first period, explaining the monotonic behaviour of output.

Model 3 does display hump shaped response of hours worked and output again, since the path of individual hours is smoother than in model 2, reflecting preferences over hours and employment, yielding smaller decreases in hours at every date.

The most important feature of mid term dynamics of model 3 is the third period response of output (figure 3). First period basically consists of the impact response of hours, the second one includes the delayed response of employment. But output here still grows during the third period, even though hours worked have already started to decline (figure 1). Fastened capital accumulation appears to be the engine of this transitory growth once initial effects on factor utilization begin to vanish: thanks to greater factor utilization during the first two periods, the large increase in output has enabled a boost of investment, which in turn causes a substantial change in the path of the capital stock. Five or six periods after the shock, the capital stock exceeds its steady state
value by 1% or more in this third model, which is more than 50% higher than in the benchmark one. Production therefore keeps growing even if labour services decline. This positive effect of capital does of course last many periods, being a strong persistence mechanism.

Table 5 presents the autocorrelation of output growth from order one to four. It appears that model 1, although it embeds labour hoarding, does not display significantly positive autocorrelation of output growth: output growth appears to be close to a white noise. On the contrary, first order autocorrelation of output growth in model 2 and 3 is highly significative, as in after-war quarterly US data. Propagation mechanisms of the model 3 are even strong enough to significantly exceeds the observed first-order autocorrelation of output growth.

<table>
<thead>
<tr>
<th>$\Delta \log \hat{Y}$</th>
<th>Order 1</th>
<th>Order 2</th>
<th>Order 3</th>
<th>Order 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>US data</td>
<td>.45</td>
<td>.32</td>
<td>.23</td>
<td>.14</td>
</tr>
<tr>
<td></td>
<td>(.091)</td>
<td>(.086)</td>
<td>(.080)</td>
<td>(.090)</td>
</tr>
<tr>
<td>model 1</td>
<td>.07</td>
<td>.00</td>
<td>-.04</td>
<td>-.01</td>
</tr>
<tr>
<td>model 2</td>
<td>.14</td>
<td>-.02</td>
<td>-.06</td>
<td>-.02</td>
</tr>
<tr>
<td>model 3</td>
<td>.63</td>
<td>.20</td>
<td>-.01</td>
<td>-.10</td>
</tr>
</tbody>
</table>

Note: Standard error into parenthesis

Lastly, let us emphasize the timing of responses to the different factor margin available. Individual hours, which is the utilization degree of labour, and capital utilization rate appear to be the most versatile margins: they jump instantaneously and return to their steady state level quite quickly (in 5 periods for models 1 and 2). If the shock persists, employment will respond as well, peaking in the third period. Its response is positive between 10 to 15 periods, which is substantially longer than the response of individual hours. Finally, a very persistent shock (as the standard technological or government spending shock) causes the capital stock to move above its steady state level, this deviation being very long lasting.

This timing is consistent with two well-known findings concerning factor adjustment (Cueva [1995] among others): factor utilization degrees adjust quicker than factor stocks; labour services adjust more rapidly than capital services\(^{(9)}\). This framework therefore reproduces

\(^{(9)}\) Empirical literature has not a unique answer to the question of ordering the responses of capital utilization rates and employment. Because of the specification of capital utilization adopted, this issue is not relevant here.
the traditional pattern of factor utilization, which could be used to justify *ex post* their introduction.

### 3.3 Second order moments

Most existing models neglect variations in the workweek of capital; either employment or individual hours is the only margin allowed to vary. However, Shapiro [1986]'s data on capital operating time show that its cyclical fluctuations are even slightly larger than those of aggregate hours worked (respectively 1.48 and 1.32% on the 1959-1982 sample). Capital productive service flows are therefore likely to appear more volatile than labour service flows. This justifies our focus on such fluctuations.

On US data, the workweek of capital varies three times as much as the workweek of labour. This finding supports the emphasis put on the number of shifts, since fluctuations in overtime hours account for only one third of total variations in capital operating time. From this point of view, even our third specification underestimates the volatility of capital operating time.

Column 3 of table 6 shows that our baseline reproduces fairly well usual stylized facts, and specially those concerning the labor market. The fact that this model displays simultaneously variations in employment and individual works has already been underlined. But respective volatilities of these two margins appear quite close to US data, as well as overall variability of total hours worked.

### Table 6: Autocorrelation of output growth. Second order moment

<table>
<thead>
<tr>
<th>Moment</th>
<th>US data&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{\tilde{Y}} )</td>
<td>1.59</td>
<td>1.58</td>
<td>2.66</td>
<td>2.60</td>
</tr>
<tr>
<td>( \sigma_{\tilde{C}/\tilde{Y}} )</td>
<td>.52</td>
<td>.44</td>
<td>.22</td>
<td>.22</td>
</tr>
<tr>
<td>( \sigma_{\tilde{H}/\tilde{Y}} )</td>
<td>.34</td>
<td>.42</td>
<td>.15</td>
<td>.20</td>
</tr>
<tr>
<td>( \sigma_{\tilde{N}/\tilde{Y}} )</td>
<td>.63</td>
<td>.66</td>
<td>.87</td>
<td>.84</td>
</tr>
<tr>
<td>( \sigma_{\tilde{NH}/\tilde{Y}} )</td>
<td>.87</td>
<td>.73</td>
<td>.90</td>
<td>.88</td>
</tr>
<tr>
<td>( \sigma_{I/\tilde{Y}} )</td>
<td>2.71</td>
<td>2.59</td>
<td>2.79</td>
<td>2.76</td>
</tr>
<tr>
<td>( \sigma_{\tilde{H}/\tilde{Y}} )</td>
<td>.95&lt;sup&gt;b&lt;/sup&gt;</td>
<td>.00</td>
<td>.15</td>
<td>.88</td>
</tr>
<tr>
<td>( \sigma_{\tilde{P}/\tilde{Y}} )</td>
<td>.59</td>
<td>.72</td>
<td>.26</td>
<td>.33</td>
</tr>
</tbody>
</table>

<sup>a</sup> Data are described in appendix B.

<sup>b</sup> Data on the workweek of capital range from 1959:1 to 1982:4.

Comparing the competing models, the sharp increase in the volatility of output due to the introduction of varying capital utilization (model
2 and 3) is fully consistent with the analysis undertaken in the previous section, as well as the moderate increase in investment. The volatility of consumption goes down and is now far below its observed level, because the household can adjust its labour supply in many different margins and therefore smoothes its consumption path more easily.

Individual hours are surprisingly less volatile relative to output in model 2 and 3 (with variable capital utilization), although instantaneous response was largely magnified. Larger returns to hours explain that the volatility of output increases more than the volatility of hours, yielding a decrease of the ratio. This puzzle is therefore linked to the very strong propagation of shocks to the output dynamics, which causes the standard error of cyclical output to exceed its observed value.

From the point of view of second order moments, variable capital utilization has failed to improve the accuracy of the model, even though the relative standard deviation of aggregate hours worked and investment match remarkably well their empirical counterpart for models 2 and 3. Plausible explanations of such discrepancies include both the specific design of this model and measurement issues.

First, it should be clear that these results rely heavily on the specifications of capital operating time we used. As emphasized earlier, these remain approximations of the true endogenous variable based on assumptions regarding evolutions in shiftworking over the cycle. Further work towards purely endogenous determination of the number of shifts is required. Such work is in progress.

Secondly, the volatility of output is highly dependent on the calibration of exogenous shocks. The strategy adopted is only consistent with the mispecification of the production function studied here. But this calibration of the technological shock is likely to overestimate the volatility of a "true" productivity shock if the Solow residual is affected in different ways (as in Hall [1989] among others).

4 Conclusion

This paper studies a stochastic dynamic general equilibrium model with quasi-fixed and weakly complementary factor stocks. It underlines that instantaneous variations of factor stocks are costly, and that most of the initial adjustment after shocks affecting the economy is achieved through changes in factor utilization rates. Specifically, it describes variations in individual hours, employment and capital operating time. Such margins are shown to substantially magnify the propagation mechanisms of the economy, and to amplify the response of the traditional Solow residual to various kinds of shocks.
Moreover, this model displays highly persistent output growth, instead of a white-noise as in many other RBC models. It therefore contradicts Cogley ans Nason [1995]'s intuition according to which capital dynamics cannot play any role in this persistence; one key feature highlighted in this paper is the much fastened accumulation enabled by the sharp increase in capital operating time and output.

Models presented here follow Kydland and Prescott [1988] and Bils and Cho [1994] by assuming that the workweek of capital increases with individual hours. We however, in contradiction to Bils and Cho [1994], believe that the capital operating time may increase with employment as well, as soon as new workers operate new shifts. In line with these two papers, we find that, although varying capital utilization amplifies the responses of hours worked and output to aggregate shocks, it does not drastically improve the replication of the volatility of output, hours and employment. This last failure suggests that our calibration of the technological shock still overestimates the volatility of the “true” productivity shock, and that our theoretical approximations of capital operating time are misspecified. Further work on the endogenous determination of the number of shifts should therefore receive much attention.

APPENDIX A

Unemployment

In the presentation of the program of the social planner, we argued that the structure of adjustment costs might lead to unemployment, even if individual hours are flexible and would therefore ensure full employment otherwise. The underlying idea is that firms should prefer to smooth the path of capitalistic intensity, because of the adjustment costs, by increasing (respectively decreasing) employment during periods of over (respectively under) accumulation. This of course requires employment ratio to lie below one, at least during some dates.

We try here to show why we can rule out the existence of unemployment in the absence of adjustment costs but no longer when they exist. Let us consider here the program of the social planner when there exist no insurance facility such as those associated with an employment lotteries market; in order to simplify exposition, we abstract from information issues, assuming that all decisions are
taken simultaneously:

\[
\max_{\{C_t, N_t, H_{L,t}, H_{K,t}, K_{t+1}\}} \mathbb{E} \left\{ \sum_{r=t}^{+\infty} \beta^r \left[ \log C_r + \theta \log \left[ T - N_r (\zeta + H_{L,r}) \right] \right] \right\}
\]

\[
\begin{align*}
K_{t+1} &= (1 - \delta) K_t + A_t \left[ H_{L,t} N_t \right]^{\alpha} \left[ H_{K,t} K_t \right]^{1-\alpha} \\
- C_t - G_t - \gamma \left( \frac{K_t / N_t}{K_{t-1} / N_{t-1}} - \gamma \chi_N \right)^2 K_t &= (\lambda_t \geq 0) \\
N_t &\leq 1 \quad (\mu_t \geq 0).
\end{align*}
\]

Notice that the commuting cost \( \zeta \) does not concern agents who decide not to work.

Deriving the Lagrangian of this constrained maximization with respect to employment and individual hours gives the two following optimality conditions:

\[
\frac{\partial \mathcal{L}_t}{\partial N_t} = -\frac{\theta (\zeta + H_{L,t})}{T - N_t (\zeta + H_{L,t})} + \left[ \alpha A_t H_{L,t}^{\alpha} N_t^{\alpha-1} (H_{K,t} K_t)^{1-\alpha} + 2\gamma \frac{K_t / N_t}{K_{t-1} / N_{t-1}} \left( \frac{K_t / N_t}{K_{t-1} / N_{t-1}} - \gamma \chi_N \right) K_t \right] \lambda_t - \mu_t = 0
\]

\[
\frac{\partial \mathcal{L}_t}{\partial H_{L,t}} = -\frac{\theta N_t}{T - N_t (\zeta + H_{L,t})} + \alpha A_t H_{L,t}^{\alpha-1} N_t^{\alpha} (H_{K,t} K_t)^{1-\alpha} \lambda_t = 0
\]

and the second exclusion relation writes:

\[
\mu_t \left[ 1 - N_t \right] = 0.
\]

As usual, this relation means that the Lagrange multiplier must equal zero if the constraint is not binding. Let us assume that the economy experiences underemployment. It necessarily implies that \( \mu_t = 0 \). The ratio of the first order conditions on employment and hours thus gives:

\[
\frac{\zeta + H_{L,t}}{N_t} = \frac{\alpha A_t H_{L,t}^{\alpha} N_t^{\alpha-1} (H_{K,t} K_t)^{1-\alpha} + 2\gamma \frac{K_t / N_t}{K_{t-1} / N_{t-1}} \left( \frac{K_t / N_t}{K_{t-1} / N_{t-1}} - \gamma \chi_N \right) K_t}{\alpha A_t H_{L,t}^{\alpha-1} N_t^{\alpha} (H_{K,t} K_t)^{1-\alpha}}.
\]

If \( \gamma = 0 \), i.e. in the absence of adjustment costs, equation (8) turns to \( \zeta + H_{L,t} = H_{L,t} \), which is wrong as soon as \( \zeta \) differs from 0. This means that
our assumption $\mu_t = 0$ is wrong. From equation (7), we see that $N_t = 1 \forall t$: the
absence of adjustment costs involving employment insures a full employment
equilibrium path. But the case $\mu_t = 0$ can no longer be ruled out when $\gamma \geq 0$.
This explains why we might expect some degree of underemployment when it is
costly to adjust the capitalistic intensity.

APPENDIX B

Data

US quarterly data on the sample 59:1 to 92:4 were taken from the Citibase.
Acronyms are given in parentheses.

Private consumption is the sum of personal consumption expenditures
of non-durable goods (GCNQ) and services (GCSQ). Government consumption
is composed of federal spending in non-durable goods (GGNNQ+GGONQ)\(^{(10)}\)
and services (GGNSAQ+GGOSAQ) and state and local expenditures in non-
durable goods (GGSNQ) and services (GGSSAQ). Total investment is defined
as non-residential fixed investment (GINQ), residential investment (GIRQ) and
consumption of durable goods (GCDQ), plus federal expenditures in structures
(GGNCQ+GGOCQ) and durable goods (GGNDQ+GGODQ), and local expenditures
in structures (GGSCQ) and durable goods (GGSDQ). Output is measured
as the sum of private, governmental consumption and total investment. Em-
ployment (LHEM) is obtained from the establishment survey whereas weekly
hours worked (LCH) comes from household survey. Data are deflated by total
civilian population aged 16 and over (P16) and can therefore be interpreted as
per capita measures.

Data on capital operating time were computed by Shapiro [1986], using the
Bureau of Labor Statistics data on shift work. Let $L_1$, $L_2$ and $L_3$ respectively
denote employment on the first, second and third shifts, and $H$ the average
workweek of labour. Capital operating time is measured as:

$$COT = \frac{H(L_1 - L_2) + 2H(L_2 - L_3) + 3HL_3}{L_1}.$$

By assuming that overtime work on one shift overlaps with that on another
shift, it helps to ensure that such measure of the workweek of capital is not
spuriously correlated with the workweek of labour.

\(^{(10)}\) After 1972, defense and non-defense spending were disaggregated.
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