Launhardt and Hotelling

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1 Introduction

Hotelling's model is certainly one of the most popular instruments of modern economic analysis. Introduced in the rightly famous paper "Stability in Competition", published in the *Economic Journal* in 1929, it was designed as a weapon against Bertrand's [1883] objection to Cournot, that a solution to the duopoly problem is impossible, since competition between two producers who undercut each other's price necessarily leads to a process of price reduction without limit. Hotelling wanted to show that this instability pointed out by Bertrand disappears as a consequence of product differentiation of any kind. Positive demand for the more expensive product may then subsist, in spite of the principle of uniformity of price in a perfect market, because "for some purposes it is better to consider a market [not as a point, without length, breadth or thickness, but] as an extended region". Thus, *space* gets in.

By contrast, space was in, from the beginning — in a more pedestrian way, to tell the truth — in Launhardt's work, mainly devoted to the planning of transportation networks. In the last section of his 1885 book *Mathematical Foundations of Economics*, Launhardt develops an economic analysis of transportation, after a first section on exchange (partly influenced by Jevons and Walras) and a second section on production. This analysis is closely related with Hotelling's work, a relation that has sometimes been viewed as an *anticipation* by the earlier author of the later's independent contribution. The anticipation seems to be neat enough to justify referring, as some German speaking authors do, to the "Launhardt-Hotelling model" (Selten [1979]), "problem" (Schneider [1963], foreword to Launhardt [1885]), or "solution" (Niehans [1987]). Contrary to Hotelling, Launhardt is however widely ignored in the economic literature, and indeed completely absent from the standard reference books and papers on microeconomics and industrial organisation. His contribution to economic theory is mainly acknowledged only in the field of spatial economics, and then almost exclusively for his

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concept of market area and his analysis of optimal location, not for his place in oligopoly theory (see for instance Blaug [1985]). And even that limited recognition cannot be taken as sure, as shown by the neglect of his name in the entry "spatial economics" of *The New Palgrave*.

In addition to the language problem — his 1885 book has waited until 1993 for an English translation — Pinto [1977], who discovered a 1900 English edition of some of Launhardt's papers on railway location, including the analysis of market areas, suggests three explanatory factors for Launhardt's obscurity, in particular in his own time. He was an engineer not an economist, he published in engineering journals not in economic journals, he used abstract mathematical and geometrical analyses. We can make these factors more precise. The first factor may account for his peculiar and somewhat unattractive style, for "his almost ruthless use of particular forms of function — by which he produces results of disconcerting definiteness" (Schumpeter [1954], p. 948n.10), very much in contrast with, say, Cournot's style. The third factor is reinforced by his dominant historicist environment in Germany, which was rather hostile to the mathematical method, and more generally to an analytical approach in economics.

In any case, and quite independently of the reasons explaining the discrepancy between Hotelling's celebrity and Launhardt's obscurity, it remains to evaluate the proximity of the two authors' independent contributions to the theory of spatial competition. Does Launhardt anticipate Hotelling? Is there a sufficient convergence of their respective models, problems, solutions, to justify the name "Launhardt-Hotelling"? Or should we go still farther and, by inverting so to speak the time arrow, view the earlier writer as generalising and completing the work of the later? We shall argue in section 2 in favour of this extreme position. But, if we are right, what is objectively left of Hotelling? Does he transcend Launhardt's achievement? We discuss this second question in section 3, where we shall contend that, besides a genuine strategic approach to the choice of locations, Hotelling's major contribution is the conception of an abstract characteristics space, of which the ordinary geographical space is but a rather insignificant instance. This conception is really what has made of Hotelling's (or rather Launhardt's) model one of the most popular instruments of economic analysis.

2   Did Launhardt anticipate Hotelling?

In order to assess the potential status of Launhardt as a forerunner of Hotelling, we shall compare the theories of oligopolistic competition developed, principally in the simple case of a duopoly, by the two authors, first by describing their *models*, then by discussing their respective

approaches to *market areas* and *market power*, finally by examining their analyses of *price equilibria*. We end up by considering their positions with respect to the well known question of *prices versus quantities as strategic variables*.

2.1 The models

In both models a single good is produced, either at the same point or at two distinct locations, and it is marketed in a whole region over which is distributed a population of identical consumers. The distribution is basically assumed to be uniform, although Hotelling mentions the possibility of a "varying density", and Launhardt looks, for the sake of comparison, on the case in which the customers are concentrated at the producer's location. The first significant difference between the two models is that Launhardt considers from the beginning a region in the plane, referring to the case of a road only as a particular case, whereas Hotelling sticks to his linear example of the Main Street, waiting until the end of his paper to allude to a space of higher dimension, as an extension of the basic model.

In Hotelling's model consumers' demand is rigid, the case of an elastic demand being again mentioned in the end as a possible extension. On the contrary, rigid demand is assumed by Launhardt only in particular instances of his model. Otherwise, he uses a linear demand function, derived from utility maximisation, assuming the utility function to be separable with respect to the different goods and (approximately) quadratic.

Finally, technology is linear in both models, more precisely, unit production and transportation costs are constant. But Hotelling adds a symmetry assumption: the good can be produced at the same cost by both firms (in fact for free, without loss of generality, because of demand rigidity) and it can be transported also at the same rate, independently of the identity of its producer. By contrast, Launhardt accepts the possibility of two asymmetries, with respect to production costs and to transportation rates. The former corresponds to a rather insignificant generalisation, but the latter is quite important. By distinguishing two *qualities*, the "better" product being the lighter and hence the easier to transport, Launhardt introduces *vertical differentiation* in his model, in addition to the *horizontal differentiation* implied by distinct producers' locations and consumers' spreading out in the space (see Dos Santos Ferreira and Thisse [1996]).

To conclude on this first point of our comparison, it is clear that Hotelling's model is nothing but a particular case of the more general and much earlier Launhardt's model.

2.2 Market areas

Because of product differentiation, a firm is not necessarily deprived of all its potential customers as soon as one of its competitors undercuts its price, as argued by Bertrand. The reason is that, instead of a single (perfect) market for the products of the different firms, product differentiation generates a system of related markets, possibly one for each firm, as in the framework of monopolistic competition. In the spatial model, each market appears as the *market area* for one firm's product. In the case of a uniform distribution of consumers, this market area is an index of the mass of customers of the firm, given the prices set by its competitors.

If there is only one firm (§ 27 in Launhardt [1885]), its market area is the intersection of the region occupied by the consumers and the disc, centred on the producer's location, over which the delivered price, including the transportation cost, does not exceed the consumer's reservation price (taken as infinite in Hotelling's model). If there are two firms located at points A and B (same notation in § 28 of Launhardt [1885] and in Hotelling [1929]), the market area of each firm is in general restricted by the presence of its competitor. The two areas are separated by a boundary corresponding to the equality of the delivered prices:

$$p' + f'x = p'' + f''y$$

(eq. (91) of Launhardt [1885]), where p' and p'' are the two mill prices, f' and f'' are the two freight rates, and x and y are the two distances of the boundary point from A and B, respectively, or

$$p_1 + cx = p_2 + cy$$

(eq. (5) of Hotelling [1929]), with essentially the same notations, except for c, the common transportation rate. It should nevertheless be noticed that, with equal transportation rates, the intersection of the two market areas may in fact be a non-degenerate region of the relevant space, instead of a common boundary. This happens whenever there is equality between the delivered prices of both products at some producer's location.

The boundary separating the two market areas is clearly just a point in Hotelling's Main Street but, as the author indicates, it becomes an arc of hyperbola when the customers are spread out on the plane. Launhardt makes the same statement, which however applies only to the particular case of symmetric transportation costs (and different mill prices, the hyperbola degenerating otherwise into a straight line). By contrast, in the case of asymmetric freight rates, the boundary is an ellipse surrounding the point where the worse (or heavier) good is produced.

To conclude on this point, we see that, as a natural consequence of the greater generality of his model, Launhardt's characterisation of market areas is more complete than that of Hotelling.

2.3 Market power

The market segmentation into distinct market areas for the products of the different firms is a source of market power, viewed here as the extent to which a firm resists the adverse effects of a price cut by one of its competitors. Launhardt aptly uses his concept of market area to describe these effects in the duopoly context, in particular when differentiation is vertical only, that is, when the locations A and B of the two firms coincide (§ 29). Take as an example the consequences of the behaviour of the firm producing the better good, the one whose market area lies outside. When it lowers its price, this firm widens its market area, both *outwards*, by bringing to the market potential customers, heretofore kept away by delivered prices exceeding their reservation price, and *inwards*, by diverting from its competitor part of its clientele. The former is the sole effect which, together with the possible increase in individual demand, is taken into account in Cournot's homogeneous duopoly, with instantaneous price alignment by the assailed competitors and hence impossibility of undercutting. On the other hand, the latter is the sole effect that is stressed by Bertrand, in his objection against Cournot. Both are carefully distinguished and put into perspective by Launhardt. By contrast, as a consequence of his assumption of demand rigidity, Hotelling can only contemplate the clientele effect, again a limitation of his analysis.

Using the equation of the boundary between the two market areas, it is easy to establish that its intersection with the segment, of length ℓ, linking the two locations A and B is a point at a distance x from A, satisfying:

$$x = \frac{p'' - p' + f''\ell}{f' + f''}$$

(eq. (93) of Launhardt [1885]). Hence, the lower the freight rates the more severe the contraction undergone by the market area of the victim of a competitive price cut, expressed by a decrease (increase) of x, if it is $p''(p')$ that is lowered. If we consider the intersection of the boundary of the two market areas with the continuation of the line beyond, say, location B, a situation that is excluded in Hotelling's symmetric model, we similarly obtain a point at a distance y from B, such that:

$$y = \frac{p'' - p' - f'\ell}{f' - f''}$$

(eq. (95) of Launhardt [1885]). We may consider two cases: if the first (second) firm, located at A (B), is the one producing the worse good, that is, if $f' > f''(f' < f'')$, a decrease in y, caused by the lowering of $p''(p')$ means a contraction of the market area of the first (second) firm. We see that a high level and also a high difference of transportation costs make firms' market areas less sensitive to price cuts by their competitors. In other words, market power is enhanced through both horizontal and vertical product differentiation.

Launhardt comments on such effects of bad transport conditions: "the improvement of the means of transport is dangerous to the more expensive good, [since] it loses the most effective of all protective tariffs, the protection through bad roads". As a seeming echo, we find Hotelling's well-known remark that the "merchants would do well, instead of organising improvement clubs and booster associations to better the roads, to make transportation as difficult as possible", and that "still better would be their situation if they could obtain a protective tariff to hinder the transportation of their commodity between them".

The same conclusion applies on this point as on the preceding one: by using a more general model, Launhardt succeeds to give a more satisfactory account than Hotelling of the impact of product differentiation on market power.

2.4   Price equilibria

Hotelling's analysis of oligopolistic competition is based on the choice of prices as the relevant strategic variables, according to the approach which we are used to associate with the names of Bertrand and Edgeworth: "each competitor adjusts his price so that, with the existing value of the other price, his own profit will be a maximum". He thus gets to a concept of strategic equilibrium in prices, which he opposes, very much like Cournot, to the unstable collusive solution and, thanks to the stabilising effect of product differentiation, to Bertrand's process of indefinite price cutting. He then calculates this equilibrium, for any locations of the two firms in Main Street. We know, since the well-known paper by d'Aspremont, Gabszewicz and Thisse [1979], that Hotelling's price equilibrium does not exist for too close (but distinct) locations of the two firms. When horizontal product differentiation is too weak, there subsists an incentive for one of the two firms to undercut its competitors' price, and to destroy Hotelling's equilibrium, based on local profit maximisation only. Bertrand's objection against Cournot is not averted.

Launhardt has, at first sight, fundamentally the same approach to oligopolistic competition. The aggressive character of this kind of

competition as opposed to perfect competition is stressed through the opposition between the terms *Wettbewerb*, denoting the latter, and *Wettkampf* (where *Kampf* means *struggle*), denoting the former, with the corresponding terms for equilibrium: *Gleichgewicht* (with the same etymological sense as *equilibrium*) in perfect competition and *Friedensgrundlage* (meaning *peace basis*) in oligopolistic competition. Contrary to Hotelling, whose model is only a particular case of his own, Launhardt does not try to cover all the possible configurations of locations and transportation rates, when he calculates the equilibria. In two instances of his model, he assumes, like Hotelling, both a one-dimensional space and demand rigidity, and characterises the "peace basis" by a couple of unit profits, that is, of mark-ups on unit production costs. As these costs are assumed constant, this is however tantamount to characterising equilibrium in terms of mill prices. The definition itself is formulated in approximately the same terms as by Hotelling: "each one of the two entrepreneurs would lose some of his total profit, by going above or below the [equilibrium] mark-up, while his competitor keeps his price constant".

The first case leading to the determination of an equilibrium almost coincides with the extreme case of *maximum differentiation* in Hotelling's model. Consumers are supposed to be uniformly distributed along the line connecting the locations of the two firms. However, as we already know, asymmetry is not assumed. Equilibrium mark-ups are given by equations (99) in Launhardt [1885]:

$$g' = \frac{1}{3}(p'' - p') + \frac{1}{3}(2f'' + f')\ell$$

$$g'' = \frac{1}{3}(p' - p'') + \frac{1}{3}(2f' + f'')\ell,$$

where g' and g'' are the two firms' mark-ups, and p' and p'' now denote the corresponding production *costs*. In the symmetric case, we get: $g' = g'' = f\ell$, which is precisely the result given by equations (10) in Hotelling [1929], when the two firms are located at the two extremes of Main Street.

The second instance of price equilibrium under the assumptions of a one-dimensional space and demand rigidity is the opposite case of *minimum horizontal differentiation*: firms are now located at the same end of the line segment occupied by the consumers. Launhardt calculates the equilibrium mark-ups:

$$g' = \frac{1}{3}(p'' - p') + \frac{1}{3}(f' - f'')\ell$$

$$g'' = \frac{1}{3}(p' - p'') + \frac{2}{3}(f' - f'')\ell,$$

where the worse good is produced by the first firm, that is, $f' > f''$. A quick comparison with the preceding equilibrium shows that, as horizontal differentiation vanishes, unit profits both shrink by $f''\ell$, a comparison which gives rise to Launhardt's remark that "domestic competition is far more dangerous to the entrepreneur than foreign competition".

To conclude, Launhardt's analysis of price equilibria, although fragmentary is again more complete than that of Hotelling, except for the calculation of equilibria with arbitrary locations on the line. On the last point, we must however recall that Hotelling, by disregarding the possibility of an effective incentive to undercutting, did not manage to carry out his calculation quite correctly.

2.5 Quantities as strategic variables

In a somewhat obscure passage at the end of his paper, Hotelling comments on "the difficulty as to whether prices or quantities should be used as independent variables", a "question [that] has troubled many readers of Cournot". He points out that, contrary to Cournot's duopoly example and also his own approach with rigid demand, assuming elastic demand would make equivalent the use of prices or quantities as independent variables. This is certainly not so, unless the firms are confronted with demand functions which are independent of each other, that is, unless there is a simple juxtaposition of monopolies.

In Cournot's homogeneous duopoly, each producer behaves as a monopolist facing the residual demand, calculated at a *uniform* price and given the competitor's choice of output. As a monopolist, he chooses concomitantly the quantity to produce and the *common* price. How the price decisions are coordinated is not clear. In other words, Cournot does not describe what is going on behind his "convenient" use of the inverse demand function to determine the common price. But he quite explicitly states that each producer is "able to accomplish [his quantity choice] by properly adjusting his price". The difficulty is not in determining whether prices or quantities should be used as choice variables, but in determining which choices are conjectured to be independent and which to be adjusted in response to one's own choices.

Without having read Cournot at the time he wrote his book, Launhardt seems to sustain fundamentally the same position on oligopolistic competition in the paragraph where he discusses the case of an equal location for both firms (§ 29). He supposes that a price cut by one producer, designed to obtain an expansion of his sales, will be followed by a comparable price cut by his competitor, who will however accept a contraction of his market area, in so far as the increase of individual demand induced by his price cut will allow him to sell, over a smaller

area, an "unchanged quantity of good". And, before giving his character-
isation of an equilibrium, he concludes that "in the competition between
the two entrepreneurs, each one depends upon the price that his com-
petitor must set, in order to find an outlet for the quantity of good he
has produced".

The "peace basis" for the "competitive struggle" explicitly involves
prices *and* output quantities as strategic variables, but is unfortunately
unclear concerning duopolists' conjectures on their competitors' reac-
tions. When choosing a quantity and the corresponding price, does each
producer admit that the other firm will keep its output constant and ad-
just its price accordingly, as the preceding discussion suggests? If this is
so, how can we reconcile such quantity equilibrium with the former price
equilibrium, in which price decisions were taken as given, with market
areas, and hence quantities, adjusting accordingly? Launhardt does not
answer to these questions, but he does calculate an equilibrium in the
case of an oligopoly with N firms located at the same point on the plane
and selling *at the market price* products of equal quality, that is, products
which are transportable at the same cost. He assumes, in other words,
that there is neither horizontal nor vertical differentiation. Thus, we
are back to Cournot homogeneous oligopoly, and it appears that Laun-
hardt's solution precisely coincides with Cournot's. A slight difference of
approach lies in the calculation itself, which does not resort to Cournot's
"convenient" inversion of the demand function, but goes straight to the
maximisation of a monopoly profit, taking into account the residual de-
mand. Launhardt's analysis ends up with a sketchy examination of the
questions of entry (in an asymmetric oligopoly) and optimal scale.

The conclusion on this point is that neither Hotelling nor Laun-
hardt develop a satisfactory analysis of the relation between price and
quantity competition in oligopoly. A remark of the former incorrectly
suggests that the difference between the two forms of competition is im-
material except in the extreme cases of a homogeneous oligopoly or, in
presence of product differentiation, of demand rigidity. The discussion
of the latter puzzlingly suggests that producers take their competitors'
quantities as given and prices as accordingly adjustable, whereas he
defines and determines equilibria with each firm taking as given the
other's price, when demand is rigid. But Launhardt's analysis, redis-
covering Cournot in a spatial context, actually when there is no product
differentiation, is again more complete.

A general conclusion emerges: Launhardt should not be looked
on as a forerunner of Hotelling. From the point of view of the preced-
ing discussion, involving their respective models, market analyses and
equilibrium concepts and calculations, Hotelling is much more a late
(unconscious) epigone of Launhardt. But is he just that?

3   Did Hotelling transcend Launhardt?

We shall concentrate on two significant contributions of Hotelling, not foreshadowed by Launhardt, which concern the strategic choice of locations and, more fundamentally, the conception of an abstract space.

3.1   The strategic choice of locations

Launhardt gave a penetrating though fragmentary account of the influence of producers' locations and transportation rates on the conditions under which they compete strategically. He discussed in particular the effects on market power of an improvement in the means of transport. However, he did not explicitly consider the strategic choice of either location or transport technology, that is, of either product variety or product quality.

Hotelling did. In the second part of his paper, he supposes, "as a further problem", that " A's location has been fixed but that B is free to choose his place of business". And he makes firm B choose this place in view of maximising its profit function, as resulting from the already calculated price equilibrium. In other words, he comes close to a new concept of equilibrium: the *subgame perfect equilibrium* of a two-stage game, in which locations are first chosen, and then prices. Actually, he does neither explicitly define nor try to determine such equilibrium, and he does not go beyond the optimal choice of location by one firm, given the other's location.

But this is sufficient for Hotelling to point out the "gravitation" of one seller towards the other, in spite of "the sharper competition due to proximity", an effect which is "offset by the greater body of buyers with whom he has an advantage". Such "agglomerative tendency" has three consequences, all discussed by Hotelling. First, it endangers the stability of the duopoly, by increasing the incentive to undercutting, which is the stronger the lower the degree of differentiation. Second, by weakening firms' market power, it depresses their profits, a seemingly paradoxical result, which is however not necessarily different from Cournot's observation that firms would do better by colluding than by engaging in a non-cooperative behaviour. Third, it happens to be inefficient, by compelling some customers to have their goods transported from a greater distance than necessary. Hotelling shows indeed that minimisation of the social cost of transportation requires the two firms to occupy the first and the third quartiles of Main Street, instead of "crowding together as closely as possible", thus "contributing an argument to the socialist side" in the question of capitalism versus socialism.

The first consequence, concerning the attractiveness of undercutting when differentiation is weak, is crucial. It provides an explanation

of Hotelling's reluctance to determine a perfect equilibrium in locations and prices. When both firms are located at the same point, "we have Cournot's case, and Bertrand's objection applies". As a matter of fact, the difficulty is still greater, since the price equilibrium is destroyed before the firms get into contact, as we know from d'Aspremont, Gabszewicz and Thisse [1979], who also show that Hotelling's conjecture about the tendency to minimum differentiation is not robust, since it does not apply to the case of quadratic transportation costs, in which the price equilibrium exists for any locations. But, perhaps more significantly, Hotelling's conjecture can be vindicated in the case of linear transportation costs, if we admit that the weakening of differentiation is compensated by some tacit coordination device, making undercutting impracticable (see *e.g.* d'Aspremont, Dos Santos Ferreira and Gérard-Varet [1995]). The conjecture that the competitor will react to a price cut, by adjusting his price so as to keep constant the saleable quantity of his output, works as such a device, precisely the one imagined by Cournot — and Launhardt.

In any case, Hotelling's strategic approach to the determination of product variety was a significant step beyond Launhardt. That step had yet to be followed, in the more general model of the latter, by a similar strategic approach to the determination of product quality (see Dos Santos Ferreira and Thisse [1996]).

3.2   The conception of an abstract space

The most important novel contribution of Hotelling is however his perception of the possibility of extending the spatial analysis of strategic competition, with product differentiation, from a geographical to a much more abstract space. Again in the second part of his paper, he stresses that geographic space has only been used "for illustration":

> "Instead of sellers of an identical commodity separated geographically we might have considered two competing cider merchants side by side, one selling a sweeter liquid than the other. If the consumers of cider be thought of as varying by infinitesimal degrees in the sourness they desire, we have much the same situation as before. The measure of sourness now replaces distance, while instead of transportation costs there are the degrees of disutility resulting from a consumer getting cider more or less different from what he wants" (Hotelling [1929/1953^R], p. 481).

Thus, space becomes an abstract *characteristics space*, here with one single characteristic, sourness, but ready to be extended in the following page to a multi-dimensional "symbolic space". Commodities are bundles of characteristics, represented in this symbolic space by points which may also be viewed as their producers' locations. Consumers' util-

ity is now directly defined on the characteristics space, although in the simple form of a distance with respect to consumers' addresses, rather than on the commodity space. Hotelling's approach is clearly the starting point of that of Lancaster [1966].

Hotelling conjectures that the so-called principle of minimum differentiation applies to this extended framework, so that "buyers are confronted everywhere with an excessive sameness". As a matter of fact, the same tendency to standardisation "appears in the most diverse fields of competitive activity", and not only in economic life: "Methodist and Presbyterian churches are too much alike". The most famous application suggested by Hotelling of the principle of minimum differentiation, and possibly the most robust, concerns politics, with the example of the remarkable convergence of the Republican and Democratic programmes, imposed by the competition for votes between the two parties.

To conclude, we may observe that this impressive and probably relevant extension of the analysis of competition outside the field of economic activity is designed by Hotelling through his conception of an abstract space, that leaves far behind the geographical space which concerned Launhardt. With Hotelling, we move in fact away from spatial economics as such and enter into the spatial approach of strategic competition.

4 Conclusion

We have by now plainly answered to the two questions we have asked in this paper.

Launhardt did not anticipate Hotelling, because in many respects he actually carried out the same research programme farther than his successor. His model contained Hotelling's as a particular case. This greater generality was far from trivial, since admitting asymmetric transportation costs opened up the way to the consideration of vertical differentiation, in addition to the horizontal differentiation described by Hotelling. Also, as a result of the greater generality, Launhardt's analysis of markets and strategic competition was often finer and more complete.

However, Hotelling did transcend Launhardt, by endogenising locations, explicitly considered as the result of strategic choices, and above all by adopting an abstract approach to space as a device to model attitudes towards differentiation. The transfiguration of geographical space into a generalised characteristics space, thus achieved by Hotelling, was a decisive step in rescuing the spatial model from its original specialised assignment, and turning it into one of the most versatile tools in the theory of imperfect competition.

REFERENCES

D'ASPREMONT, C., J.J. GABSZEWICZ & J.-F. THISSE, [1979], On Hotelling's stability in competition, *Econometrica*, 47(5), pp. 1145–1150.

D'ASPREMONT, C., R. DOS SANTOS FERREIRA & L.-A. GÉRARD-VARET, [1995], Concurrence et coordination dans le modèle de Hotelling, *Revue d'Economie Industrielle*, numéro exceptionnel: "Economie industrielle: développements récents", pp. 287–303.

BERTRAND, J. [1883], [Review of] "Théorie mathématique de la richesse sociale" par Léon Walras, professeur d'économie politique à l'académie de Lausanne, 1883; "Recherches sur les principes mathématiques de la théorie des richesses" par Augustin Cournot, Paris, 1838, *Journal des Savants*, pp. 499-508. English translation by J. W. Friedman, *in* A. F. Daughety (Ed.) [1988], *Cournot oligopoly: characterization and applications*, Cambridge, Cambridge University Press, ch.2.

BLAUG, M. [1985], *Economic Theory in Retrospect*, 4th edition, Cambridge, Cambridge University Press.

COURNOT, A. [1838], *Recherches sur les principes mathématiques de la théorie des richesses*, Hachette, Paris. English translation [1897^R], *Researches into the mathematical principles of the theory of wealth*, New York, Macmillan.

DOS SANTOS FERREIRA, R. and J.-F. THISSE [1996], Horizontal and vertical differentiation: The Launhardt model, *International Journal of Industrial Organization*, 14(4), pp. 485–506.

HOTELLING, H. [1929], Stability in competition, *Economic Journal*, 39, pp. 41–57. Reproduced [1953^R] *in* G.J. Stigler & K.E. Boulding (Eds.) [1953], *Readings in Price Theory*, London, George Allen & Unwin, ch. 23.

LANCASTER, K. [1966], A new approach to consumer theory, *Journal of Political Economy*, 74(2), pp. 132–157.

LAUNHARDT W. [1885], *Mathematische Begründung der Volkswirtschaftslehre*, Leipzig, B.G. Teubner. Reprint with a foreword by E. Schneider [1963^R], Aalen, Scientia. English translation [1993^R], *Principles of mathematical economics*, Gloucester, E. Elgar.

NIEHANS, J. [1987], Launhardt, Carl Friedrich Wilhelm (1832–1918), *in* J. Eatwell, M. Milgate & P. Newman (Eds.), *The New Palgrave: A Dictionary of Economics*, London, Macmillan.

PINTO, J.V. [1977], Launhardt and location theory: Rediscovery of a neglected book, *Journal of Regional Science*, 17(1), pp. 17–29.

SELTEN, R. [1979], Oligopoltheorie, *in* M.J. Beckmann, G. Menges & R. Selten (eds.), *Handwörterbuch der Mathematischen Wirtschaftswissenschaften*, 1 (*Wirtschaftstheorie*), Wiesbaden, Gabler.

SCHUMPETER J.A. [1954], *History of Economic Analysis*, London, George Allen & Unwin.