Trigger values for (non-)residential structures and equipment investment

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1 Introduction

In many countries (non-)residential investment is a substantial part of national investment. This is shown in table 1 where national investment is disaggregated into structures and equipment (1). Structures comprise the residential or "housing" investment and non-residential or "other buildings" investment. Equipment is investment in all other projects.

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The average investment in structures was 58% in the US during 1960-1993. In Canada, West-Germany and the UK these shares were 73%, 64% and 64% respectively. So in all these countries investment in structures was higher than equipment investment.

Several studies focus on structures projects in particular, see for example Mayer [1960], Kydland and Prescott [1982], Majd and Pindyck [1987], Altug [1989] and Peeters [1995]. From these studies it follows that structures differ from equipment in the important respect that they need time to be built. Structures are necessarily constructed in consecutive stages whereas equipment like machinery, computers, furniture

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(1) Further details are given in section 4.1.
etc. is often delivered without long delays. Structures have thus longer investment gestation lags in comparison with equipment.

In this study the impact of investment decision rules for time-to-build (TTB, for short) projects are compared with those for no-TTB projects. For both types of projects the main focus is on the impact of uncertainty concerning the future returns, according to the “option” theory for real investment decisions of Dixit and Pindyck [1994](2).

Dixit and Pindyck [1994] emphasize(3) that an investment decision is influenced by the expected volatility of future returns since uncertainty entails a high expected volatility and consequently an incentive to delay investment decisions. Waiting to invest implies that the option to invest is not exercised.

This theory of Dixit and Pindyck [1994] is important in several respects. As the optimal moment “to trigger” the investment is derived under uncertainty, the decision to invest is described as a decision between investing and delaying the investment decision instead of investing and not investing. The investment equation derived under the option theory differs therefore from the neoclassical investment equation. In addition to the first moments of the variables that determine (expected) returns, an investment decision depends on second moments. In estimating investment equations thus not only the levels of revenues, prices etc. are to be considered as done in studies like Jorgenson [1963] or q-studies, but also statistics on the dispersion.

The main aim of this study is to investigate the effects of TTB and uncertainty on investment decisions. Investment equations are derived for TTB projects in a similar way as the derivations for no-TTB projects in Dixit and Pindyck [1994]. By comparing no-TTB and TTB projects it is shown that the effect of both TTB and uncertainty renders trigger values that are disproportionally higher than trigger values for projects with rather certain returns or projects with no-TTB. The main message of Dixit and Pindyck [1994] and the main message of Majd and Pindyck [1987], being that neglecting uncertainty is “wrong” and neglecting TTB is “wrong”, respectively, is therefore emphatically corroborated for TTB-projects under uncertainty. Another contribution of this study is the empirical testing of the importance of uncertainty. Aggregate investment equations with a measurement for the “uncertainty” as an explanatory variable are estimated with time series on (non-)residential structures

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(2) See also earlier work, for example Dixit [1989] and Dixit [1991], where the main principles of triggering decisions and thus determining optimal decision moments are outlined and the link with financial investment decisions is made.

(3) See for instance also Bernanke [1983] or Dixit [1992].
(TTB projects) and equipment (no-TTB projects) from the US, Canada, West-Germany and the UK.

The outline is as follows. Section 2 derives investment rules for TTB-projects under the option theory. Section 3 discusses aggregate investment rules. Section 4 presents some estimation results. Section 5 concludes.

2 Single investment projects

In this section Net Present Value ($NPV$) rules for a single investment project are discussed. In the first subsection $NPV$ rules for a TTB project are derived according to the option theory (see Dixit [1992], pp. 110–116). In the second subsection the triggers under TTB and no-TTB are compared. In the third subsection the case of negative waiting values is considered. The last subsection summarizes.

2.1 The effect of waiting under time-to-build

If an investment project needs time to be built, a construction period exists during which money is spent to build the project. During this period no returns are obtained since the unfinished project is not useful. Let us assume that the length of the construction period equals $J$ and that the scheme of investment during construction is a uniform distribution. Then, in each period $1/J$ of the total investment sum, denoted $Q$, is invested. Returns from the project are obtained after completion of the construction and equal $R$ in each period. Under these assumptions, the $NPV$ of the project under TTB is

$$NPV_{ttb} = -\sum_{t=0}^{J-1} \frac{Q/J}{(1 + \rho)^t} + \sum_{t=J}^{\infty} \frac{R}{(1 + \rho)^t} = -r_1 \frac{Q}{J} + \frac{R}{r_2}, \quad (1)$$

where

$$r_1 = \frac{(1 + \rho)^J - 1}{(1 + \rho)^J - (1 + \rho)^{J-1}}$$

$$r_2 = \rho(1 + \rho)^{J-1}.$$

2.1.1 No TTB

If $J = 1$, $NPV_{ttb}$ equals the $NPV$ of a project without TTB. In this case $r_1 = 1$ and $r_2 = \rho$. From the $NPV$ theory follows that investment should take place if $NPV \geq 0$. The value of the investment without
TTB is thus

\[
\begin{align*}
0 & \quad \text{if } R \leq M_{\text{no-ttb}} \\
-\frac{Q + \frac{R}{\rho}}{\rho} & \quad \text{if } R \geq M_{\text{no-ttb}}, \quad M_{\text{no-ttb}} \equiv \rho Q.
\end{align*}
\]

(2)

An investor is indifferent between investing and not investing if \( R = \rho Q \). This amount of revenues is called the "Marshallian investment trigger", and denoted \( M_{\text{no-ttb}} \) (following Dixit [1992]). An investor thus "pulls the trigger" if \( R \geq M_{\text{no-ttb}} \) and does not pull the trigger if \( R \leq M_{\text{no-ttb}} \).

If the investor in this example can delay the decision to invest, the decision rule alters. In this case the investor does not have a value of zero when the trigger is not pulled. He has a (fictive) option to invest in the future and this option has a value. The decision is thus not whether to invest or not at this moment, but whether to invest now or to wait.

In many cases waiting will have a positive value because information arrives over time. For instance, consider an investment in a production input like a new assembly line that is needed to make a (final) product. More information on sales possibilities of the final product and thus on (potential) returns, can be gathered when time passes. The costs of waiting to invest in the assembly line are thus the (additional) returns that could have been obtained if the project would have been started. The benefit of waiting is the obtained information. More downside uncertainty(4), like a larger expected fall in demand for the final product, makes waiting to invest in the assembly line more worthwhile. Projects with high downside uncertainty can even abstain potential investors from investing. In other cases there can be a moment where waiting does no longer have a value that exceeds the returns obtainable when investment takes place. At this moment the investment will be triggered.

The amount of returns where the trigger is pulled will be called trigger value \( H_{\text{no-ttb}} \). If the returns \( R \) exceed trigger value \( H_{\text{no-ttb}} \), the investment takes place. Elsewise, the investment will not take place.

An investor’s decision rule with the possibility of waiting, (see Dixit [1992] or Dixit and Pindyck [1994]) is

(4) Bernanke [1983] argues that only downside uncertainty matters to investors since there is no value of waiting when investing is the best decision. Upside uncertainty thus not influences the decision to wait. This is referred to as "the bad news principle".
\[
\begin{cases}
BR^\beta & \text{if } R \leq H_{\text{no-ttb}} \\
-Q + \frac{R}{\rho} & \text{if } R \geq H_{\text{no-ttb}}
\end{cases}
\] 

(3)

where

\[H_{\text{no-ttb}} = \frac{\beta}{\beta - 1} \rho Q\]

and

\[\beta = \frac{1}{2} \left[ 1 + \sqrt{1 + \frac{8\rho}{\sigma^2}} \right] > 1 \quad \text{and} \quad B = \frac{(\beta - 1)^{\beta - 1}}{\beta^\beta \rho^{\beta - 1}} > 0.\]

If \( R \geq H_{\text{no-ttb}} \), investment takes place. In this case the value of the project is positive and equal to the value under the \( NPV \)-rule (see (2)). If \( R < H_{\text{no-ttb}} \), investment does not take place. In this case the project has a positive waiting value \( BR^\beta \), being the waiting or "option" value of the project. It can be shown that the \( NPV \)-line and the \( BR^\beta \)-curve are tangential where \( H_{\text{no-ttb}} \) is the tangential point. From this the expressions for \( H_{\text{no-ttb}} \) and \( B \) in (3) are derived. The expression for \( \beta \) follows from the dynamic programming problem\(^{(5)}\).

It turns out that \( \beta > 1 \) by which \( \beta/(\beta - 1) > 1 \). The \( H_{\text{no-ttb}} \)-trigger in (3) thus always exceeds the \( M_{\text{no-ttb}} \)-trigger. In this example this is intuitively evident since investment will never take place at lower revenue levels than in case of no delaying possibilities. Counterarguments for this reasoning will be discussed in section 2.3.

\( \sigma^2 \) represents the volatility of the expected returns \( R \). \( \beta \) depends negatively on \( \sigma^2 \). This implication is very important. If \( \sigma^2 \) increases, the returns on the project are more uncertain. And if \( \sigma^2 \to \infty \) it follows from (3) that \( \beta \to 1 \). The trigger point thus increases. In the opposite case, i.e. when \( \sigma^2 \to 0 \), \( \beta \to \infty \) and the trigger point \( H \to M = \rho Q \). Less (more) uncertainty thus induces a lower (higher) trigger value by which the probability that investment takes place increases (decreases).

### 2.1.2 TTB

In case of time-to-build, i.e. \( J > 1 \) in (1), the initial investment \( Q \) is lower under TTB than under no-TTB since investment takes place stagewise during construction. During the first \( J - 1 \) periods no returns

\(^{(5)}\) This option value is the solution of a dynamic programming problem, being the expansion of the Bellman equation of expected returns according to Ito's lemma, in addition to some boundary conditions (see Dixit and Pindyck [1994], pp. 140-142), the appendix of Dixit [1992] or appendices A-B here.
can be obtained. From \( NPV \)-theory follows that investment takes place if \( NPV_{ttb} > 0 \). The value of the investment is thus

\[
\begin{cases}
0 & \text{if } R \leq M_{ttb} \\
-r_1 \frac{Q}{J} + \frac{R}{r_2} & \text{if } R \geq M_{ttb}
\end{cases}
\]

\[ M_{ttb} \equiv r_1 r_2 \frac{Q}{J} = \left[ (1 + \rho)J - 1 \right] \frac{Q}{J}. \]  

(4)

and as shown in appendix A, similar to the no-TTB case in (3), it follows that the decision rule for an investor who has the possibility to wait is

\[
\begin{cases}
B^* R^{\beta^*} & \text{if } R \leq H_{ttb} \\
-r_1 \frac{Q}{J} + \frac{R}{r_2} & \text{if } R \geq H_{ttb}
\end{cases}
\]

(5)

where

\[ H_{ttb} = \frac{\beta}{\beta - 1} r_1 r_2 \frac{Q}{J} \]

and

\[ \beta^* = \frac{1}{2} \left[ 1 + \sqrt{1 + \frac{8r_2^2}{\sigma^2}} \right] > 1, \quad B^* = \frac{(\beta^* - 1)J}{(\beta^*)^{\beta^*}} = 1 > 0. \]

2.1.3 Illustration

![Figure 1: The value of waiting and investing](image)

Both the no-TTB and TTB case (with \( J = 5 \)) are illustrated in figure 1. The straight lines are the \( NPV \)'s. The intersection of these
lines with the x-axis gives the Marshallian trigger $M$ and $M_{ttb}$. The curved lines are the values of waiting. The intersection of these lines with the straight lines are the trigger values $H$ and $H_{ttb}$. The no-TTB lines equals figure 2 of Dixit [1992]. As $\beta/(\beta - 1) > 1$, the figure shows that $H_{(no-\text{ttb})} > M_{(no-\text{ttb})}$. It further follows that $M_{ttb} > M_{no-\text{ttb}}$ and $H_{ttb} > H_{no-\text{ttb}}$, inequalities that will be further investigated now.

2.2 A comparison between the triggers under TTB and no-TTB

Majd and Pindyck [1987] and Pindyck [1993] pay attention to TTB projects. In their views, projects that need series of expenditures that cannot exceed a maximum rate (being $1/J$ in the example here) are like a compound option: each unit of investment buys an option to the next unit. Traditional discounted cash flow criteria, like (1) with $J = 1$, lead to incorrect investment rules. The incorrectness of these rules is thus not only due to the negligence of uncertainty, as Dixit [1992] and Dixit and Pindyck [1994] emphasize, but also to the negligence of the existence of a multi-period TTB. One of their most important findings is that uncertainty is likely to have a depressive effect on the level of investment, an effect which is likely to be magnified when there is time to build. This finding is investigated in more detail in our discrete time framework here.

In order to investigate the separate effects of uncertainty and TTB, the $M$—as well as the $H$-triggers under TTB and no-TTB will be compared. The difference between the $M$-triggers, as follow from (2) and (4), is

$$DM \equiv M_{ttb} - M_{no-\text{ttb}} = (\tau_{ttb} - 1)M_{no-\text{ttb}} \quad (6)$$

where

$$M_{ttb} \equiv \tau_{ttb}M_{no-\text{ttb}}$$

$$M_{no-\text{ttb}} \equiv \rho Q$$

$$\tau_{ttb} \equiv \frac{r_1r_2}{\rho J}$$

and the difference between the $H$-triggers, as follows from (3) and (5), is

$$DH \equiv H_{ttb} - H_{no-\text{ttb}} = \tau_\text{unc}(\tau_{ttb} - 1)M_{no-\text{ttb}} \quad (7)$$

where

$$H_{ttb} \equiv \tau_\text{unc}\tau_{ttb}M_{no-\text{ttb}}$$

$$H_{no-\text{ttb}} \equiv \tau_\text{unc}M_{no-\text{ttb}}$$

$$\tau_\text{unc} \equiv \frac{\beta}{\beta - 1}.$$
Trivially, it follows that $DM = DH = 0$ if $J = 1$ and $DM = DH$ if $\sigma^2 = 0$. If both $J > 1$ and $\sigma^2 > 0$, the four triggers $M_{nottb}$, $M_{ttb}$, $H_{nottb}$, $H_{ttb}$ differ.

The difference due to TTB only, see (6), is always positive since $\tau_{ttb} > 1$. If $J$ increases, $DM$ increases. If $M_{nottb}$ instead of $M_{ttb}$ is calculated in case of TTB projects, the error that is made is thus "worse" the larger the TTB is.

The difference due to TTB and uncertainty, see (7), is also positive since $\tau_{unc} > 1$. If $J$ increases, $DH$ increases. An important finding is further that an increase in $DH$ always exceeds the increase in $DM$. The statement of Majd and Pindyck [1987], mentioned earlier, is thus corroborated. The TTB increases the depressive effect of uncertainty. But the reverse also holds since uncertainty increases the depressive effect of TTB (6). $DH$ with varying $\sigma^2$ and $J$ is shown in figure 2. At a certain $\sigma^2$, $DH$ increases at an increasing rate in $J$. At a certain $J$, $DH$ increases at an increasing rate in $\sigma^2$.

\[ \text{Figure 2: } H_{ttb} - H_{nottb} \text{ with varying TTB and uncertainty} \]

Dixit and Pindyck [1994] concentrate mainly on the multiplier $\tau_{unc} = [\beta/(\beta - 1)]\rho$, to which they refer as the "hurdle rate" and find

(6) Consider for example the investment in a house, a factory and a machine. Assume that they each have the same expected stream of returns. The house and the factory need a TTB of, let say four periods, whereas the machine can be obtained at once. If TTB increases to (for instance) five periods, the theory here predicts that the trigger value of the factory will be more affected than the trigger value of the house when the returns on the factory are more uncertain than those (i.e. "the utility") of the house. The theory also predicts that the trigger value of the factory will be more affected than the trigger value of the machine, if uncertainty increases.
values of about 1.5 to 3. This implies that the investment projects have hurdle rates that are 1.5 to 3 times higher than predicted by the standard $NPV$ theory. As follows here, if projects need TTB, the actual hurdle rate equals $\tau_{unc}T(Tb) > \tau_{unc}T$ and is thus even more underestimated by using standard $NPV$ rules.

2.3 A negative (or no) value of waiting?

The value of waiting in the previous subsections is assumed to be positive\(^{(7)}\), which suggests that uncertainty always depresses investment. Some studies cast however doubt on the negative sign of the relationship between investment and uncertainty.

An example is Caballero [1991]. His main conclusion obtained with a model including asymmetric adjustment costs is that, besides the irreversibility of the investment, a certain degree of imperfect competition is crucial to obtain this negative relationship. The intuition is as follows. The larger uncertainty is, the more probable are "very good" and "very bad" news. The more imperfect competition then is, the more probable a positive return is and thus investment takes place\(^{(8)}\). In this case the opportunity costs of not investing are relatively high.

Also Majd and Pindyck [1987] pay attention to the possible effect of opportunity costs on the decision to invest in TTB projects. Higher opportunity costs increase the incentive to invest under no-TTB, though, have an ambiguous effect on TTB projects. They increase the incentive to invest but, as they entail high foregone cash flows during the construction period, they decrease investment in turn (see Majd and Pindyck [1987], p. 20).

Bar-Ilan, Sulem and Zanello [1993] also concentrate on TTB projects and, unlike Majd and Pindyck [1987], assume an endogenous opportunity cost. They conclude that the cost of waiting-to-invest is likely to be much higher than the benefit of delaying the investment.

\[ \ldots \] Inaction today precludes the exploitation of profitable opportunities were they to materialize, and thus implies an irreversible loss. The possibility of future "good news" affects today's decisions. \[ \ldots \] Time-to-build thus creates a tension between the desire to undertake projects in spite of current conditions and in expectation of future payoffs, ("the good news principle") and the standard "bad news principle" [\ldots ].

\(^{(7)}\) This follows from figure 1 where $BR^T > 0$.

\(^{(8)}\) If investment is irreversible, investing is less costly than rectifying the investment. The more irreversible the investment is, the more expensive the bad news is in comparison to the good news. Caballero [1991] specifies this with asymmetric adjustment costs.
In their model the relationship uncertainty-investment is more likely to be positive for TTB projects the larger the TTB is.

These studies concentrate on the opportunity costs of not investing or the negative value of waiting, unlike Dixit and Pindyck [1994]. This case is derived in appendix B. In figure 3 the differences of the TTB and the no-TBB triggers are given for different opportunity costs, represented by $W$. The TTB is assumed to be fixed. Like in figure 2, the differences increase if uncertainty increases. In contrast with figure 2, though, the differences can be negative since TTB projects are triggered more quickly than no-TTB projects if opportunity costs are high. This happens in this figure when the opportunity cost $W$ is highest, i.e. $W = 0.6$.

![Figure 3: $H_{ttb} - H_{no-ttb}$ with varying opportunity costs and uncertainty](image)

### 2.4 Summary

The most important results of the option theory can be summarized as follows.

- An investor can have the possibility to delay an investment decision or an "option" to invest at a moment in the future. Exercising the option implies that the option cannot be exercised in the future.

- The value of waiting is assumed to be positive. The trigger value under uncertainty, the "$H$"-trigger for short, then exceeds the $M$(arshallian)-trigger, being the trigger derived from standard $N_{NPV}$ rules.
• In contrast to the $M$-trigger, the $H$-trigger depends on the dispersion of the returns. The higher this dispersion is, the larger is the error made when applying the $M$-triggers (see Dixit (1992)).

• Trigger values increase when uncertainty increases and the multiplier under TTB equals the multiplier under no-TTB. In the terminology of this section: $H_{\text{no- TTB}}/M_{\text{no- TTB}} = H_{\text{ttb}}/M_{\text{ttb}} = \tau_{\text{unc}} > 1$ and $\tau_{\text{unc}} \to \infty$ if $\sigma^2 \to \infty$.

• The trigger values for projects with (un)certain increase when TTB increases. Also these multipliers are equal: $M_{\text{ttb}}/M_{\text{no- TTB}} = H_{\text{ttb}}/H_{\text{no- TTB}} = \tau_{\text{ttb}} > 1$ and $\tau_{\text{ttb}} \to \infty$ if $J \to \infty$.

• The trigger values for a TTB project under uncertainty consists of a TTB effect multiplied by an uncertainty effect and both effects equal or exceed 1. TTB has thus more impact on projects that are faced with more uncertainty and vice versa.

• Results are all derived under the assumption that opportunity costs are zero. If opportunity costs exist, the relationship uncertainty-investment can be positive for (no-)TTB projects. At high opportunity costs, investment in a TTB project can even be triggered more quickly than investment in a no-TTB project.

3 Aggregate investment

A main difference between this section and the previous section is that investment in different projects at one moment in time are considered, and under TTB thus investment in projects of different construction stages. Moreover, the capital stock consists of investment projects that were completed in different periods (i.e. the depreciation is not 100%). In the first subsection a neoclassical intertemporal factor demand model is presented. In the subsequent four subsections trigger values are calculated for projects faced with (no-)TTB and (un-)certainty.

3.1 The factor demand model

An entrepreneur is assumed to maximize the profit stream of the firm over an infinite horizon. The function to be maximized is specified as

$$V_t = \mathcal{E}_t \sum_{h=0}^{\infty} b_{t+h} \Pi_{t+h}.$$  

(8)

$\mathcal{E}_t$ is the rational expectations operator. $b_{t+h}$ represents the discount
factor at period \( t + h \) that equals

\[
b_{t+h} \equiv \prod_{i=0}^{h-1} \frac{1}{1 + \rho_{t+i}}.
\]  

Like in the previous sections, \( \rho_t \) represents the nominal discount rate, but is time-dependent here. \( \Pi_t \) are the profits defined as

\[
\Pi_t \equiv \mathcal{F}(A_t, K_t) - Q_t I_t.
\]  

\( \mathcal{F}(A_t, K_t) \) is the production function, assumed to depend on capital stock \( K_t \) and a technology shock \( A_t \). This shock can also be interpreted as being an index of business conditions (see for instance Bertola and Caballero [1994]). Inada conditions are assumed to be satisfied. Production factors like labour, materials etc. are omitted since the main focus in the model is on the trigger values for capital projects. \( Q_t \) is the real price of the capital good \( K_t \). Like in the previous section, this price is to be paid when the investment takes place. \( I_t \) is total investment.

### 3.2 No uncertainty of returns and no-TTB

The depreciation rate, \( \kappa \), is assumed to be constant. Under no-TTB, investment consists of the investment in net capital \( K_{t+1} - K_t \) and the replacement investment \( \kappa K_t \), i.e.

\[
I_t = K_{t+1} - K_t + \kappa K_t \quad \iff \quad K_{t+1} = (1 - \kappa)K_t + I_t.
\]  

Note that the investment at time \( t \) determines the capital stock at time \( t + 1 \), which is consistent with the assumption in (1) with \( J = 1 \) in that return \( R \) is obtained one period after the investment \( Q \). The capital stock that can be influenced at period \( t \) is \( K_{t+1} \) and not \( K_t \). Substitution of (9)-(11) in (8) and differentiation with respect to \( K_{t+1} \) renders the Euler equation

\[
\frac{\partial \mathcal{F}(A_{t+1}, K_{t+1})}{\partial K_{t+1}} = (1 + \rho_t)Q_t + (\kappa - 1)E_t Q_{t+1}.
\]  

The expression on the right side of the equality sign are the marginal costs. They equal

\[
Q_t [1 + \rho_t + (\kappa - 1)](1 + \text{infl}_{t+1}) \approx Q_t \text{ucc}_t
\]  

where

\[
\text{infl}_{t+1} \equiv \frac{E_t Q_{t+1}}{Q_t} - 1
\]

\[
\text{ucc}_t \equiv \rho_t + \kappa - \text{infl}_{t+1}.
\]
infl_t is the inflation rate. The expression unc_t is the user cost of capital as defined in Jorgenson [1963]. The marginal cost of capital increases (decreases) if the interest rate or the depreciation increases (decreases) or inflation decreases (increases).

According to the Marshallian rule the entrepreneur invests if the value of the project exceeds the investment price, i.e. \( V_t > Q_t \) (see (8)), or if the marginal returns exceed the expected marginal costs, i.e.

\[
\frac{\partial F(A_{t+1}, K_{t+1})}{\partial K_{t+1}} \geq (1 + \rho_t)Q_t + (\kappa - 1)\epsilon_t Q_{t+1}
\]

(see (12)). The trigger value for the (aggregate) investment equals thus the marginal costs, i.e.

\[
\text{Trig}_{t}^{\text{no-unc}, \text{no-ttb}} = (1 + \rho_t)Q_t + (\kappa - 1)\epsilon_t Q_{t+1}.
\]

(14)

If \( \kappa = 0 \), \( Q_t = Q_{t+1} = Q \) and \( \rho_t = \rho \), this trigger equals the Marshallian trigger \( M_{\text{no-ttb}} = \rho Q \) in (2).

3.3 Uncertainty of returns and no-TTB

According to (14) the investor only considers the level of relative investment prices. Even if these prices are very volatile, investment can take place.

As described in the previous section, however, according to the theory of Dixit and Pindyck [1994] uncertainty matters. Taking uncertainty into account, leads us to the \( H \)-trigger as described in (4). The trigger with uncertainty equals

\[
\text{Trig}_{t}^{\text{unc}, \text{no-ttb}} = \epsilon_t \left( \text{Trig}_{t}^{\text{no-unc}, \text{no-ttb}} + \frac{1}{2} \beta_{t+1} \sigma_t^2 Q_t \right).
\]

(15)

The derivation is described in Dixit and Pindyck [1994] (pp. 144–145) for the case of no depreciation, no inflation and a constant discount rate. It follows from \( H \equiv [\beta/(\beta - 1)]\rho Q = (\rho + 1/2\beta \sigma^2)Q \) (see (3) and (43)) and is given for period \( t \) here. More information on the derivation is given in the last part of appendix A. As \( \beta > 1 \) it follows that

\[
\text{Trig}_{t}^{\text{no-unc}, \text{no-ttb}} < \text{Trig}_{t}^{\text{unc}, \text{no-ttb}}.
\]

3.4 No uncertainty of returns and TTB

If there is a multi-period TTB of \( J \) periods \( (J > 1) \), the standard capital accumulation rule (11) does not hold. \( I_t \) consists of investment in projects that were started \( J \) periods ago, \( J - 1 \) periods ago... and
projects that started in (this) period \( t \). Following Kydland and Prescot [1982], the capital accumulation process can be specified as

\[
\begin{align*}
K_t &= (1 - \kappa)K_{t-1} + S_{1,t-1} \\
I_t &= \sum_{j=1}^{J} \delta_j S_{j,t} \\
\sum_{j=1}^{J} \delta_j &= 1 \\
S_{j,t} &= S_{j+1,t-1}, \quad j = 1, 2, \ldots, J.
\end{align*}
\]

(16)

\( S_{j,t} \) represents investment in projects that need \( j \) periods to be completed at the end of period \( t \). \( S_{1,t-1} \) are thus projects that are finished at the end of period \( t \). At the end of period \( t \) \( S_{1,t-1} \) is added to the productive capital stock \( K_t \), see (16). During period \( t \) the total value of initiated projects equals \( S_{J,t} \). During \( t \), \( \delta_j S_{j,t} \) is invested in this new project. At \( t+1 \), \( \delta_{j-1} S_{j-1,t+1} \) is invested in this new project. Etc. The final part, \( \delta_I S_{I+t,J}, \) is invested at \( t + J \). During the construction period there is thus a fixed investment scheme \( \delta_j, \delta_{j-1}, \ldots, \delta_1 \). This scheme is assumed to hold for each project. Gross investment during \( t \), \( I_t \), thus consists of investment in the projects \( S_{J,t}, S_{J-1,t}, \ldots, S_{1,t} \). During \( t \) the part \( \delta_j S_{j,t} \) is invested for each current project.

From the last equality in (16) it follows that initiated projects cannot be changed during construction. If there is no-TTB there is only one current project, \( S_{1,t} \), that equals \( I_t \). In this case (16) reduces to (11) since \( J = 1 \) and \( \delta_1 = 1 \). Notice further that both \( J \) as well as this scheme are assumed to be constant in time.

By substituting the last and the first equation of (16) in the second equation it follows that

\[
I_t = \sum_{j=1}^{J} \delta_j S_{1,t+j-1} = \sum_{j=1}^{J} \delta_j \left( K_{t+j} + (\kappa - 1)K_{t+j-1} \right). \tag{17}
\]

If the entrepreneur in subsection 3.1 is faced with TTB, (8) is maximized subject to (9)-(10) and (17). The decision variable is \( K_{t+j} \) since the initiated project at period \( t \), \( S_{J,t} \) determines \( K_{t+j} \). The Euler equation is

\[
\frac{\partial F(A_{t+j}, K_{t+j})}{\partial K_{t+j}} = \mathcal{E}_t \sum_{j=0}^{J} \phi_{j-t} b_{t+j} Q_{t+j} \tag{18}
\]

where
\[
\phi_0 \equiv (\kappa - 1)\delta_1 \\
\phi_j \equiv (\kappa - 1)\delta_{j+1} + \delta_j, \quad j = 1, 2, \ldots, J - 1 \\
\phi_J \equiv \delta_J
\]

since
\[
\frac{\partial I_{t+j}}{\partial K_{t+j}} = \phi_{j-j}, \quad j = 0, 1, \ldots, J. \tag{19}
\]

Because of the TTB, the "user cost of capital" does not only contain prices one period in the future but also prices until period \(t + J\), see also Altug [1993] and Peeters [1995]. The future capital stock \(K_{t+j}\) is determined at time \(t\) because changes during construction are not allowed. The future prices \(Q_{t+j}\) for \(j > 1\) are unobserved and thus accompanied by the rational expectation sign.

The trigger value for projects under TTB without uncertainty thus equals
\[
Trig_{t}^{\text{no-unc,ttb}} = E_t \sum_{j=0}^{J} \phi_{j-j}Q_{t+j} \prod_{i=j}^{J-1} (1 + \rho_{t+i}). \tag{20}
\]

3.5 Uncertainty of returns and TTB

If uncertainty matters, the \(H\)-trigger under TTB holds according to (3). This trigger equals
\[
Trig_{t}^{\text{unc,ttb}} = E_t \left( Trig_{t}^{\text{no-unc,ttb}} + \frac{1}{2} \beta_{t+J}\sigma_{t+J}^2 Q_t \right). \tag{21}
\]

The value of projects that is decided upon at \(t\) is \(S_{t,t}\) and only renders returns from \(t + J\) onwards. For this reason uncertainty of returns at \(t + J\), \(\sigma_{t+J}^2\), influences the current investment trigger. A heuristic derivation of the trigger is given in appendix A.

4 Empirical evidence on trigger values

The aim of this section is to obtain empirical evidence on the trigger values for no-TTB and TTB projects. In the first subsection the investment data are described. In the second subsection trigger values are presented. In the third subsection investment equations that include dispersion statistics are derived and their estimation results are discussed.
4.1 (Non-)residential structures and equipment data

The data are quarterly time series of the National Accounts (OECD) on national aggregate investment (in volumes) of the US, Canada, West-Germany and the UK. Further time series used are the official discount rates as the interest rate $\rho$, product prices to calculate real investment prices and production series as instrumental variable. These series are from the International Financial Statistics (IMF).

Three types of investment are distinguished, being residential structures, non-residential structures and equipment. Structures are assumed to need time to be built since they are evidently construction projects\(^{(9)}\). Residential structures are mainly consumer goods whereas non-residentials are demanded by the government or producers. Equipment on the other hand, are goods demanded by consumers, producers or the government. In table 1 in the introduction the average shares of the three types of investment are presented. In this table also the sample period for each country is given. For West-Germany and the UK all investment series only exist from 1968 and 1965 onwards, respectively.

To calculate the trigger values that were derived in the previous sections investment prices are needed. They are calculated by dividing the current investment series by the constant investment series. Real investment prices are obtained by deflating these nominal series by the product prices.

All series are seasonally unadjusted and turn out to be trending over the sample period. Most series have a unit root, whereas the other series have nearly a unit root, according to unit root tests. To correct for seasonal patterns, which is in particular difficult in multivariate frameworks, fourth difference growth rates are taken.

4.2 Trigger values for (non-)residential structures and equipment

Dixit and Pindyck [1994] find $H$-trigger values that are two to three times larger than the $M$-triggers. They do not calculate or estimate $\sigma^2$ but fix this measurement of uncertainty as well as the interest rate $\rho$ at some intuitive values. Here, on the contrary, trigger values are calculated for the (no-)TTB projects by using the information on the investment series under investigation, observed interest rates and a calculated $\sigma_i^2$ for each period of the sample period.

\(^{(9)}\)Mayer [1960] and Peeters [1995] give evidence on the TTB for these housing and plant projects whereas Altug [1989] already made the distinction between structures and equipment projects as being TTB and no-TTB projects.
In order to calculate the trigger values $\text{Trig}_{\text{unc}}^{(\text{no-})\text{ttb}}$ in (14), (15), (20) and (21) several assumptions are to be made. First, the observed variable is taken to be the best prediction of the unobserved variables by which the rational expectation operators thus disappear. Second, $J$ is to be determined. Similar as in Mayer [1960] and Peeters [1995] it will be assumed that $J = 4$ for both non-residential as well as residential structures. Third, the investment scheme $\delta_j$ for $j = 1, 2, 3, 4$ is to be determined. The most obvious scheme $\delta_j = 0.25$ will be assumed here. Fourth, the depreciation rate $\kappa$ will be assumed to equal 0.05. By these assumptions $\phi_j$ is (20) are determined.

Finally, a measurement for $\beta_t \sigma_t^2$ is needed. The approximation of Driver and Moreton [1992] is taken here by which a moving variance is used for $\sigma_t^2$. The variable considered is the growth rate of the real price of the investment series under investigation. The window for the average is taken to be two years, being four quarters directly ahead of the observation and four quarters that precede the observation.

In table 2 the correlation between $\sigma_t^2$ and the growth rate of investment is given. $\sigma_t^2$ is the sample eight quarter moving average variance of the own investment price. Except for only three cases, the correlation is negative. This negative correlation corroborates the negative relation between uncertainty and investment (growth). Experiments with other moving average lengths and other investment growth measures show that the sign of the correlations does not alter.

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>Canada</th>
<th>West-Germany</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>-28</td>
<td>-12</td>
<td>-21</td>
<td>-14</td>
</tr>
<tr>
<td>Non-residential</td>
<td>-15</td>
<td>-30</td>
<td>8</td>
<td>-23</td>
</tr>
<tr>
<td>Equipment</td>
<td>-21</td>
<td>13</td>
<td>-35</td>
<td>1</td>
</tr>
</tbody>
</table>

$\beta_t$ is calculated according to (38) using this “estimate” for $\sigma_t^2$ and the observed interest rate $\rho_t$. As the investment prices and interest rates are observed, the four triggers can be calculated for each type of investment, for each country, at each moment in time. The absolute triggers, though, do not give much information. For this reason all triggers are divided by the $M$-trigger under no-TTB and row-wise presented in table 3. The TTB triggers are not presented for equipment because equipment is not assumed to need TTB.

The upper part of table 3 shows the trigger values for residential structures. The first row gives the $H$-triggers in comparison with the
### Table 3: Average relative trigger values

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>Canada</th>
<th>West-Germany</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Residential structures</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\text{Trig}<em>{t}^{\text{unc}, \text{no-ttb}}}{\text{Trig}</em>{t}^{\text{no-unc}, \text{no-ttb}}}$</td>
<td>2.46</td>
<td>2.16</td>
<td>1.23</td>
<td>2.92</td>
</tr>
<tr>
<td>$\frac{\text{Trig}<em>{t}^{\text{no-unc}, \text{ttb}}}{\text{Trig}</em>{t}^{\text{no-unc}, \text{no-ttb}}}$</td>
<td>1.14</td>
<td>1.20</td>
<td>1.08</td>
<td>0.95</td>
</tr>
<tr>
<td>$\frac{\text{Trig}<em>{t}^{\text{unc}, \text{ttb}}}{\text{Trig}</em>{t}^{\text{no-unc}, \text{no-ttb}}}$</td>
<td>2.35</td>
<td>2.35</td>
<td>1.28</td>
<td>2.13</td>
</tr>
<tr>
<td><strong>Non-residential structures</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\text{Trig}<em>{t}^{\text{unc}, \text{no-ttb}}}{\text{Trig}</em>{t}^{\text{no-unc}, \text{no-ttb}}}$</td>
<td>2.88</td>
<td>3.04</td>
<td>1.56</td>
<td>283.08</td>
</tr>
<tr>
<td>$\frac{\text{Trig}<em>{t}^{\text{no-unc}, \text{ttb}}}{\text{Trig}</em>{t}^{\text{no-unc}, \text{no-ttb}}}$</td>
<td>1.14</td>
<td>1.17</td>
<td>1.09</td>
<td>1.73</td>
</tr>
<tr>
<td>$\frac{\text{Trig}<em>{t}^{\text{unc}, \text{ttb}}}{\text{Trig}</em>{t}^{\text{no-unc}, \text{no-ttb}}}$</td>
<td>2.99</td>
<td>3.07</td>
<td>1.57</td>
<td>275.30</td>
</tr>
<tr>
<td><strong>Equipment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\text{Trig}<em>{t}^{\text{unc}, \text{no-ttb}}}{\text{Trig}</em>{t}^{\text{no-unc}, \text{no-ttb}}}$</td>
<td>7.61</td>
<td>1.22</td>
<td>10.10</td>
<td>5.71</td>
</tr>
</tbody>
</table>

Trig$_{t}^{x,y}$ represents the trigger under certainty ($x = \text{no-unc}$) and uncertainly ($x = \text{unc}$) and no-TTB ($y = \text{no-ttb}$) or TTB ($y = \text{ttb}$).

$M$-triggers for projects without TTB. The $H$-triggers are 123% (West-Germany) to 292% (UK) times higher than the $M$-triggers. From the second row follows that the $M$-trigger without TTB is in the range of 95% to 120% times the standard $M$-trigger. From the third row follows that the $H$-trigger under TTB is much higher since it ranges from 213% to 235%.

The second part of table 3 concerns the non-residential structures and the third part equipment. Due to the calculation method it holds that

$$\frac{\text{Trig}_{t}^{\text{unc}, \text{no-ttb}}}{\text{Trig}_{t}^{\text{no-unc}, \text{no-ttb}}} > 1 \quad \text{and} \quad \frac{\text{Trig}_{t}^{\text{unc}, \text{ttb}}}{\text{Trig}_{t}^{\text{no-unc}, \text{no-ttb}}} \geq \frac{\text{Trig}_{t}^{\text{no-unc}, \text{ttb}}}{\text{Trig}_{t}^{\text{no-unc}, \text{no-ttb}}}.$$

For residential structures it follows that

$$\frac{\text{Trig}_{t}^{\text{no-unc}, \text{ttb}}}{\text{Trig}_{t}^{\text{no-unc}, \text{no-ttb}}} \leq 1$$

in the UK. This result is possible because of the growth in real investment prices, the inflation, that can be high in comparison with the depreciation and interest rate (see (13) and (20)).

Three points are important.
1. The trigger values for residential structures are smaller than those for non-residential structures. This result follows from the fact that (i) the uncertainty measure for residential is less than the uncertainty measure for non-residential and/or (ii) inflation of the associated investment price is higher. In particular the trigger values for non-residential under uncertainty are extremely high (283% and 275%) in the UK. The reason for these extremes is the volatile growth of the price of non-residential. This result shows that uncertainty can influence the trigger values considerably.

2. For residential structures in the US and (non-)residential structures in the UK the $H$-trigger values with no-TTB exceed the $H$-trigger with TTB. As the latter is assumed to be the most correct trigger value because structures need TTB, these results imply that TTB does not necessarily increase the hurdle rate. This is because the inflation in investment prices is high by which the user cost of capital under time-to-build, as defined by (18) and (13), is low. The trigger values are thus depressed.

3. Dixit and Pindyck [1994] calculate the triggers $H_{no-ttb}/M_{no-ttb}$ and find values in the range 2 to 3 whereas the triggers in table 3 vary between 0.95 and 283.08. So the error made by calculating the $M$-trigger instead of the $H$-trigger can be lower or much higher than Dixit and Pindyck [1994] suggest. The main difference with their calculations is that $\beta_t$ is calculated by using the information on investment prices, the interest rate, $\sigma_t^2$ and $\rho_t$. Both $\beta_t$ and $\rho_t$ compensate the positive influence that $\sigma_t^2$ has on the trigger values\(^{(10)}\). It is thus important to calculate $\beta_t$ and $\sigma_t$ correctly.

4.3 The impact of uncertainty on (no-)TTB investment

The theory in the sections 2-3 suggests that uncertainty affects investment under TTB more than under no TTB. For structures projects in most countries this is confirmed by the results in table 3. What we would like to know now is (i) whether the effect is significant and (ii) whether it holds that uncertainty affects (non-)residential structures more than equipment. To find an answer to these questions investment equations are derived and estimated.

According to the theory of Dixit and Pindyck [1994] the level of investment depends negatively on the trigger values. If this relation is

\(^{(10)}\) The "estimate" of $\sigma_t^2$ for most series of investment here does not differ much from the average values used in Dixit and Pindyck [1994].
assumed to be linear it holds that

$$I_t = \gamma \text{Trig}_t + \varepsilon_t, \quad \gamma < 0,$$

(22)

where \( \text{Trig}_t \) represents the trigger value under investigation, \( \gamma \) is a parameter to be estimated and \( \varepsilon_t \) is a disturbance term. This disturbance results from the substitution of the observed variables for the (unobserved) future investment prices and future interest rates.

The investment equations for the three types of investment can be estimated, for each country. In order to do so, \( J \), the investment scheme \( \delta_j \) and the depreciation rate \( \kappa \) are fixed at the values mentioned in the previous subsection and \( \beta_1 \sigma_i^2 \) is calculated as discussed before. It should be kept in mind that measuring \( \beta_1 \sigma_i^2 \) is only a rough approximation of the expected dispersion of the returns. Measurement errors are assumed to be captured by \( \varepsilon_t \).

The impact of the dispersion on investment is to be investigated. For this reason, and similar to the studies of Caballero and Pindyck [1992] and Meersman and Cassimon [1994][11], this “uncertainty” term is associated, in the following analyses, by a parameter that is estimated.

The investment equation with trigger (21) is transformed from

$$I_t = \gamma_1 X_{1t} + \gamma_2 X_{2t} + \varepsilon_t,$$

(23)

where

$$X_{1t} \equiv \sum_{j=0}^{J} \phi_{j} Q_{t+j} \prod_{i=j}^{J-1} (1 + \rho_{t+i}),$$

$$X_{2t} \equiv \beta_1 \sigma_i^2 Q_t,$$

into

$$i_t = \gamma_1 x_{1,t} + \gamma_2 x_{2,t} + \eta_t,$$

(24)

where

$$i_t \equiv \ln I_t - \ln I_{t-1},$$

$$x_{i,t} \equiv \ln X_{i,t} - \ln X_{i,t-1}, \quad i = 1, 2,$$

$$\eta_t \equiv \ln \varepsilon_t - \ln \varepsilon_{t-1},$$

(25)

and \( \eta_t \) is assumed to be i.i.d.

For each country, the system of the three investment equations is estimated by the Generalized Method of Moments. This method is

[11] The two studies mentioned calculate \( \sigma_i^2 \) but not \( \beta_1 \), and fix \( \rho_t \). In addition to this they investigate the marginal productivity of capital instead of investment.
used since the disturbance term can be a moving average and, in addition to this, instruments are necessary to correct for possible correlation between the disturbance and the explanatory variables. The instruments used are a constant, the investment price of residential, non-residential and equipment, the price of production and production. These six variables are taken in fourth differences and two periods lagged. A moving average of four quarters is accounted for when calculating the weighting matrix and corrections for heteroscedasticity are made. Furthermore, a constant is included in the regressions to account for a (possible) linear trend in the original relationship between investment \( (i_t) \) on one side, and costs \( (x_{1,t}) \) and uncertainty \( (x_{2,t}) \) on the other side.

The results are presented in table 4. They are striking in several respects.

**Table 4: GMM-results of investment equations (23)**

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>Canada</th>
<th>West-Germany</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residential structures</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>0.4</td>
<td>-0.4</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>(4.5)</td>
<td>(-1.0)</td>
<td>(4.9)</td>
<td>(0.1)</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>-0.1</td>
<td>0.2</td>
<td>-0.03</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>(-2.5)</td>
<td>(2.8)</td>
<td>(-0.9)</td>
<td>(0.1)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.07</td>
<td>0.05</td>
<td>0.28</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-residential structures</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>-0.3</td>
<td>-0.4</td>
<td>-0.3</td>
<td>-0.4</td>
</tr>
<tr>
<td></td>
<td>(-4.7)</td>
<td>(-1.8)</td>
<td>(-5.3)</td>
<td>(-10.8)</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>1.0</td>
<td>0.9</td>
<td>0.9</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>(64.3)</td>
<td>(10.2)</td>
<td>(41.9)</td>
<td>(44.9)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equipment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>0.4</td>
<td>-0.4</td>
<td>0.3</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(6.2)</td>
<td>(-2.6)</td>
<td>(3.9)</td>
<td>(-0.2)</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>-0.05</td>
<td>0.1</td>
<td>-0.04</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>(-2.4)</td>
<td>(1.8)</td>
<td>(-1.3)</td>
<td>(2.7)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.15</td>
<td>0.02</td>
<td>0.15</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( J )-statistic</td>
<td>10.7</td>
<td>9.1</td>
<td>10.5</td>
<td>10.4</td>
</tr>
<tr>
<td>( p )-value</td>
<td>0.30</td>
<td>0.43</td>
<td>0.32</td>
<td>0.30</td>
</tr>
</tbody>
</table>

First, if there is no uncertainty \( (\sigma^2 = 0) \) the coefficient \( \gamma_1 \) is ac-
cording to the theory expected to be negative. For both residential structures and equipment $\gamma_1$ is positive and highly significant in the US and West-Germany. The level of investment prices as measured by $X_{1t}$ seems thus not to influence investment negatively. Second, the coefficient $\gamma_2$ is extremely significant, in particular for structures. This implies that $X_{2t}$, being the uncertainty measure times the price $Q_t$, affects non-residential structures investment. As a consequence of the high explanatory power of the uncertainty measure, the $R^2$ -given in the last rows- are high for non-residential structures. The uncertainty effect thus influences non-residential structures more significantly than residential structures and equipment.

<table>
<thead>
<tr>
<th></th>
<th>$p$-value</th>
<th>J-statistic</th>
<th>US</th>
<th>Canada</th>
<th>West-Germany</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model (23)</strong></td>
<td>0.30</td>
<td>0.43</td>
<td>0.32</td>
<td>0.32</td>
<td></td>
<td>0.32</td>
</tr>
<tr>
<td><strong>Model (23) without TTB</strong></td>
<td>0.29</td>
<td>0.43</td>
<td>0.33</td>
<td>0.35</td>
<td></td>
<td>0.17</td>
</tr>
<tr>
<td><strong>Model (23) without prices</strong></td>
<td>0.19</td>
<td>0.32</td>
<td>0.18</td>
<td>0.23</td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Model (23) without uncertainty</strong></td>
<td>0.20</td>
<td>0.42</td>
<td>0.23</td>
<td></td>
<td></td>
<td>0.25</td>
</tr>
</tbody>
</table>

In order to investigate the effects of TTB and uncertainty further, the model (23) is estimated (i) without TTB for structures, i.e. $\delta_j = 0$ for $j = 2,3,4$ and $\delta_1 = 1$, (ii) without the level of prices, i.e. $\gamma_1 = 0$, and (iii) without uncertainty, i.e. $\gamma_2 = 0$. The $R^2$'s and $p$-values of the $J$-statistics are given in table 5.

A comparison of the $p$-values of the model (23) and model (23) without TTB shows that $p$-values do not differ very much. TTB does thus not crucially influence the estimation results. This, of course, depends on the way TTB was incorporated by the choice of investment scheme and TTB-length here.

A comparison of the model (23) with the model (23) without prices and with model (23) without uncertainty shows, however, that $p$-values are much lower for the last two models. This holds for all countries. It indicates that including investment prices as well as the uncertainty measure is important for the whole system of three types of investment.

In particular for non-residential structures the uncertainty measure is important. This follows from the graphs in figures 4 and 5. For each country and for each type of investment, the growth rate of the investment, its prediction by the model (23), and the prediction by the model (23) without uncertainty are graphed. As the $R^2$'s in table 4 also
show, the uncertainty measure $\beta_t \sigma_t^2 Q_t$ leads to much better predictions for non-residential structures investment.

5 Conclusions

In this study trigger values are derived for projects that need TTB. The main issue investigated is the effect of uncertainty on both TTB and no-TTB investment projects.

An individual investment project that needs TTB and renders uncertain returns is found to have a trigger value that is the multiplication of (i) an uncertainty-factor (ii) a TTB-factor and (iii) the trigger value if there were no TTB and no uncertainty. If the investment price, the project return and interest rate are constant, both the TTB-factor and
the uncertainty-factor exceed one. According to these findings standard Net Present Value rules for TTB projects under uncertainty overestimate investment because of both uncertainty and TTB. But if opportunity costs are taken into account these conclusions may no longer be true. Large opportunity costs can even entail investment in TTB projects to be triggered at lower returns than investment in no-TTB projects with the same returns.

Empirically, the significance of uncertainty on TTB and no-TTB investment is investigated with national structures and equipment times series data from the US, Canada, West-Germany and the UK. The moving average dispersion of investment prices is assumed to be the uncertainty measure. The estimation results show that uncertainty influences non-residential structures more significantly than residential
structures and equipment. For these projects the uncertainty measures even turn out to have a higher explanatory power than the investment prices in levels.

Investment models should thus account for uncertainty effects, in particular when TTB projects are considered. The importance of uncertainty in combination with TTB, emphasized by Majd and Pindyck [1987], are evidently corroborated.

Some difficulties are left for future research.

The time to build analyses should be extended to more realistic assumptions. An unknown construction period, an unknown amount of investment needed to complete the projects and possibilities to abandon the project once construction is started could be considered. These extensions are, though, theoretically difficult to tackle (see Pindyck [1993]).

Furthermore, a major difficulty in drawing inferences on uncertainty is the construction of an appropriate measure for the dispersion of returns. The variance of returns changes in time and might be better measured by using cross section data, as in Caballero and Pindyck [1992] and Meersman and Cassimon [1994], to calculate the dispersion in time for each "individual". Unfortunately, an additional problem remains since the "uncertainty" relevant to the investor is not the observed dispersion, but the expected uncertainty. So information on the expectations of investors should be gathered. Also, different types of uncertainty that firms are confronted with in different market structures need further investigation.
APPENDIX A

The derivation of the $H$-trigger under (no-)TTB

In this appendix the trigger under uncertainty is derived along the lines of Dixit and Pindyck [1994] (chapter 5). The main difference with Dixit and Pindyck [1994] is that TTB is considered by which, as shown in section 3.4, the user cost of capital differs.

Returns $R$ are assumed to follow the geometric Brownian motion,

$$dR = \alpha Rdtd + \sigma Rdz,$$

like the value of the investment project, represented by $V$,

$$dV = \alpha Vdt + \sigma Vdz.$$

The value of the opportunity to wait is given as

$$F(V) = \max \{ (V_T - \hat{\tau}_1 Q) e^{-\rho T} \}.$$  

$T$ is the (unknown) moment where the investment takes place and $\rho$ is the (constant) discount rate. $\hat{\tau}_1$ is a parameter where

$$\hat{\tau}_1 = 1 \quad \text{in case of no-TTB}$$
$$\hat{\tau}_1 > 1 \quad \text{in case of TTB.}$$

$F(V)$ must satisfy the conditions

$$\begin{align*}
\lim_{V \to 0} F(V) &= 0 \\
F'(\hat{V}) &= \hat{V} - \hat{\tau}_1 Q \\
F'(\hat{V}) &= 1.
\end{align*}$$

The first equation in (30) certifies that the value of the option is zero if the returns are zero. According to the second condition in (30) the value of the option where it is optimal to invest, denoted by $\hat{V}$, is equal to the value of the project minus the investment.

If the investment takes place, $\hat{\tau}_1 Q$ is paid and $V$ is obtained but the "opportunity value" is lost. In the optimum it thus holds that $\hat{\tau}_1 Q = \hat{V} - F(\hat{V})$. $F'(V)$ in (30) represents the first derivative of $F(\hat{V})$. The condition $F'(\hat{V}) = 1$ is called the "smooth pasting" condition (see Dixit and Pindyck [1994], chapter 4) that is known from the option theory.

The Bellman equation for $F(V)$ is given by

$$\hat{\tau}_2 F(V)dt = \mathcal{E}(dF).$$

Like $\hat{\tau}_1$, $\hat{\tau}_2$ is a parameter. It holds that

$$\hat{\tau}_2 = \rho \quad \text{in case of no-TTB}$$
$$\hat{\tau}_2 > \rho \quad \text{in case of TTB.}$$
According to Ito's lemma, \( F(V) \) is expanded as
\[
dF = F'(V)dV + \frac{1}{2} F''(V)(dV)^2.
\]
(33)
Substituting (27) in (33) and using \( \mathcal{V}(dz) = \mathcal{E}(dz^2) = dt \) and (31) renders
\[
\frac{1}{2} \sigma^2 V^2 F''(V) + \alpha V F'(V) - \hat{r}_2 F(V) = 0.
\]
(34)
A solution to this dynamic programming problem that satisfies the boundary conditions (30), has the form
\[
F(V) = BV^\beta + CV_1^\beta.
\]
(35)
Because of condition (30(ii)) it must hold that \( C = 0 \) and the solution
\[
F(V) = BV^\beta
\]
(36)
remains\(^{12}\). Substituting this solution into (34) renders
\[
\frac{1}{2} \sigma^2 \beta (\beta - 1) + \alpha \beta - \hat{r}_2 = 0,
\]
(37)
where the only non-explosive solution is
\[
\beta^* = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left[ \frac{\alpha}{\sigma^2} - \frac{1}{2} \right]^2 + 2 \frac{\hat{r}_2}{\sigma^2} > 1.}
\]
(38)
From the last two equations in (30) then follows
\[
\beta BV^{\beta - 1} = 1 \Leftrightarrow B = \frac{1}{\beta} V^{1-\beta},
\]
(39)
and substitution of (36) and (39) in the second equation of (30) renders
\[
\hat{V}^* = \frac{\beta^*}{\beta^* - 1} \hat{r}_1 Q,
\]
(40)
by which
\[
B^* = \frac{(\beta^* - 1)^{\beta^* - 1}}{(\beta^*)^{\beta^*}(\hat{r}_1 Q)^{\beta^* - 1}}.
\]
(41)
Note that \( \hat{r}_1 = r_1/J \) and \( \hat{r}_2 = r_2 \) in section 2.

The calculations that are carried out in a continuous time in this appendix, are adopted in section 3 in discrete time. Instead of only the interest rate as "cost

\(^{12}\) Instead of (36) \( F(V) = (\hat{V} - Q)(V/\hat{V})^{\beta} \) could be written. This follows from finding an expression for \( B \) from (36) and (30(ii)) and its substitution in (36). The expression for \( F(V) \) has an intuitive meaning since it represents the expected discount profit; \( \hat{V} - Q \) is the return when the option is exercised and \( (V/\hat{V})^\beta \) is "a weighted discount rate" (see Karlin and Taylor [1975], pp. 363-365, and Mella-Barral [1995], footnote 5).
of capital", both the depreciation rate and the inflation are taken into account. This follows from (13).

Furthermore, the stream of returns is given by

\[ V_t = \mathcal{E} \int_t^\infty R_s e^{-\tilde{r}_2(s-t)}ds = \frac{R_t}{\tilde{r}_2 - \alpha}. \]  

(42)

The condition \( \rho > \alpha \) must be satisfied. If this condition is violated \( V_t \) is not determined and investment will never occur.

The optimal moment to pull the trigger according to the neoclassical theory is where \( V_t \geq \tilde{r}_1 Q \), though, according to the option theory \( V_t \geq |\beta^*/(\beta^* - 1)|\tilde{r}_1 Q \) (see (40)). Together with (42) and (37) it then follows

\[ R_t \geq \frac{\beta^*}{\beta^* - 1} (\tilde{r}_2 - \alpha) \tilde{r}_1 Q = \left( \tilde{r}_2 + \frac{1}{2}\beta^* \sigma^2 \right) \tilde{r}_1 Q. \]  

(43)

\section*{APPENDIX B}

\textbf{The derivation of the H-trigger for negative waiting values}

In this section the trigger value for a project that has an opportunity cost is derived. The derivations are similar to the those in the previous appendix.

Suppose that the opportunity cost is a lump sum that equals \( W \). The first equation in (30) is then redefined as

\[ F(0) = -W \]  

(44)

and (36) is redefined as

\[ F(V) = -W + BV^{\beta}. \]  

(45)

(39) still holds. (45) then becomes

\[ V^* = \frac{\beta^*}{\beta^* - 1} [\tilde{r}_2 Q - W] \]  

(46)

and the trigger value (see (43))

\[ \frac{\beta^*}{\beta^* - 1} \tilde{r}_1 Q - W. \]  

(47)

It then follows that

\[ DH^* \equiv H^*_{ttb} - H^*_{ntottb} = \tau_{unc}[\tau_{ttb} - 1]M_{ntottb} + \tau_{unc}(\rho - \tilde{r}_2)W. \]  

(48)

If \( W = 0 \), \( DH^* \) equals \( DH \) in (7). It follows that high opportunity costs make \( DH^* \) negative since \( \rho < \tilde{r}_2 \) if \( J > 1 \). \( DH^* \) is shown in graph 3 for different values of \( \sigma^2 \) and becomes negative for \( W = 0.6 \).
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