On capital, increasing returns and long-run growth in a model of overlapping generations

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1 Introduction

This contribution illustrates the role of an imperfectly competitive process of capital accumulation in the growth process. It emphasizes the central role of investment in long-run outcomes.

One of the most successful contributions of the new growth literature has been interested in the role of innovation in growth. In the line of Schumpeter, innovation is viewed as a *creative destruction* process[(1)]. Growth results from the occurrence of temporary monopolies. One of the main problems with such a view of the growth process is its empirical relevance. No physical investment being considered, the sole engine of growth becomes the research and development activity. This contradicts with the conclusions of Long & Summers [1991]. In a study concerned with long-run experiences since the second world war, these authors have indeed stressed the central role of a decreasing motion for the price of capital in the growth miracles. At the opposite, this trend is tackled by the other main approach in the recent growth literature that refers to *historical* capital stock externalities. However, this alternative has not a lot to say about the incentives that underlie entrepreneurship. Moreover and from a more general point of view; although the *historically- driven* externalities' view appears as both relevant and plausible from a local point of view, it is not as attractive at the aggregate level.

The relaxation of the perfect competitition assumption on capital markets significantly improves the understanding of investment mo-

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(1) Well-known contributions in such a perspective are, *e.g.*, Aghion & Howitt [1990] or Grossman & Helpman [1991].

tives and may hence provide an attractive basis for the role of investment in long-run growth. However, a difficult issue in the consideration of an imperfectly competitive setting with a capital accumulation process has to do with the great number of state variables and shadow prices. It usually becomes difficult to characterize the properties of the dynamical system(2).

At that stage, the present study simplifies matters by the focus on an overlapping generations structure with finite lives for the firms. This in particular gives rise to the possibility of a thorough understanding of the dynamical system(3). Moreover, the decreasing motion for the price of capital is explicitly based on profit incentives.

More precisely, the economy under study builds from a multisectoral structure with a unique final good and a continuum of capital goods. Its properties can be described as follows: the existence of monopolistic positions within the capital good sectors provides incentives for firms to invest in research. This research activity will improve the productivity of the investment technology. This will imply a decreasing path for the price of capital and so the possibility of a long-run growth path of the economy. With respect to the recent growth literature, several specific features of this solution should be pointed out. First, the growth path does not rely on *aggregate externalities* (as in the early Romer's [1986] contribution) but on local ones. Hence, spillovers will be *dynamic* and *intrasectoral.* This is particularly appealing in regard to the appearance of synergy effects between research areas during the eighties(4). Otherwise stated, whilst there is clearly case for *complementaries* between sectors at a given date of time, the diffusion of *historically driven* external effects is more likely to occur only within a given industry.

Another noticable feature of the model comes from a non-homothetic growth solution; indeed, all components of a growth path do not exhibit the same growth rate. This results from the non-linear dependence of the growth rate with respect to the capital stock upon the aggregate technological knowledge. In this perspective, it is worth emphasizing that, contrary to the recent growth literature, the current contribution analyses an imperfectly competitive capital goods market within the capital accumulation activity.

(2) An illustration of this difficulty was given in Romer [1990] where the author's analysis was based on the assumption of a unique *saddle-point* stable stationary growth path.

(3) Cf. Chou & Shy [1990] or Wigniolle [1991] for related contributions that also make use of the overlapping generations framework — without a capital accumulation process.

(4) The most famous example is the Sillicon Valley for the software industry.

Section 2 specifies the model, Section 3 characterizes the equilibrium and depicts the properties of a long-run stationary growth path.

2 The model

There exists a unique sector of final goods in the economy, together with a continuum of sectors producing capital goods. Whereas the production of final good holds under perfect competition, the production of capital goods is characterized by temporary monopolies that last for two periods. The final good is retained as a numéraire.

The production technology of the representative producer of the final good is given by[5]:

$$Y_t = K_t^{\alpha} W_t^{1-\alpha}, \alpha \in (0,1), \tag{1}$$

with K_t as the amount of capital employed in the final good sector and W_t as the amount of labour employed at time t.

It is assumed that K_t corresponds to the aggregate of capital services provided by the equipment goods producers (distributed over $[0,1]$):

$$K_t = \left[\int_0^1 K_t(j)^{\alpha} dj\right]^{1/\alpha}, \tag{2}$$

with $K_t(j)$ as the amount of capital services of the sector j used by the final good producer at date t. Capital services of type j are rented at rate $p_{K,t}(j)$.

Denoting w_t as the wage rate that the final good producer pays at date t, his program can be written as:

$$\begin{gathered}\operatorname*{Max}_{\{W_t, K_t(j)\}} \int_0^1 K_t(j)^{\alpha} W_t^{1-\alpha} dj - \int_0^1 p_{K_t}(j) K_t(j) dj - w_t W_t \\ \text{s.t. } W_t \geqslant 0, K_t(j) \geqslant 0 .\end{gathered} \tag{3}$$

Due to the perfect competition assumption on the final good market, this program is static. Its first order conditions state as:

$$p_{K_t}(j) = \alpha K_t(j)^{\alpha-1} W_t^{1-\alpha}, \tag{4a}$$

$$w_t = (1-\alpha) \int_0^1 K_t(j)^{\alpha} dj W_t^{-\alpha}. \tag{4b}$$

[5] Had a general specification been retained, no crucial feature of the remaining analysis would have been modified. Along the purpose of the contribution that is to provide a simple illustration, it has been prefered to hinge from the very beginning of the analysis on basic formulations.

Finally, the depreciation rate of the capital stock is assumed to be unitary.[6] The next assumption will further simplify the analysis without modifying the essential argument:

Assumption 1 *Firms in the investment goods sector live for two periods.*

The rationale that underlies Assumption 1 is the following one. During the first period of its life, the firm in sector $j \in [0,1]$ will have to pay wages and to invest $i_t(j)$ units of final good. This will allow the firm to sale capital services and to make profits at the beginning of the following period. Expressed in prices of date $t+1$, the actualised profits of a monopolist operating in sector j at date t will thus be given by:

$$\pi_{t+1}(j) = p_{K,t+1}(j)K_{t+1}(j) - \mathcal{R}_{t+1}\left[w_t z_t(j) + i_t(j)\right], \tag{5}$$

with $\mathcal{R}_{t+1}$ as the interest factor on the borrowed funds and $z_t(j)$ as the employed labour in sector j. More explicitly, the program solved by the monopolist of sector j is as follows:

$$\begin{aligned} \max_{\{z_t(j), i_t(j), K_{t-1}(j)\}} \quad & p_K\left[K_{t+1}(j)\right]K_{t+1}(j) - \mathcal{R}_{t+1}\left[w_t z_t(j) + i_t(j)\right] \\ \text{s.t.} \quad & K_{t+1}(j) = A_{t+1}(j) i_t^{\zeta}(j), \\ & A_{t+1}(j) = A_t(j)\psi(z_t(j)), \\ & z_t(j) \geqslant 0, i_t(j) \geqslant 0, K_{t+1}(j) \geqslant 0, \forall j \in [0,1], \end{aligned} \tag{6}$$

with $\zeta > 0$[7] and the aggregate value of knowledge specific to sector j, *i.e.*, $A_t(j)$ perceived as given at the beginning of period t. This notation reflects that, when he solves his program, the monopolist benefits from the previous improvements of the internal knowledge of the industry. He perceives this level of knowledge as exogenously given by the history of sector j: in that sense, this phenomenom features a *dynamic externality* that remains internal to sector j. Furthermore, the research technology is assumed to satisfy the following properties:

Assumption 2 $\psi(\cdot) : \mathbb{R}_+ \to [1, +\infty)$, $\psi(0) = 1$, $\psi' > 0$ *for any* $z > 0$[8].

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[6] This assumption allows us to reach simple analytical results. Had it been relaxed, the dimension of the dynamical system would have been greater because the optimization problem of the monopolist would have been fully dynamic.

[7] Note that one could have contained the exposition to $\zeta = 1$ without modifying any essential result. This specification has been retained because it illustrates that the linearity of the investment technology is not essential to the occurrence of a sustained growth solution.

[8] Note that, in equilibrium, $\psi(\cdot)$ will only assume finite values since the latters

Given Assumption 2, it is obvious that with a null *research activity*, the productivity of the investment technology would remain constant. Hence, for any strictly positive amount of research in sector j, an increasing returns to scale property appears in the production technology of the capital goods. Finally, note that the linearity of the law of motion of the *internal knowledge* of industry j, *i.e.*, $A_{t+1}(j)$, will be essential to the possibility of a sustained growth solution.

The intuition that underlies the monopolist program is as follows: at the beginning of period t, the new monopolist of sector j inherits a technological knowledge $A_t(j)$ specific to sector j. By investing in research — *i.e.*, $z_t(j) > 0$ —, he can improve the productivity of his investment and make positive profits. During period $t+1$, the improvement of the technological knowledge resulting from his investment will spread through the sector and the newly entered monopolist will benefit in his turn from a level of knowledge $A_{t+1}(j)$. He will then be able to increase this level of knowledge by investing in research like the preceding *generation* in industry j.

Integrating the form of the demand curve for capital services — see (4a)—, the objective of the monopolist of sector j can be reformulated as :

$$\begin{aligned} &\text{Max}_{\{z_t(j),i_t(j),K_{t+1}(j)\}} \quad \frac{p_K\left[K_{t+1}(j)\right]K_{t+1}(j)}{\mathcal{R}_{t+1}} - \left[w_t(j)z_t(j) + i_t(j)\right] \\ &\text{s.t.} \qquad K_{t+1}(j) = A_t(j)\psi\left(z_t(j)\right)i_t^{\zeta}(j)\,, \end{aligned} \tag{7}$$

where $\zeta \in [0,1]$.

Denoting λ_t as the shadow price associated to the constraint of the above program at date t, the first order conditions of this problem derive as:

$$w_t = \lambda_t A_t(j)\psi'\left(z_t(j)\right)i_t^{\zeta}(j)\,, \tag{8a}$$

$$1 = \zeta\lambda_t A_t(j)\psi\left(z_t(j)\right)i_t^{\zeta-1}\,, \tag{8b}$$

$$\lambda_t = \frac{p'_{K_{t+1}}(j)\left[K_{t+1}(j)\right] + p_{K_{t+1}}(j)}{\mathcal{R}_{t+1}}\,. \tag{8c}$$

Let us already precise that condition (8c) will never be considered explicitly in the subsequent analysis because of the specific savings'

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are univocally determined by the instantaneous flow of research, itself being upper-bounded by the finite value of labour supply available at each date of time.

behaviour we will assume. Indeed, there will be no role for the interest rate whilst, in the general case, it is precisely this relation which would have ruled the fixation of this price.(9) Expressing the ratio of (8a) and (8b), it derives:

$$w_t = \frac{\zeta^{-1}\psi'\big(z_t(j)\big)i_t(j)}{\psi\big(z_t(j)\big)}. \tag{9}$$

The holding of increasing returns in the technology suggests that special attention must be paid to the examination of second order conditions. It is shown in the appendix that, for $\zeta \in (0,1)$, sufficient conditions for a maximum back to:

$$\frac{\psi''}{\psi'} < \frac{\psi'}{\psi}\left[1 + \frac{\alpha}{-1+\alpha\zeta}\right]. \tag{10}$$

Interestingly, this does not imply that ψ must be necessarily concave. For $\alpha < (1+\zeta)^{-1}$, it may be convex but still satisfy this condition.

As usual in overlapping generations frameworks, any consumer is assumed to live for two periods,(10) to supply labour but not to consume when young and, conversely, not to work but to consume during his old age. From the above analysis, it is immediate that the essential issue lies in the allocation of labour between the two sectors. It is assumed that the *representative* member of the young generation divides his working time between the two sectors(11). In order to simplify further, it is assumed that the young generation will supply one unit of labour inelastically. It is characterised by a utility function $u(c^t_{t+1})$; $u(\cdot)$ is a standard concave increasing instantaneous utility function that satisfies Inada conditions.

It is further assumed that the representative member of generation $t \geqslant 1$, whatever time he might have spent in the research area, will not discriminate between the monopolists hiring firms in the capital goods. The above described preferences further imply that, for any value of the rate of interest, the representative consumer will save the totality of his first period earnings(12).

(9) A more general savings behaviour with positive amounts of consumption during both periods would integrate the latter. Unfortunately, it disables any analytical treatment in the general case.

(10) But to the initial *old generation* whose treatment is omitted in the subsequent analysis.

(11) Equivalently, another possibility would be to assume a continuum of consumers distributed over the interval $[0,1]$ and to consider the number of consumers Z_t working in the research sector at date $t \geq 1$ and the remaining fraction $1 - Z_t$ employed in the production of the capital goods.

(12) This implies that the condition (8c) will eventually play no role in the definition of a dynamic equilibrium.

3 Equilibrium growth with increasing returns

In order to further simplify the analysis, only a specific class of equilibria will be considered:

Assumption 3 $A_0(j) = A_0(j')$, $\forall j, j' \in [0,1]$.

The focus will thus be on symmetric market outcomes. This in particular implies the holding of the following equality on the labour market:

$$\int_0^1 z_t(j)dj + W_t = Z_t + W_t = 1\,. \tag{11}$$

Parallely, on the market of investment goods, Assumption 3 implies that, $\forall j \in [0,1], \forall t \geqslant 0$:

$$p_{K,t+1}(j) \quad = \quad p_{K,t+1}, K_{t+1}(j) \;=\; K_{t+1}, \tag{12a}$$

$$z_t(j) \quad = \quad z_t, i_t(j) = i_t, A_{t+1}(j) \;=\; A_{t+1}, \tag{12b}$$

Due to Assumption 3, the equilibrium on the capital market can be expressed as:

$$\int_0^1 \left[i_t(j) + w_t z_t(j)\right]dj = w_t\,. \tag{13}$$

i.e., the total indebtness of the monopolists must be equal to the aggregate supply or savings of the youngs, or[13]:

$$i_t + Z_t w_t = w_t\,. \tag{14}$$

Assuming that a symmetric equilibrium under monopolistic competitition exists, the first order conditions of the maximization program of the monopolists then lead in equilibrium to the following equation:

$$1 = \frac{\psi'(Z_t)}{\psi(Z_t)}(1 - Z_t)\,. \tag{15}$$

A solution to (15) being a constant $Z_t = Z_0$, it follows from the equilibrium condition on the labour market that W_t $(= 1 - Z_t)$ is a constant term W_0. Furthermore, under the previous requirements on $\psi(\cdot)$ (*i.e.*, Assumption 2 plus a concavity requirement in order for a maximum), a sufficient condition for the existence of at least one solution to (15) is that $\psi(0)/\psi'(0) < 1$, which is not restrictive. Assume that at least

[13] Note that the simplicity of the subsequent determination of the growth rate is a direct corollary of the explicit expression for i_t/z_t that derives from this formula.

one interior solution exists. Remark that, in symmetric equilibrium, $w_t = (1-\alpha)K_t^{\alpha}W_0^{\alpha}$, and equilibrium conditions on the labour market and on the capital market state as:

$$\begin{aligned} Z_0 + W_0 &= 1 . & (16a) \\ Z_0 w_t + i_t &= w_t . & (16b) \end{aligned}$$

A substitution between the two lines of this system gives:

$$W_0 w_t = i_t . \tag{17}$$

Incorporating the above equation in the aggregate equilibrium condition of capital accumulation gives finally:

$$\begin{aligned} K_{t+1} &= A_{t+1}\left[(1-\alpha)K_t^{\alpha}W_0^{1-\alpha}\right]^{\zeta} , & (18a) \\ &= A_t\psi(Z_0)\left[W_0(1-\alpha)K_t^{\alpha}W_0^{-\alpha}\right]^{\zeta} , & (18b) \end{aligned}$$

The ratio of (18b) taken respectively at dates $t+1$ and t gives:

$$\frac{K_{t+1}}{K_t} = \psi(Z_0)\left(\frac{K_t}{K_{t-1}}\right)^{\alpha\zeta} . \tag{19}$$

For $\alpha, \zeta \in (0,1)$, the growth dynamics of the capital stock will be characterized by a stable constant rate in the long run.

Besides, letting $\kappa_A \triangleq A_{t+1}/A_t$, a long-run path being characterized by $\kappa_A = \psi(Z_0)$ and by:

$$\begin{aligned} K_{t+1} &= A_{t+1} i_t^{\zeta} , & (20a) \\ i_t &= W_0(1-\alpha)K_t^{\alpha}W_0^{-\alpha} , & (20b) \end{aligned}$$

one gets $\left(\kappa_K\right)^{1-\alpha\zeta} = \kappa_A$, with $\kappa_K \triangleq K_{t+1}/K_t$. Hence, the growth path is not homothetic and $\kappa_K > \kappa_A$. This is an implication of the increasing returns property in the production technology of the capital goods.

Finally, consider the uniqueness issue. Due to increasing returns, is the multiplicity of long-run growth paths an admissible outcome?

Recall the equilibrium expression of the optimality condition:

$$\frac{\psi(Z)}{\psi'(Z)} = \zeta^{-1}(1-Z) . \tag{21}$$

The R.H.S. of (21) being associated with a straight line of negative slope, the possibility of multiple long-run paths at a positive constant rate requires that function $\varphi(Z) \triangleq \psi(Z)/\psi'(Z)$ be decreasing over some intervals. This implies:

$$\frac{(\psi')^2 - \psi\psi''}{(\psi')^2} < 0, \tag{22}$$

which can only be satisfied if $\psi(\cdot)$ is strictly convex. Remark however that (22) is not compatible with the class of convex specifications for $\psi(\cdot)$ satisfying the second-order condition (10). Hence, even for a convex specification for the research technology, there does exist at most one equilibrium growth path with increasing returns.

4 Conclusion

The purpose of this contribution was to reexamine the results reached by the recent literature on economic growth in the light of a classical dichotomy between final goods and investment goods. In the line of the recent growth literature, the existence of monopolistic positions led to the possibility of a sustained long-run growth path. This growth process was however distinct from the ones that have been analyzed previously. First, it was based on dynamic and intrasectoral externalities instead of the usual aggregate ones. Secondly, the capital stock was shown to grow more rapidly than the technological know-how within each of the capital goods sectors.

APPENDIX

Second order conditions

After simplifications, the profit function of the monopolist of type j backs to :

$$\pi_{t+1}(j) = \frac{\alpha\left\{A_t(j)\psi\left(z_t(j)\right)i_t^{\zeta}(j)\right\}^{\alpha} . W_{t+1}^{1-\alpha}}{\mathcal{R}_{t+1}} - \left\{w_t z_t(j) + i_t(j)\right\}.$$

After simplifications, the hessian matrix can be expressed as follows:

$$\mathcal{H} = \alpha^2 K^{\alpha} W^{1-\alpha} \begin{bmatrix} (\zeta/i^2)(\alpha\zeta - 1) & \alpha(\psi'/\psi)(\zeta/i) \\ \alpha(\psi'/\psi)(\zeta/i) & (\psi''/\psi) + (\psi'/\psi)^2(\alpha - 1) \end{bmatrix}.$$

It is negative definite if and only if its principal minor determinants $\mathcal{M}_n$ are of sign $(-1)^n$, for $n = 1, 2$. The restriction on $\mathcal{M}_1$ backs to $-1 + \alpha\zeta < 0$. The corresponding condition on $\mathcal{M}_2$ is summarized by $\left(\psi''/\psi'\right) < \left(\psi'/\psi\right)\left[\left(-1 + (\zeta + 1)\right)/(\alpha\zeta - 1)\right] = \left(\psi'/\psi\right)\left\{1 + \left[\alpha/(\alpha\zeta - 1)\right]\right\}$. □

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