Common trends and common cycles in Belgian sectoral GDP

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1 Introduction

The fact that real variables, such as output, can be decomposed into cyclical or transitory movements and a trend or permanent component has played an important role in thinking about economic phenomena. This type of time series' decomposition is a common practice in the Business Cycle theory. Lucas [1977] defines business cycles as "repeated fluctuations in employment, output, and the composition of output, associated with a certain typical pattern of comovements in prices and other variables". Long and Plosser [1983] consider that "the term business cycles refers to the joint time series behaviour of a wide range of economic variables such as prices, output, employment, consumption, and investment". They also refer to the fact that outputs in different sectors move together: when one sector is above (below) its trend, other sectors are also, in general, above (below) their trends.

In the case of output, the trend component is viewed as being in the domain of growth theory with real factors such as capital accumulation, population growth and technological change as the main determinants. These common stochastic trends are principally the cumulative effects of permanent productivity shocks and they explain the most important part of output evolution. On the other hand, the cyclical component is assumed to dissipate over time *i.e.* to be transitory or stationary.

It is convenient to use appropriate econometric techniques to analyse comovements among economic variables. Cointegration is an indicator of comovements among non stationary series. Engle and Granger [1987] provide a unified theoretical framework to deal with multivari-

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ate data sets where the variables contain a reduced number of stochastic trends. Johansen [1988,1991] addresses the estimation and testing problem of long run relations in economic modelling. He provides test statistics for the hypothesis of a given number of cointegration vectors and estimates of these vectors. Gourieroux and Peaucelle [1990] propose an indicator of comovements among stationary series, the codependence. The introduction by Engle and Kozicki [1993] of the concept of common features provides a unified way to discuss data sets in which variables display both long run comovement (common stochastic trends) and short run comovement (common serial correlation).

Techniques proposed by Vahid and Engle [1993] and Engle and Issler [1995] allow to a better analysis of the role of cyclical components in Real Business Cycle theory. They study the number of common cycles, the amplitude of the cycles in pro and counter cyclical sectors, the correlation between trend and cyclical components, and the importance of trend and cyclical innovations. Therefore, the attraction of my article is that provides new empirical evidence on Belgian sectoral GDP cycles, replicating the work of Engle and Issler [1995].

The plan of this paper is as follows: after the definitions of cointegration and common features in Section 2, the applied methodology is described in Section 3. A unique trend-cycle decomposition of the data is discussed as a special case. Sections 2 and 3 are based on the papers of Vahid and Engle [1992] and Engle and Issler [1995]. Readers interested solely in how these techniques are applied may want to begin with the empirical application of common trends and common cycles in Belgian sectoral output series analysed in Section 4. Conclusions are presented in Section 5.

2 Common features

2.1 Cointegration and common features

Recent developments in the study of comovements among time series reveal the existence of common components that give more information about the economic structure behind the models. Therefore, *cointegration* is an indicator of comovement among non stationary variables. When the variables are integrated of order one and cointegrated they share long run relationships, and at least one linear combination of them is stationary.

Codependence(1) is an indicator of comovement among stationary series. A set of codependent variables presents at least one linear

(1) Gourieroux and Peaucelle [1990].

combination with lower autoregressive order than the others. According to Engle and Kozicki [1993] the *serial correlation common feature* is a strong form of codependence. In this case, a linear combination of the stationary variables eliminates all correlation with the past and is completely unpredictable from the past information set. A formal definition is important to clarify this concept.

Definition 2.1 (Vahid and Engle [1993]) *Let y_t denote an N-vector of $I(1)$ variables, and Δy_t the vector of $I(0)$ first differences of y_t. Then, the elements of Δy_t have s $(s < N)$ serial correlation common features if there are s linear independent combinations of them which are innovations.*

These linear combinations are called the cofeature combinations and the vectors which represent them are called the cofeature vectors. The set of cofeature vectors forms a $(N \times s)$ matrix of rank s, the cofeature matrix $\tilde{\alpha}$. This matrix provides a transformation that eliminates serial correlation in Δy_t.

The N-vector y_t could be decomposed into a random walk (the trend) and a stationary part (the cycle). When r cointegrating vectors exist, r independent linear combinations of y_t eliminate the trends, so there are $N - r$ common trends. Common cycles exist when there are some linear combinations of y_t which do not contain cycles.

A very important relationship between common features and cointegration is summarised in the following theorem.

Theorem 2.1 *Let y_t be an N-vector of $I(1)$ variables with r linearly independent cointegrating vectors $(r < N)$. Then, if some elements of y_t have common cycles, there can at most exist $N - r$ linearly independent cofeature vectors that eliminate the common cycles. Moreover, these linear combinations must be linearly independent of the cointegration vectors.*

Proof: Vahid and Engle [1993], page 345. □

2.2 VAR representation

To test for the presence of common cycles and trends a finite vector autoregressive (VAR) representation is helpful. Let y_t be a column vector of N variables, then a VAR system can be written as

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_k y_{t-k} + \mu + \varepsilon_t, \tag{1}$$

where $\varepsilon_t \sim IN(0, \Omega)$, μ is a constant vector, and k is the length required for residuals to be white noise.

Generally, economic time series are non-stationary processes, and finite order VAR systems like equation (1) are usually expressed in the first difference form. Using $\Delta = 1 - L$, where L is the lag operator, the model can be rewritten as

$$\Delta y_t = \Pi y_{t-1} + B_1 \Delta y_{t-1} + B_2 \Delta y_{t-2} + \ldots + B_{k-1} \Delta y_{t-k+1} + \mu + \varepsilon_t, \quad (2)$$

where $B_i = -(A_{i+1} + \ldots + A_k)$ and $\Pi = -(I - A_1 - \ldots - A_k)$.

The coefficient matrix Π contains information about long run relationships between the variables in the data vector. When $rank(\Pi) = r$, $r < N$, there are two $(N \times r)$ matrices β and α such that $\Pi = \beta\alpha'$. The columns of the cointegrating matrix α are the r independent cointegrating vectors which have the property that $\alpha' y_t$ is stationary even though y_t itself is non stationary. Under the assumptions of the Granger representation theorem, any cointegrated system can be written as a Vector Error Correction Model (VECM)[(2)]:

$$\Delta y_t = \beta\alpha' y_{t-1} + B_1 \Delta y_{t-1} + B_2 \Delta y_{t-2} + \cdots + B_{k-1} \Delta y_{t-k+1} + \mu + \varepsilon_t, \quad (3)$$

where $\alpha' y_{t-1}$ denote the error correction terms. The number of error correction terms is equal to the cointegrating rank. In Section 3, this VECM is used to decompose y_t into trend and cyclical components.

2.3 A theoretical model for real business cycles

Long and Plosser [1983] developed a theoretical model to explain two stylised facts of business cycles: persistence and comovement in consumption, input, and output time series. The assumptions in their model guarantee that comovents in output arise solely from the nature of the input decision rules and the production technology chosen, but not from the existence of a common shock or shocks that are correlated across variables. It can also be shown that persistence in output must arise from the propagation mechanism in their model and not from serially correlated exogenous shocks.

As Engle and Issler [1995] noted, (the log of) the technology shocks in the model of Long and Plosser [1983] are assumed to be uncorrelated random walks, and this implies that (the log of) the sectoral outputs are not cointegrated, even if they could share common cycles. Moreover, Engle and Issler [1995] propose an alternative specification of the technology shocks requiring that (in logs) they be cointegrated. Assuming cointegrated productivity shocks, technological innovations in one sector can contribute to technological innovations in others. This assumption allows for cointegration of the sectoral output (in logs).

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(2) Engle and Granger [1987].

The model of Engle and Issler [1995] serves as a basis for exposing the characteristics of a Real Business Cycle system of sectoral outputs, since these variables (in logs) are treated as I(1), cointegrated, and sharing common cycles.

3 Econometric specification of common cycles and common trends

3.1 Cofeature rank and canonical correlations

Several approaches are available in the literature to decompose the movements in time series into trends and cycles. To standardise the notation, the Engle and Issler [1992] specification is used. They define the trend component (y_t^p) to be a random walk, and the cyclical component (y_t^c) to capture all serial correlation in the first differences of y_t such that

$$y_t = y_t^p + y_t^c\,. \tag{4}$$

To distinguish these two components the VECM of equation (3) is considered. One looks for linear combinations of Δy_t which are innovations. Since all the serial correlation of Δy_t is captured by the conditioning variables on the right hand side of (3), one should look for linear combinations of Δy_t which are uncorrelated with any linear combination of these conditioning variables. This can be performed by canonical correlation analysis.

The method of finding the cofeature rank incorporates the canonical correlation analysis as follows. Consider these two sets of variables

$$\Delta y_t' = (\Delta y_{1t}, \Delta y_{2t}, \dots, \Delta y_{Nt})'$$

i.e. the variables of the left hand side of (3), and

$$Z_t' = \left(\Delta y_{t-1}', \Delta y_{t-2}', \dots, \Delta y_{t-k+1}', (\alpha' y_{t-1})', 1\right)'$$

i.e. the conditioning variables in the right hand side of (3).

Let $u_{it} = \widetilde{\alpha}_i' \Delta y_t$ and $v_{it} = \gamma_i' Z_t$ ($i = 1, 2, , \dots, N$) be specific linear combinations of Δy_t and Z_t respectively. The set of the N orthogonal choices of $\widetilde{\alpha}_i$ and γ_i which has a maximal correlation coefficient between u_{it} and v_{it} is the canonical correlation set. Each statistically zero canonical correlation represents a linear combination of Δy_t uncorrelated with all linear combinations of Z_t, since it is uncorrelated with the one which presents maximal correlation between u_{it} and v_{it}. Therefore, $\widetilde{\alpha}_i' \Delta y_t$ ($i = 1, 2, \dots, N$) are linear combinations of Δy_t and are innovations, since they are uncorrelated with any linear combination of the conditioning variables of Δy_t. Let $\widetilde{\alpha}$ be the matrix $(\widetilde{\alpha}_1, \widetilde{\alpha}_2, \dots, \widetilde{\alpha}_s)$

built with all the $\widetilde{\alpha}_i$ which are associated with the zero canonical correlations. Then, by definition, $\widetilde{\alpha}$ is the cofeature matrix, the $(N \times s)$ full rank matrix of all independent cofeature vectors. The cofeature rank s is the number of statistically zero canonical correlations, where $s \leqslant N-r$ and r is the cointegrating rank[(3)]. The number of common cycles is the number of non zero canonical correlations and it is equal to $N-s$.

3.2 A unique trend-cycle decomposition of the data

Vahid an Engle [1992] discuss a special case about the dimensions of the cointegrating and cofeature spaces where a unique trend-cycle decomposition of the data is generated. If the cointegrating rank (r) and the cofeature rank (s) add up to the number of variables in the data set (N), then the common trend representation in equation (3) presents special characteristics. Since Δy_t is stationary it admits the following Wold representation

$$\Delta y_t = C(L)(\varepsilon_t + \mu_0), \tag{5}$$

where $\varepsilon_t \sim IN(0, \Omega)$, μ_0 is a constant vector, and

$$C(L) = \sum_{i=0}^{\infty} C_i L^i, \quad \text{with} \quad C_0 = I_N \quad \text{and} \quad \sum_{j=1}^{\infty} j\,|C_j| < \infty.$$

The cumulative or total effect of $C(L)$ is given by

$$C(1) = I_N + \sum_{i=1}^{\infty} C_i.$$

The polynomial matrix $C(L)$ can be rewritten as

$$C(L) = C(1) + C^*(L)(1-L),$$

where

$$C^*(L) = \sum_{i=0}^{\infty} C_i^* L^i \quad \text{and} \quad C_i^* = -\sum_{j=i+1}^{\infty} C_j$$

so that, $C_0^* = I_N - C(1)$.

When $\mu_0 = 0$ the vector y_t can be expressed as

$$y_t = \mu + C(1)\sum_{i=1}^{i=t} \varepsilon_i + C^*(L)\varepsilon_t, \tag{6}$$

where μ is a constant vector.

(3) Theorem 1 of Section 2.

If cointegration is assumed (*i.e.* reduced rank matrix Π in the autoregressive representation), under the assumptions of the Granger representation theorem, the matrix $C(1)$ in the moving average form has rank inferior to N (*i.e.* $N - r$). Then, the term $C(1)\sum_{i=1}^{i=t}\varepsilon_i$ represents the $N-r$ stochastic trends. Common cycles appear if $C^*(1)$ has reduced rank, since cycles are generated by $C^*(L)\varepsilon_t$.

By definition, the cointegrating matrix α is the $(N \times r)$ matrix of all independent cointegrating vectors. Then, $\alpha' y_t$ does not present stochastic trends. Thus, premultiplying equation (6) by α' one obtains

$$\alpha' y_t = \alpha'\mu + \alpha' C^*(L)\varepsilon_t.$$

Since the cofeature matrix $\widetilde{\alpha}$ has the property of cancelling the serial correlation of Δy_t, and removing the cyclical component of y_t, premultiplying equation (6) by $\widetilde{\alpha}'$ yields

$$\widetilde{\alpha}' y_t = \widetilde{\alpha}'\mu + \widetilde{\alpha}' C(1)\sum_{i=1}^{i=t}\varepsilon_i.$$

Let us stack $\widetilde{\alpha}'$ and α' as follows

$$\begin{bmatrix} \widetilde{\alpha}' \\ \alpha' \end{bmatrix} y_t = \begin{bmatrix} \widetilde{\alpha}'\mu + \widetilde{\alpha}' C(1)\sum_{i=1}^{i=t}\varepsilon_i \\ \alpha'\mu + \alpha' C^*(L)\varepsilon_t \end{bmatrix} \tag{7}$$

and define the $(N \times N)$ matrix $\mathbf{A}$ to be

$$\mathbf{A} = \begin{bmatrix} \widetilde{\alpha}' \\ \alpha' \end{bmatrix}.$$

Since $N = r + s$, matrix $\mathbf{A}$ has full rank and $\mathbf{A}^{-1}$ can be expressed as $\mathbf{A}^{-1} = \begin{bmatrix} \widetilde{a} & a \end{bmatrix}$ where $\widetilde{a}$ is an $(N \times s)$ matrix, and a is $(N \times r)$. Premultiplying equation (7) by $\mathbf{A}^{-1}$ yields

$$\begin{aligned} y_t &= \widetilde{a}\widetilde{\alpha}' y_t + a\alpha' y_t \\ &= \widetilde{a}\widetilde{\alpha}'\mu + \widetilde{a}\widetilde{\alpha}' C(1)\sum_{i=1}^{i=t}\varepsilon_i + a\alpha'\mu + a\alpha' C^*(L)\varepsilon_t. \end{aligned} \tag{8}$$

Equation (8) explicitly contains a trend-cycle decomposition of y_t as equation (4) requires. It is usual to have a cyclical component free of deterministic elements and to assign $a\alpha'\mu$ to the trend part. The terms of the right hand side of equation (4) are defined as

$$y_t^p = \widetilde{a}\widetilde{\alpha}' C(1)\sum_{i=1}^{i=t}\varepsilon_i + a\alpha'\mu + \widetilde{a}\widetilde{\alpha}'\mu \tag{9}$$

$$y_t^c = a\alpha' C^*(L)\varepsilon_t \tag{10}$$

where y_t^p contains the stochastic trends and all the deterministic components, and y_t^c contains a non deterministic cycle. Alternatively, the definition of y_t^p and y_t^c is

$$y_t^p = \tilde{a}\tilde{\alpha}' y_t + a\alpha'\mu \tag{11}$$

$$y_t^c = a\alpha' y_t - a\alpha'\mu. \tag{12}$$

The trend-cycle decomposition of equations (9)-(10) or equations (11)-(12) is unique. Linear transformations in the cointegrating space do not change the estimated trend and cycles. The cycle part (y_t^c) is a linear combination of the error correction terms ($\alpha' y_t$) which can be viewed as cycle generators. Since $C(1)$ and $C^*(L)$ have reduced ranks and $N = r + s$, it is not needed to impose any additional condition to obtain this trend-cycle decomposition.

4 Empirical results

This section applies the previously described methodology to sectoral per capita Belgian real GDP. Data consist of yearly (log) sectoral real GDP divided by the total population. Per capita real GDP is available from 1953 to 1991. The GDP and population data are provided by the IRES (UCL) data bank. Real GDP is expressed in millions of Belgian Francs (constant prices of 1985). Sectors are a sub division of GDP as follows:

1. Agriculture, Silviculture and Fisheries (AGR)
2. Mining (EXT)
3. Manufacturing (MAN)
4. Construction (CON)
5. Electricity, gas and water (ELE)
6. Wholesale and Retail Trade, Finance, Insurance and Residential Buildings (TRA)
7. Transportation and Communications (COM)
8. Services (SER)

Plots of the data in levels are presented in Figures 1 to 8 (cf. Appendix). Except Mining, all the series show an upward trend. Until the 1950s important subsidies from the Belgian government and the European Steel and Coal Community kept coal prices and production at high levels. In November 1959, fiscal and foreign pressures led the industry to run down.

Figure 9 (cf. Appendix) shows that the series are trending at different rates, especially the Electricity sector, which grows at the highest rate. This figure shows the different intersectoral dynamics after the Second World War and the "Belgian Miracle" period of rapid reconstruction[4]. Some sectors have increased their share in the GDP: Electricity, Manufacturing, Construction, Trade, Transportation and Services. Others, have reduced their shares: Agriculture, Construction, and, obviously, Mining.

4.1 Integration and cointegration results

Integration tests were performed on the data (in logs) using the Dickey-Fuller (DF) and the Augmented Dickey-Fuller (ADF) tests[5]. Optimal lags were chosen using the criteria proposed by Campbell and Perron [1991] starting with a maximum lag equal to two.

For all series in levels the null hypothesis of a unit root could not be rejected neither in models with a deterministic trend and a constant nor in models where only a constant was included. Models with and without constant were considered to analyse series in differences. In the model with a constant, series in differences reject the null hypothesis of a unit root at the 5% significance level, except from the Electricity sector that does it only at the 1%. Therefore, these data are well approximated by I(1) processes. The same results are obtained by the non parametric Phillips-Perron integration test[6].

To perform cointegration tests the Johansen [1988] technique was used. Using information criteria a VECM of order two in levels ($k = 2$) with a constant and without linear trend in the model in differences was obtained. The VECM is specified as follows

$$\Delta y_t = \beta\alpha' y_{t-1} + B_1 \Delta y_{t-1} + \mu + \varepsilon_t, \tag{13}$$

where y_t is an 8-column vector ($N = 8$). The residual correlograms do not show any significative residual serial correlation. Residuals could be considered as white-noise process. Normality is not rejected by the Doornik and Hansen [1994] test.

Results of the cointegration test are reported in Table 1. At the 5% significance level, the trace test does not reject the null hypothesis that the cointegrating rank is four ($r = 4$) which is the hypothesis kept. As there are four cointegrating vectors, the sectoral per capita

[4] Cassiers, De Villé and Solar [1993].

[5] Dickey and Fuller [1981].

[6] Phillips and Perron [1988].

real GDP share four common stochastic trends ($N - r = 4$). Looking at Figure 9 one can observe four groups of series with similar trends: 1) Manufacturing, Trade, and Services; 2) Transportation, Agriculture and Construction; 3)Electricity; and 4) Mining.

Table 1: Cointegrant results – Trace tests

Null			Critical value		
Hypothesis (r)	Constant	Statistic	90%	95%	97.5%
0	restricted	250.08	159.48	165.58	171.28
0	unrestricted	219.01	150.53	156.00	161.32
1	restricted	184.97	126.58	131.70	136.49
1	unrestricted	157.10	118.50	124.24	128.45
2	restricted	129.65	97.18	102.14	106.74
2	unrestricted	108.61	89.48	94.15	98.33
3	restricted	97.38	71.86	76.07	80.06
3	unrestricted	76.37	64.84	68.52	71.80
4	restricted	66.30	49.65	53.12	56.06
4	unrestricted	45.70	43.95	47.21	50.35
5	restricted	37.35	32.00	34.91	37.61
5	unrestricted	25.66	26.79	29.68	32.56
6	restricted	18.48	17.85	19.96	22.05
6	unrestricted	8.24	13.33	15.41	17.52
7	restricted	7.35	7.52	9.24	10.80
7	unrestricted	0.54	2.69	3.76	4.95

The hypothesis is accepted when the calculated value is smaller than the tabulated one.
Critical values are taken in tables 1* and 1 in Osterward-Lenum [1992].

4.2 Canonical correlation results

In order to infer the cofeature rank, a canonical correlation analysis was performed using the right hand side variables in the VECM as the conditioning set. As $r = 4$, α and β are (8×4) matrices. Two sets of variables are needed to calculate the canonical correlations: the LHS variables of equation (13)

$$\Delta y_t' = (\Delta y_{1t}, \Delta y_{2t}, \ldots, \Delta y_{8t})'$$

and the conditioning variables in the right hand side of equation (13)

$$Z_t' = \left(\Delta y_{t-1}', (\alpha' y_{t-1})', 1\right)'.$$

The results of the χ^2-test for zero canonical correlation considering these two sets are given in Table 2. According to the p-values of the sequential χ^2-tests, the hypothesis that the smallest four are jointly zero cannot be rejected even at the 10% significance level. The hypothesis that the four smallest canonical correlations are zero is kept. Each statistically zero canonical correlation represents a linear combination of Δy_t uncorrelated with all linear combinations of Z_t. As four smallest canonical correlations are zero there are four cofeatures vectors and the cofeature rank is four ($s = 4$). The variables in the system share four common cycles ($N - s = 4$).

Table 2: Canonical correlation analysis – Common cycles test

Roots through 8	Chi-square Statistic	*p*-value
1	233.48	0.00
2	154.72	0.00
3	108.17	0.00
4	73.18	0.01
5	46.59	0.11
6	23.82	0.47
7	11.30	0.66
8	3.18	0.78

4.3 Trend-cycle decomposition results

Since four cointegrating vectors and four cofeature vectors are observed, the eight sectors share respectively four independent stochastic trends and four common cycles. This verifies the equality $N = r + s$, and allows to find the special trend-cycle decomposition discussed in Section 3. As the matrix A is full rank, it is possible to calculate A^{-1}, where a and $\tilde{\mathrm{a}}$ are (8×4) matrices. For each sector, cycles and trends are calculated using equations (11) and (12).

Table 3 shows a summary of the statistics of sectoral outputs, cycles and trends. Cyclical components have low standard errors and, by construction, zero means. Trend standard errors are smaller than the standard errors of the series in levels. However, it cannot be said that the trend components are significantly smoother than the series in levels.

Figures 1 to 8 show each sectoral output and their respective trend. One can clearly see that trends display a very different behaviour across sectors. This result is not unexpected since the eight sectors share four common independent trends. Sectors like Manufacturing, Trade and Services seem to show trends with similar shape. The per capita Mining trend is obviously very particular, since both series and trend decrease.

Table 3: Summary statistics of sectoral GDP, cycles and trends

SECTOR	LEVELS			CYCLES		TRENDS		
	Mean	Std Dev	Mean/St Dev	Mean	Std Dev	Mean	Std Dev	Mean/St Dev
AGR	-4.63	0.11	-42.49	0.00	0.02	-4.63	0.12	-40.29
EXT	-5.53	0.49	-11.19	0.00	0.01	-5.53	0.49	-11.21
MAN	-2.62	0.44	-5.89	0.00	0.06	-2.62	0.44	-5.97
CON	-3.65	0.21	-17.05	0.00	0.08	-3.65	0.19	-18.73
ELE	-4.87	0.81	-5.98	0.00	0.05	-4.87	0.81	-6.01
COM	-2.22	0.37	-6.06	0.00	0.04	-2.22	0.37	-6.09
TRA	-3.51	0.29	-12.18	0.00	0.07	-3.51	0.28	-12.46
SER	-2.33	0.38	-6.15	0.00	0.04	-2.33	0.37	-6.22

As equation (12) shows, the estimated cycles are a linear combination of the error correction terms. Cycles can be determined calculating $Z'_{it} = \alpha'_i y_{t-1}$ $(i = 1, \ldots, 4)$ where α_i $(i = 1, \ldots, 4)$ are the 4 cointegrating vectors. Figures 10 to 13 (cf. Appendix) show Z_1, Z_2, Z_3 and Z_4. They present a typical cyclical behaviour because of the fluctuations. Their shapes are studied in the next sub section.

4.4 Pro and counter cyclical results

To determine pro cyclical and counter cyclical sectors one needs to characterise recession periods in the Belgian economy. For the purpose of this paper, it is only necessary to know the recession periods and to compare them to the sectoral cyclical components. The following recession years can be identified[7]: 1953, 1958, 1962, 1968, 1975, 1981 and 1986.

To determine pro and counter cyclical sectors the graphs of the cyclical components of sectoral GDP were compared with the recession periods (Figures 14 to 17). Except Mining, all the GDP sectors behave in accordance with the recessions years. Thus, these seven sectors could be defined as pro cyclicals. They are similar in shape but plotted separately only for differences in amplitude and scale. Construction shows the highest amplitude. The duration of these cycles is also similar across sectors and the troughs correspond to recession years. However, the duration of the cycles is not the same for each sector.

Mining is the only sector which behaves counter cyclically. To understand this evolution remember that international shocks were caused by the energy crisis. After the increase of the oil price it was

[7] Savage [1991] and Cassiers, De Villé and Solar [1993].

expected that the production of commodity substitutes (*i.e.* coal) would increase. Belgium is a small open economy, therefore, the international context plays a very important role in output evolution.

Comparing the four common cycles with the output cycles (Figures 10 to 13) one can observe that their reactions after recession years are totally different. Statistically, there are two counter cyclical common cycles and two pro cyclical ones. Nevertheless, equation (12) and Table 4[(8)] show that matrix a neutralises the counter cyclical effect of the Error Correction Term 3 (ECT 3), except for the Mining sector since the second element of the column in bold in Table 4 is the only one with positive sign, for the chosen bases for the cointegrating and cofeature spaces. The ECT 3, the most important of the error correction terms, plays in fact a pro cyclical role and it could be seen as an upside down version of Figure 14. Thus, the ECT 1 is the only common cycle which plays purely counter cyclically, and it behaves like the Mining sector does.

Table 4
The inverse matrix ($A^{-1} = [\tilde{a}\, a]$) of the cofeature ($\tilde{a}$) and the cointegrating (a) vectors

Sector	$\tilde{a}$				a			
AGR	0.042	0.109	-0.423	-0.179	0.054	0.219	**-0.053**	0.081
EXT	-0.201	0.137	0.109	0.054	0.090	0.113	**0.023**	-0.382
MAN	0.107	-0.006	0.408	-0.402	-0.099	-0.000	**-0.142**	0.775
CON	0.066	-0.209	0.384	0.219	0.176	-0.137	**-0.191**	0.627
ELE	0.156	0.103	0.608	0.287	0.307	-0.807	**-0.085**	1.386
COM	0.105	-0.056	0.284	-0.209	0.116	0.320	**-0.093**	1.004
TRA	0.104	-0.140	0.316	-0.495	0.071	-0.642	**-0.164**	0.958
SER	0.101	0.005	0.140	0.014	-0.013	-0.217	**-0.072**	1.388

4.5 Variance decomposition

To examine the relative importance of sectoral trends and cycles a variance decomposition of the total innovations of each GDP sector was performed. Trend innovations are defined as first differences of the sectors' trend. Since a cofeature matrix has the property of cancelling the serial correlation of Δy_t, $\tilde{\alpha}'\Delta y_t$ is white noise, $\tilde{\alpha}' y_t$ is random walk and the linear combination $\tilde{a}\tilde{\alpha}' y_t$ of equation (11) is also random walk. Therefore, the trend component for each sector (y_t^P) is a random walk plus a constant, and its innovation can be found by first differencing.

(8) Table 4 shows $A^{-1} = [\tilde{a}\, a]$.

Cyclical innovations are the unforecastable part of the series' cycle. Equation (12) defines y_t^c as a linear combination of the error correction terms. One possible way of finding sectoral innovations is to regress cycles on the variables of the right hand side of equation (13) and take the residuals.

Table 5: Variance decomposition of sectoral output innovations

	% of variance of sectoral output innovation attributed to			
Sectors	**Trend innovation**	**Cycle innovation**	**Covariance effect (*)**	**Sum**
AGR	78.5	6.3	15.3	100.0
EXT	106.7	1.5	-8.2	100.0
MAN	132.5	69.0	-101.5	100.0
CON	135.8	86.8	-122.5	100.0
ELE	136.2	36.7	-72.9	100.0
COM	139.3	68.9	-108.1	100.0
TRA	111.1	68.6	-79.7	100.0
SER	104.2	59.7	-63.9	100.0

Table 5 shows the results of the variance decomposition of sectoral innovations. The correlation coefficients between trend and cycle innovations were always significant at the 5% significance level. The covariance effect is negative for all sectors and it is more important in Construction. The size of the negative covariance between innovations is not trivial. A positive shock to the trend has two different effects: it sends the permanent component up, increasing the long run value of output, and it also sends the cycle down. The variance percentage of the sectoral output innovation attributed to trend innovation is always the highest. However, to better analyse the relative importance of trend and cyclical innovations one needs to orthogonalise innovations[9]. The results may depend on the order of the innovations in the orthogonalisation procedure. Table 6 shows that trend innovations have a prominent role across sectors, explaining the 52-99% of the total innovation. This is an expected result in the Real Business Cycle theory, since the trend components are viewed as the cumulative effect of permanent shocks and they explain the most important part of series' evolution.

[9] Engle and Issler [1992].

Table 6
Variance decomposition of sectoral output innovations orthogonalizing the covariance matrix

	ORDER					
	First the trend and second the cycle			First the cycle and second the trend		
	% of variance of sectoral output innovation attributed to			% of variance of sectoral output innovation attributed to		
Sectors	Trend innovation	Cycle innovation	Sum	Cycle innovation	Trend innovation	Sum
AGR	93.4	6.6	100.0	8.3	91.7	100.0
EXT	98.8	1.2	100.0	1.5	98.5	100.0
MAN	72.8	27.2	100.0	42.0	58.0	100.0
CON	69.7	30.3	100.0	48.4	51.6	100.0
ELE	83.5	16.5	100.0	26.8	73.2	100.0
COM	74.4	25.6	100.0	41.6	58.4	100.0
TRA	67.2	32.8	100.0	43.8	56.2	100.0
SER	67.6	32.4	100.0	40.6	59.4	100.0

5 Conclusions

The main results of this work can be summarised in the following remarks. It has been found that four independent common trends and four independent common cycles characterise the sectoral Belgian GDP evolution in the period 1953–1991. As the Real Business Cycles theory predicts, trend innovations explain most of the output innovations, but they are very different across sectors. The special trend-cycle decomposition used in this paper allows to obtain a classification of pro and counter cyclical sectors.

To better evaluate the usefulness of this applied technique, one can compare it with others. There are many ways to decompose output series into a trend and a cycle. In this study, the applied technique allows to integrate transitory and permanent shocks. Moreover, the empirical results also show that trend and cycle innovations are correlated. In this case, a temporary shock has also permanent effects. It is not possible to dissociate transitory from permanent shocks and transitory from permanent policies. Some macroeconometricians(10) interpret the disturbances that have a temporary effect on output as being mostly demand disturbances, and those that have a permanent effect on output,

(10) Blanchard and Quah [1989].

as mostly supply disturbances. Using the decomposition presented in this paper, this separation is meaningless.

A more complete evaluation of the used technique would require to apply it to other countries. The main restrictions are the hypothesis of I(1) cointegrated series, and the condition that the cointegrating rank and the cofeature rank add up to the number of variables. Obviously, the number of sectors, countries or regions that satisfies these hypotheses may be limited.

APPENDIX

Figures

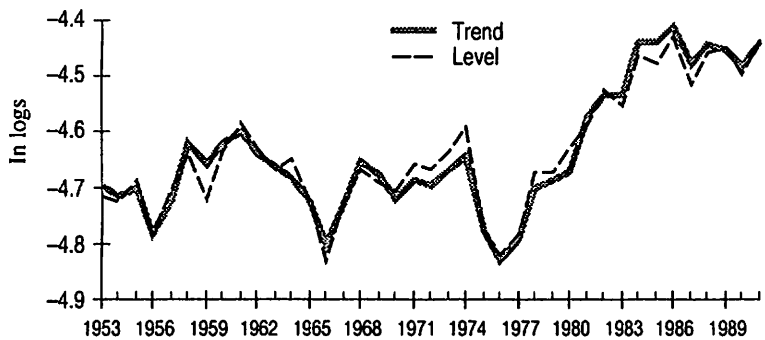

Figure 1: Agricultural GDP and its trends

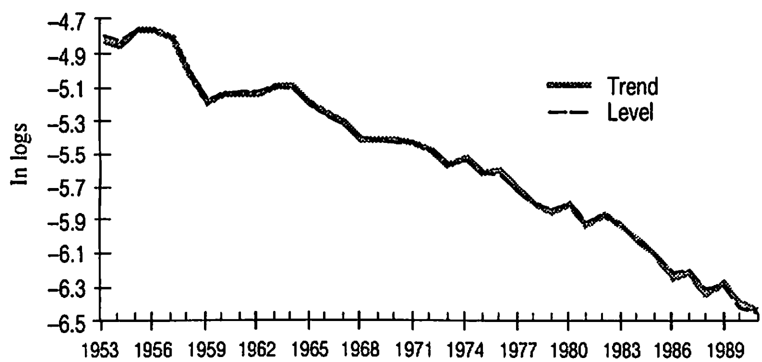

Figure 2: Mining GDP and its trends

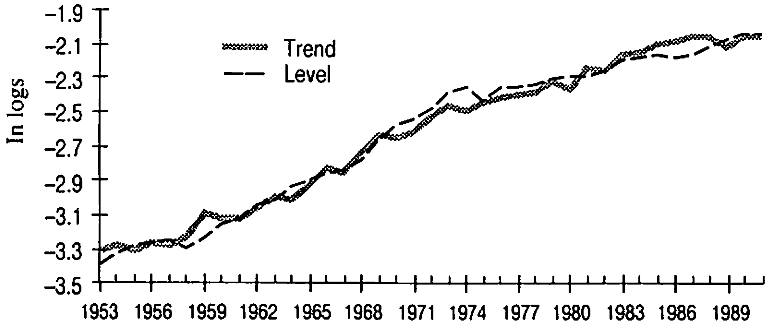

Figure 3: Manufacturing GDP and its trends

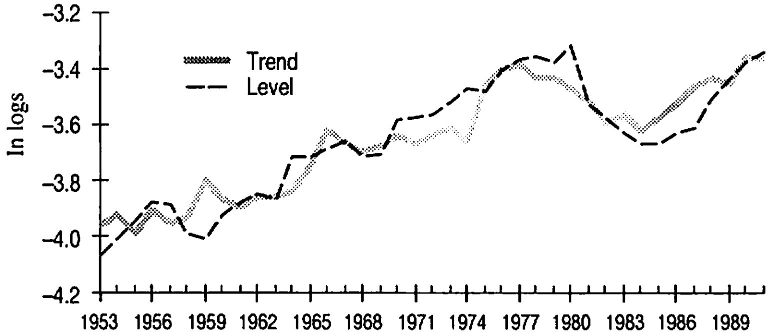

Figure 4: Construction GDP and its trends

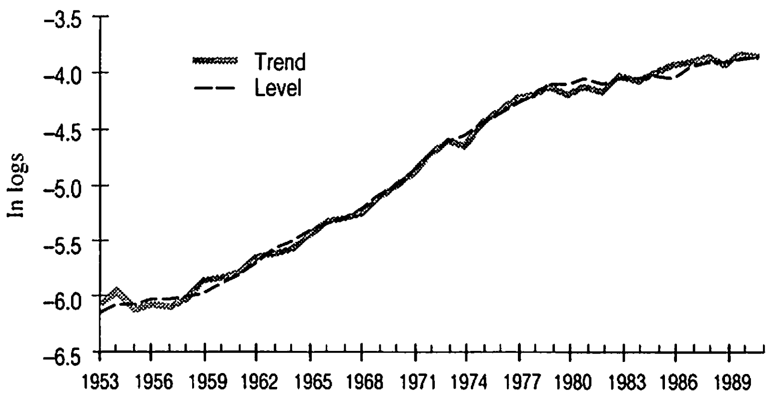

Figure 5: Electricity GDP and its trends

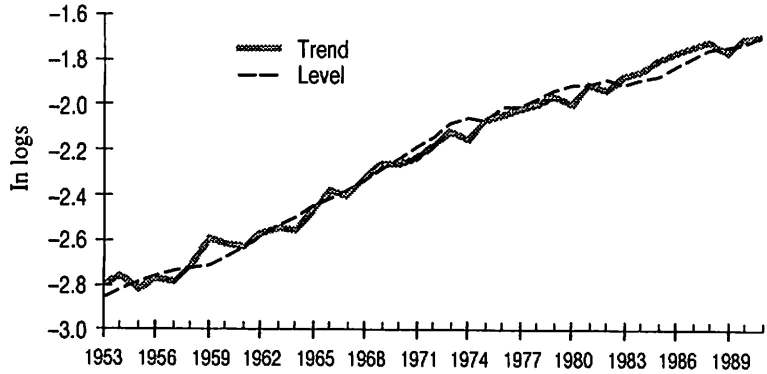

Figure 6: Trade GDP and its trends

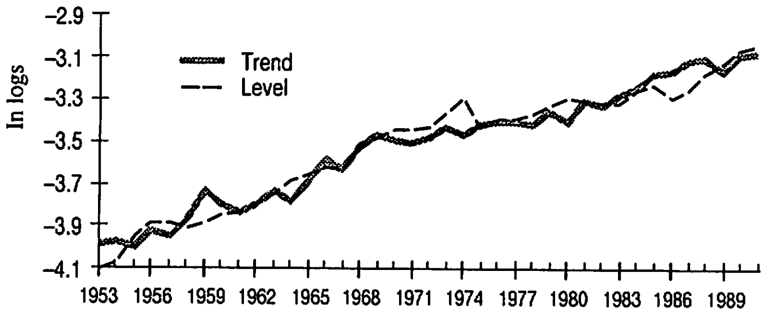

Figure 7: Transportation GDP and its trends

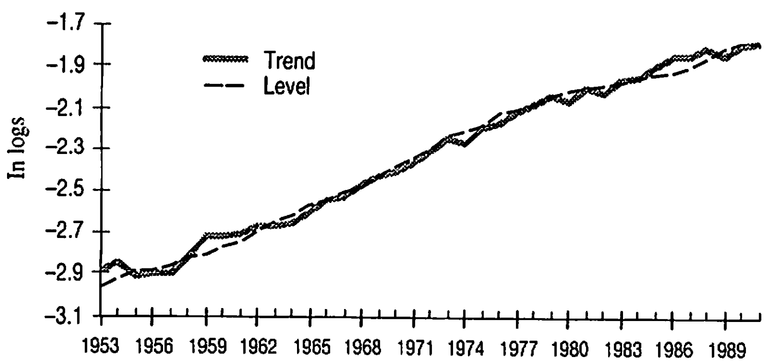

Figure 8: Services GDP and its trends

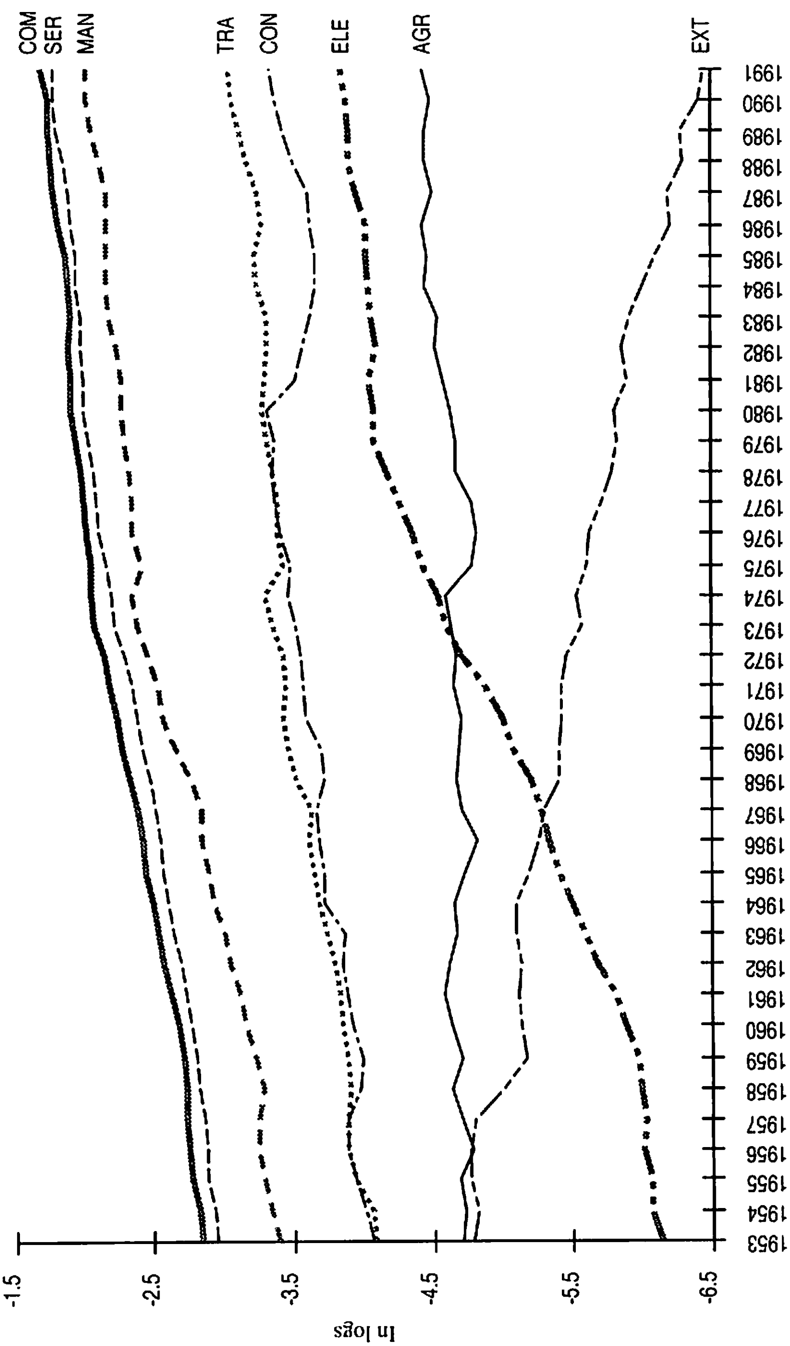

Figure 9: Sectoral per capita GDP

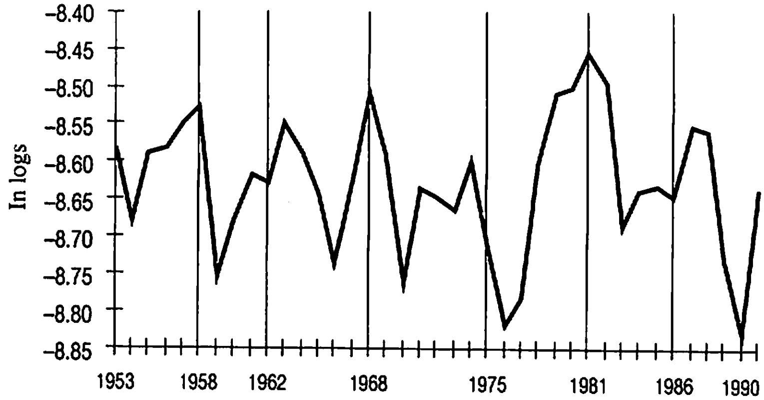

Figure 10: Error correction Term 1

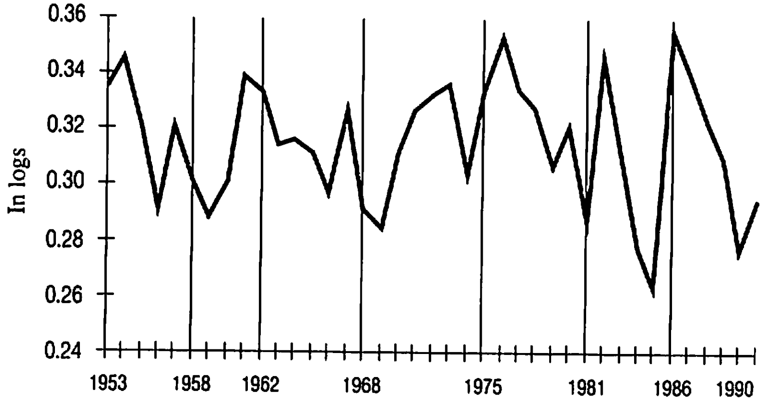

Figure 11: Error correction Term 2

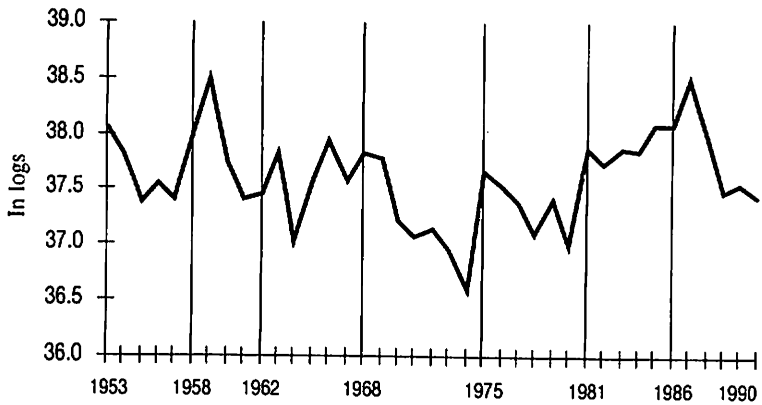

Figure 12: Error correction Term 3

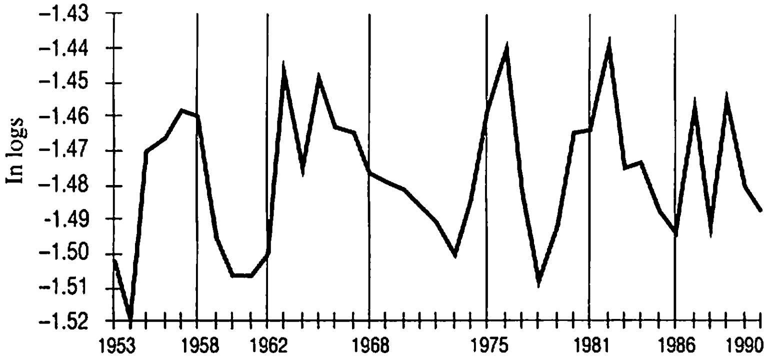

Figure 13: Error correction Term 4

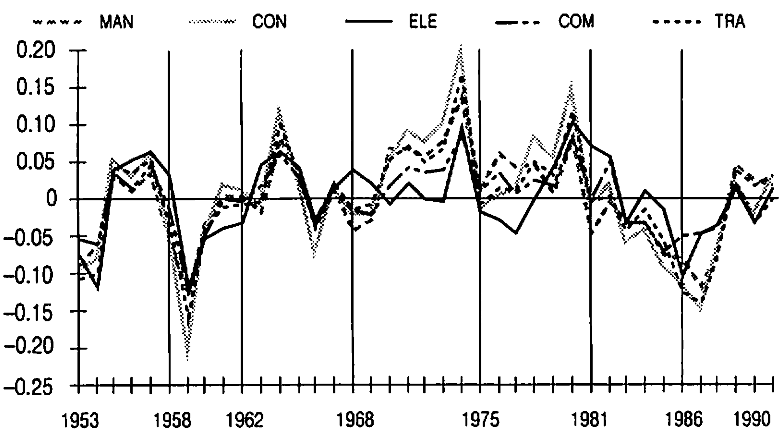

Figure 14: Manufacturing, Trade, Electricity and Transportation cycles

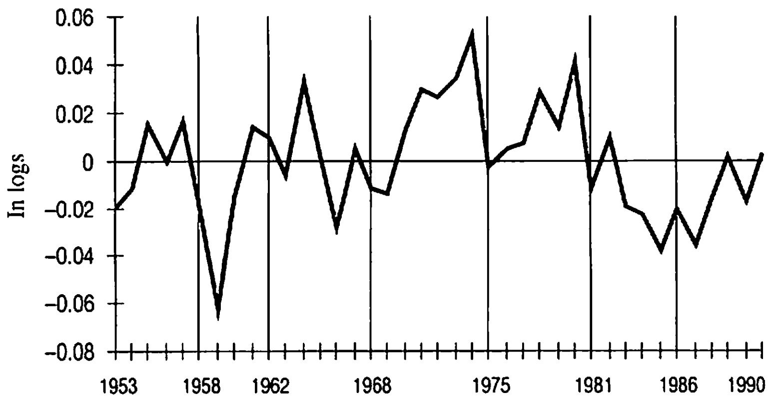

Figure 15: Per capita agricultural GDP cycle

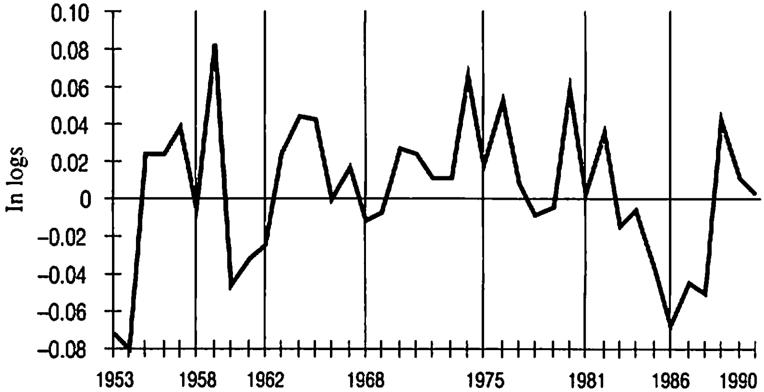

Figure 16: Per capita service GDP cycle

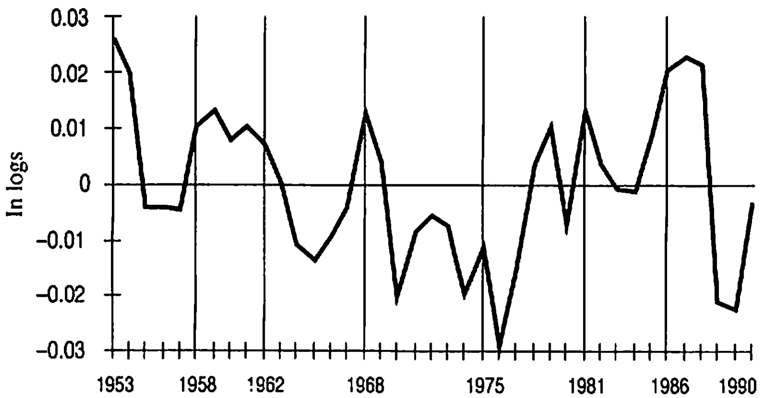

Figure 17: Per capita mining GDP cycle

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